

Evading the IR Problem of Thermal QCD

York Schröder

(Uni Bielefeld)

work with: K. Kajantie, M. Laine, K. Rummukainen, A. Vuorinen,
A. Hietanen, F. Di Renzo, V. Miccio

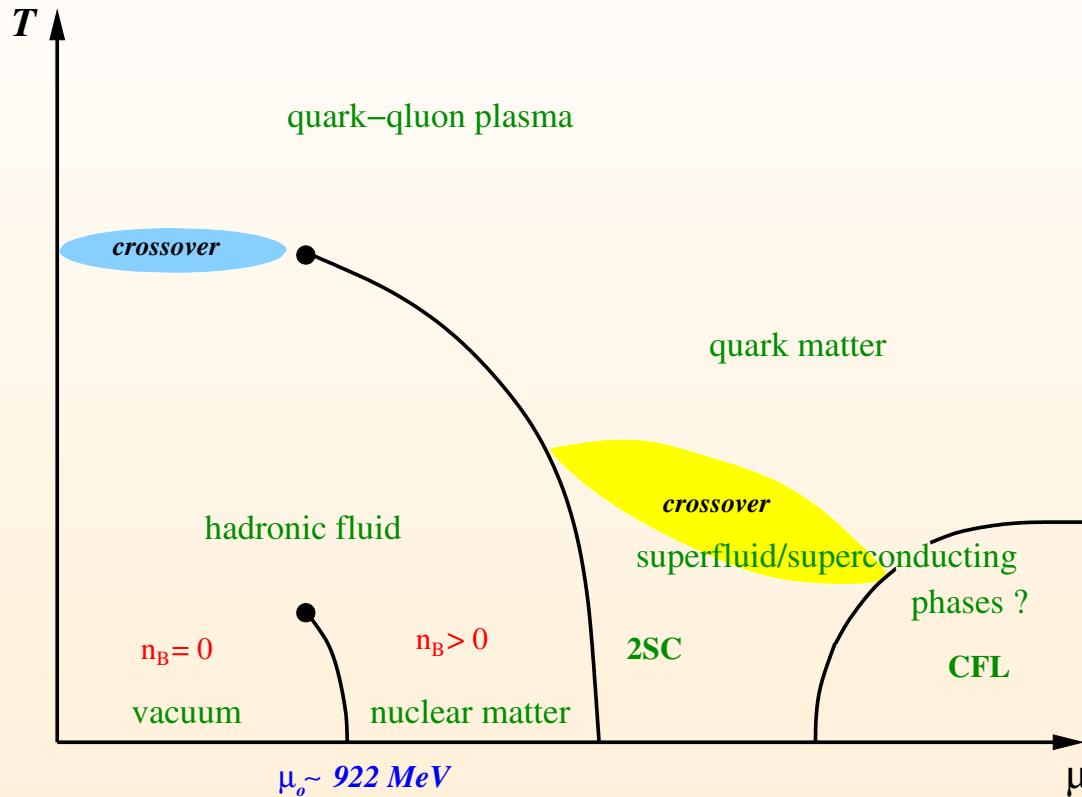
Karlsruhe, 03 Dec 2004

The QCD (equilibrium) phase diagram

$$\mathcal{L}(\psi, A_\mu) \rightarrow p, n$$

squeeze (μ_B) / heat (T) \rightarrow simpler?

The QCD (equilibrium) phase diagram



nature: early univ, μ tiny ($\sim \frac{\#baryons}{entropy}$), $T_c \sim 170 \text{ MeV} \sim 10 \mu\text{s}$
neutron/quark stars

lab expt.: SPS / RHIC $\mu_B \sim \frac{\#baryons}{pions} \sim 45 \text{ MeV}$ / LHC / GSI

Outline

The IR problem

QCD pressure: lattice

QCD pressure: perturbative

Methods

- Diagram generation
- reduction
- integration
- lattice MC
- lattice: perturbative

Conclusions

The IR problem

from now on: high T , $\mu_B \equiv 0$

⇒ asymptotic freedom!

$$g(Q \sim T) \stackrel{\text{ren}}{\propto} \frac{1}{\ln \frac{T}{\Lambda_{\overline{\text{MS}}}}} \stackrel{T \rightarrow \infty}{\rightarrow} 0$$

⇒ QGP = ideal gas of quarks/gluons?
+ perturbative corrections?

$$T > 0: \int d^4 p \rightarrow T \sum_{p_0} \int d^3 p$$

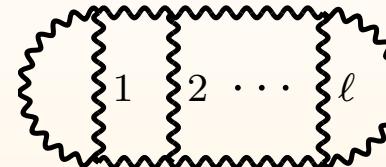
Matsubara frequencies: $p_0 = 2\pi n T$ (gluons, ghosts)
 $p_0 = 2\pi(n + 1/2)T$ (fermions)

⇒ IR divergences from bosons, $n = 0$, 'zero mode'

The IR problem

[Linde 1979; Gross/Pisarski/Yaffe 1981]

$(\ell+1)$ loops, 2ℓ vert, 3ℓ propag



$$\sim \left(T \sum_n \int d^3 p \right)^{\ell+1} \frac{(gp)^{2\ell}}{[(2\pi nT)^2 + p^2 + \Pi(2\pi nT, p)]^{3\ell}}$$

IR power counting: $n=0$, define $\Pi(0, p \rightarrow 0) \equiv m^2$

$$\sim T^{\ell+1} g^{2\ell} m^{3(\ell+1)+2\ell-6\ell} = g^6 T^4 \left(\frac{g^2 T}{m} \right)^{\ell-3}$$

- $\Pi_L(0, p) \sim (gT)^2 \Leftarrow$ OK: get series in g
- $\Pi_T(0, p) \sim (g^2 T)^2 \Leftarrow$ all orders important!

Evading the IR problem

what now?

- resummations? → not known
- change language!
 - 3 physical scales: $2\pi T$, gT , $g^2 T$ [think pure gauge for now]
 - ⇒ effective theories
 - 3 contributions [Braaten/Nieto 1995]
 - 3rd is long distance → non-pert → LAT

why not LAT from the outset?

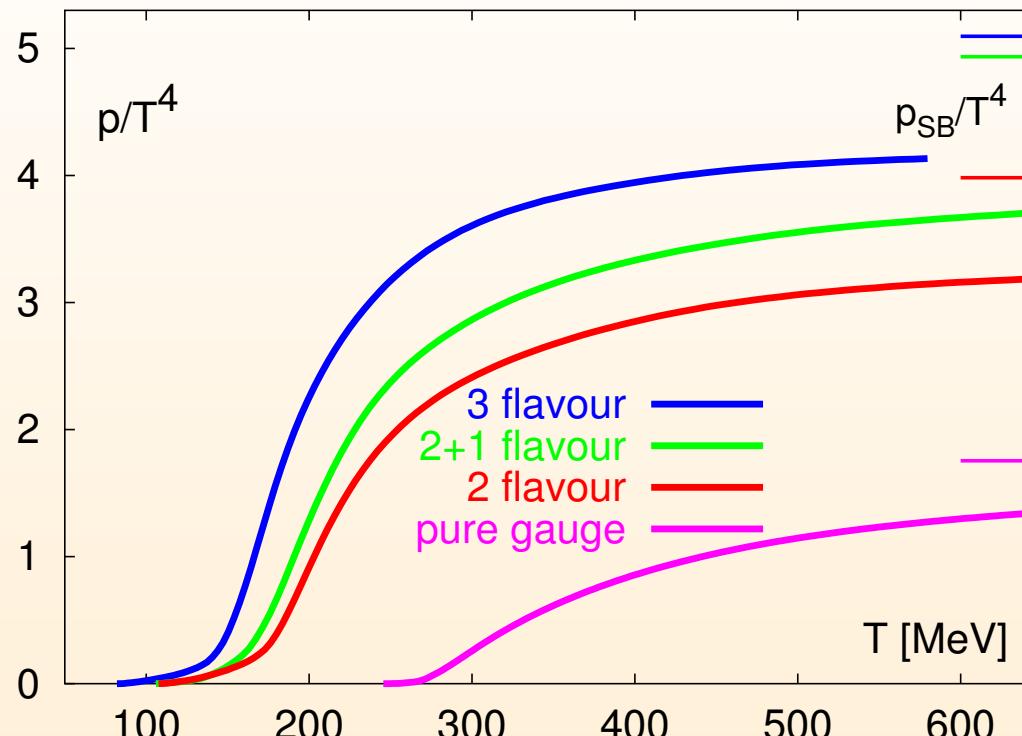
- original problem: 4d QCD, LAT
 - ▷ N_f “hard”
 - ▷ μ_B “exp. hard”
- effective theories: 4d QCD, pert
 - +3d SU(3) + adj. Higgs, pert
 - +3d SU(3), LAT
 - ▷ 3d bosonic theory! N_f OK, μ_B OK.
 - ▷ superrenormalizable!

can I see an example?

- yes - QCD pressure, $p \stackrel{V \rightarrow \infty}{=} -f$

$$\exp\left[-f(T, g)\frac{V}{T}\right] = \int \mathcal{D}[A\bar{\psi}\psi] \exp\left[-\int_0^{1/T} d\tau \int d^3x \mathcal{L}_E[A\bar{\psi}\psi]\right]$$

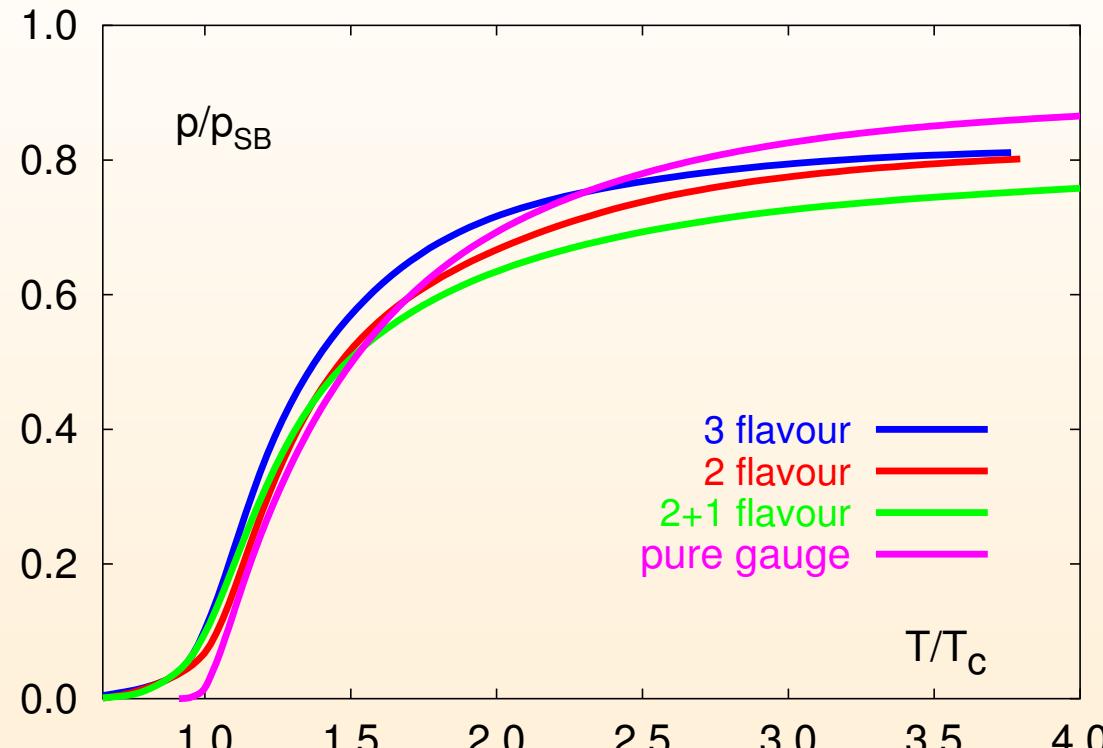
$p(T)$ on the lattice ($\mu_B = 0$)



[from Karsch et.al.]

at $T \rightarrow \infty$, expect ideal gas: $p_{SB} = \left(16 + \frac{21}{2}N_f\right) \frac{\pi^2 T^4}{90}$

$p(T)$ on the lattice ($\mu_B = 0$)



[from Karsch et.al.]

at $T \rightarrow \infty$, expect ideal gas: $p_{\text{SB}} = \left(16 + \frac{21}{2}N_f\right) \frac{\pi^2 T^4}{90}$

$p(T)$ in (thermal) perturbation theory

want to compute the QCD pressure

$$\begin{aligned} p_{\text{QCD}}(T) &\equiv \lim_{V \rightarrow \infty} \frac{T}{V} \ln \int \mathcal{D}A_\mu^a \mathcal{D}\psi \mathcal{D}\bar{\psi} \exp\left(-\frac{1}{\hbar}S_{\text{QCD}}\right) \\ S_{\text{QCD}} &= \int_0^{\beta\hbar} d\tau \int d^d x \mathcal{L}_{\text{QCD}} \\ \mathcal{L}_{\text{QCD}} &= \frac{1}{4} F_{\mu\nu}^a F_{\mu\nu}^a + \bar{\psi} \gamma_\mu D_\mu \psi + \mathcal{L}_{\text{GF}} + \mathcal{L}_{\text{FP}} \end{aligned}$$

$p_{\text{QCD}}(T)$ renormalised such that it vanishes at $T = 0$.

$p(T)$ in (thermal) perturbation theory

integrate out $|p| \gtrsim 2\pi T$: $\psi, A_\mu (n \neq 0)$

$$\begin{aligned} p_{\text{QCD}}(T) &\equiv p_{\mathbb{E}}(T) + \frac{T}{V} \ln \int \mathcal{D}A_k^a \mathcal{D}A_0^a \exp \left(- \int d^d x \mathcal{L}_{\mathbb{E}} \right) \\ \mathcal{L}_{\mathbb{E}} &= \frac{1}{2} \text{Tr } F_{kl}^2 + \text{Tr } [D_k, A_0]^2 + m_{\mathbb{E}}^2 \text{Tr } A_0^2 + \lambda_{\mathbb{E}}^{(1)} (\text{Tr } A_0^2)^2 + \lambda_{\mathbb{E}}^{(2)} \text{Tr } A_0^4 + \dots \end{aligned}$$

five matching coefficients $p_{\mathbb{E}} \sim T^4, m_{\mathbb{E}}^2 \sim g^2 T^2, g_{\mathbb{E}}^2 \sim g^2 T, \lambda_{\mathbb{E}}^{(1/2)} \sim g^4 T$.

higher order operators

$$\delta \mathcal{L}_{\mathbb{E}} \sim g^2 \frac{D_k D_l}{(2\pi T)^2} \mathcal{L}_{\mathbb{E}}$$

could contribute (most conservative estimate)

$$\frac{\delta p_{\text{QCD}}(T)}{T} \sim \delta \mathcal{L}_{\mathbb{E}} \sim g^2 \frac{(gT)^2}{(2\pi T)^2} (gT)^3 \sim g^7 T^3$$

$p(T)$ in (thermal) perturbation theory

integrate out $|p| \gtrsim gT$: A_0

$$\begin{aligned} \frac{T}{V} \ln \int \mathcal{D}A_k^a \mathcal{D}A_0^a \exp(-S_{\mathbb{E}}) &\equiv p_{\mathbb{M}}(T) + \frac{T}{V} \ln \int \mathcal{D}A_k^a \exp\left(-\int d^d x \mathcal{L}_{\mathbb{M}}\right) \\ \mathcal{L}_{\mathbb{M}} &= \frac{1}{2} \text{Tr } F_{kl}^2 + \dots \end{aligned}$$

two matching coefficients $p_{\mathbb{M}} \sim m_{\mathbb{E}}^3 T$, $g_{\mathbb{M}}^2 \sim g_{\mathbb{E}}^2$.

higher order operators

$$\delta \mathcal{L}_{\mathbb{M}} \sim g_{\mathbb{E}}^2 \frac{D_k D_l}{m_{\mathbb{E}}^3} \mathcal{L}_{\mathbb{M}} \stackrel{\text{e.g.}}{\sim} \frac{g_{\mathbb{E}}^2}{T^3} \text{Tr}[D_i, F_{i,j}]^2$$

could contribute

$$\frac{\delta p_{\text{QCD}}(T)}{T} \sim \delta \mathcal{L}_{\mathbb{M}} \sim g_{\mathbb{E}}^2 \frac{(g^2 T)^2}{m_{\mathbb{E}}^3} (g^2 T)^3 \sim g^9 T^3$$

$p(T)$ in (thermal) perturbation theory

\mathcal{L}_M only has one (dimensionful) parameter

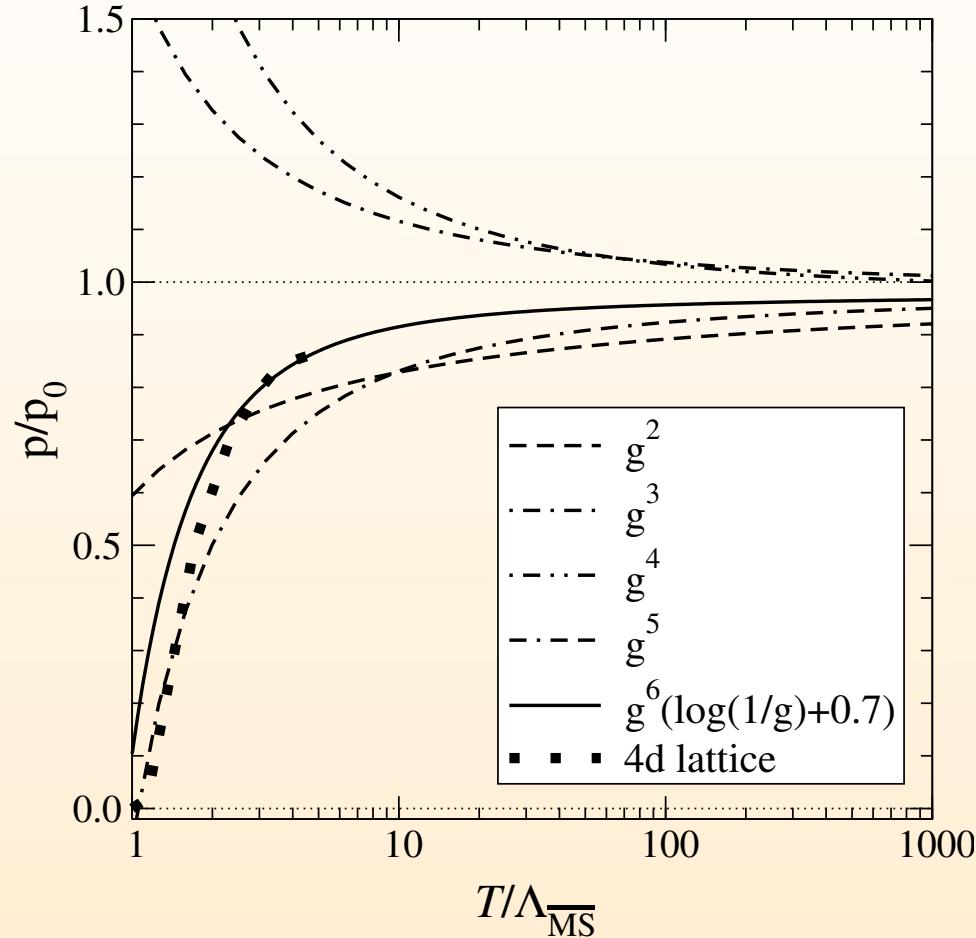
$$p_G(T) \equiv \frac{T}{V} \ln \int \mathcal{D}A_k^a \exp(-S_M) \sim T g_M^6$$

coefficient is **non-perturbative!**

$$\begin{aligned} \frac{p_{QCD}(T)}{p_{SB}} &= \frac{p_E(T)}{p_{SB}} + \frac{p_M(T)}{p_{SB}} + \frac{p_G(T)}{p_{SB}} \\ &= 1 + g^2 + g^4 + g^6 + \dots && \Leftarrow 4d \text{ QCD} \\ &\quad + g^3 + g^4 + g^5 + g^6 + \dots && \Leftarrow 3d \text{ adj H} \\ &\quad + \frac{1}{p_{SB}} \frac{T}{V} \int \mathcal{D}[A_i] \exp\left[-\int d^3x \mathcal{L}_M\right] && \Leftarrow 3d \text{ YM} \\ &= c_0 + c_2 g^2 + c_3 g^3 + (c'_4 \ln g + c_4) g^4 + c_5 g^5 + (c'_6 \ln g + c_6) g^6 + \mathcal{O}(g^7) \end{aligned}$$

[c_2 Shuryak 78, c_3 Kapusta 79, c'_4 Toimela 83, c_4 Arnold/Zhai 94,
 c_5 Zhai/Kastening 95, Braaten/Nieto 96, c'_6 KLRS 03], c_6 ??

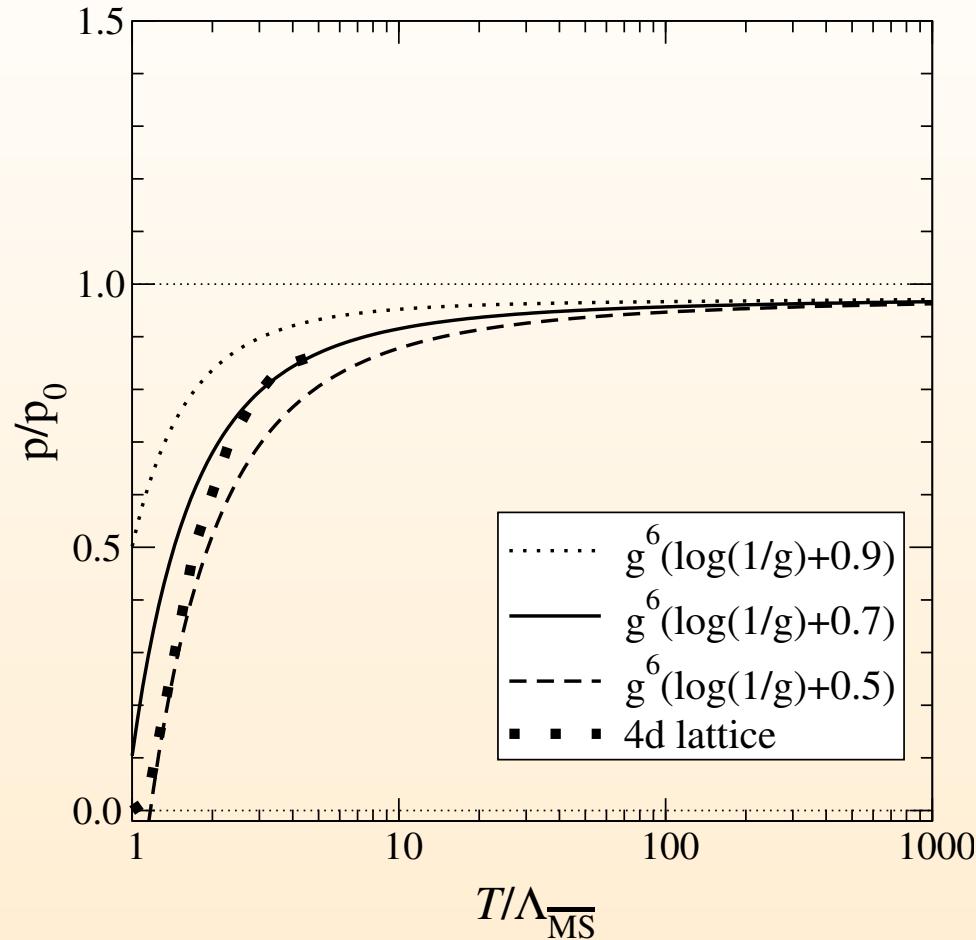
$p(T)$ in (thermal) perturbation theory



g^6 constant is a guess.

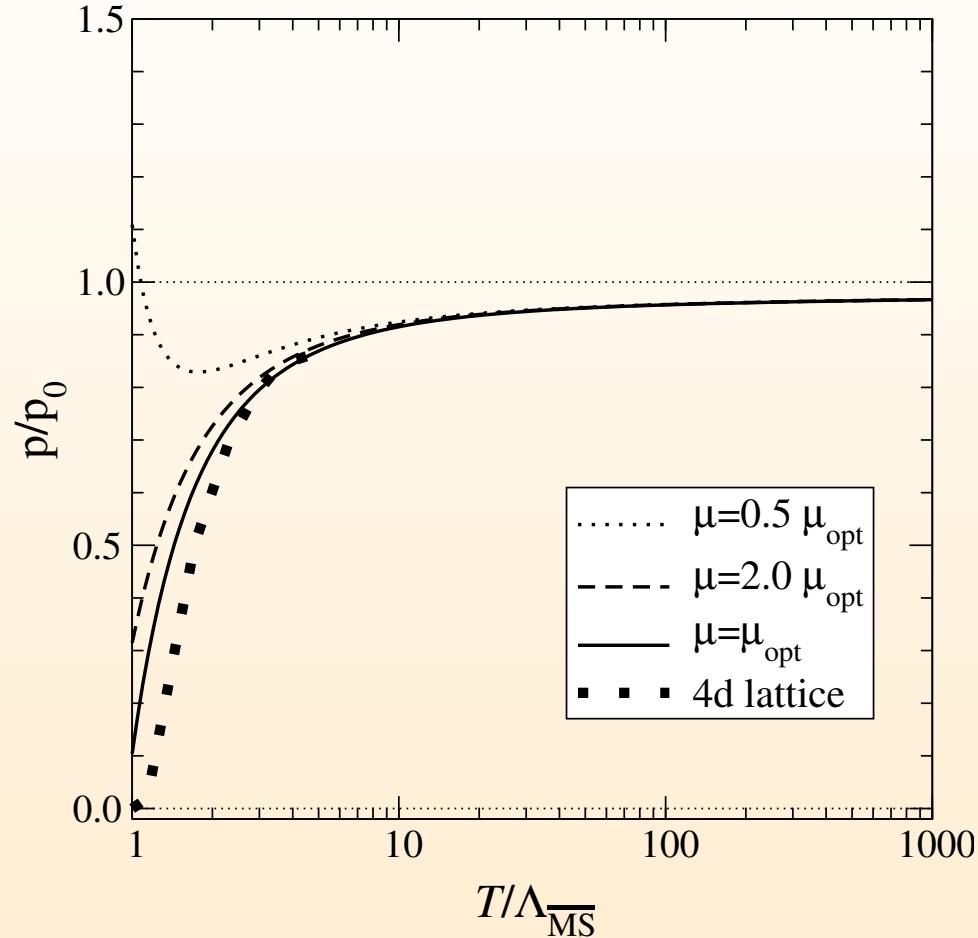
non-perturbative contrib not known, but **computable!**

$p(T)$ in (thermal) perturbation theory



depencence on g^6 constant

$p(T)$ in (thermal) perturbation theory



scale dependence

shopping list for c_6

$\dots + g^6$

- 4-loop sum-integrals needed, const term
- DOABLE?! manpower OR brainpower?

matching coeffs

- 2-loop ϵ -terms for m_E^2 , g_E^2 DONE. ML/YS, in prep.

$\dots + g^6$

- 4-loop integrals needed DONE. KLRS: reduction, master ints

match $\overline{\text{MS}}/\text{LAT}$

- 4-loop const in LAT reg
- DOABLE?! YS+Parma: NSPT

$\dots + g^6$

- measure $\langle \text{Plaquette} \rangle$ in 3d SU(N) DONE. HKLRS, in prep.

Methods I: Diagram generation

yet another generator? QGRAF [Nogueira], FeynArts [Denner/Hahn] n/a for 0-pt fcts.

skeleton (2PI) expansion [Luttinger/Ward, Baym, ...]

$$F[D] = \sum_i c_i (Tr \ln D_i^{-1} + Tr \Pi_i[D] D_i) - \Phi[D]$$

extremal property of partition function $\Rightarrow \delta_{D_i} \Phi[D] = c_i \Pi_i[D]$

$$-F = -F_0 + \Phi_2[\Delta]$$

$$\begin{aligned} &+ \left(\Phi_3[\Delta] + \sum_i c_i \left(\frac{1}{2} \textcircled{1} \textcircled{1} \right) \right) \\ &+ \left(\Phi_4[\Delta] + \sum_i c_i \left(\frac{1}{3} \textcircled{1} \textcircled{1} \textcircled{1} + \textcircled{1} \textcircled{2} + \frac{1}{2} \textcircled{1} \textcircled{2} \right) \right) \\ &+ \left(\Phi_5[\Delta] + \sum_i c_i \left(\frac{1}{4} \textcircled{1} \textcircled{1} \textcircled{1} \textcircled{1} + \textcircled{1} \textcircled{2} \textcircled{1} + \frac{1}{2} \textcircled{1} \textcircled{1} \textcircled{1} \textcircled{1} \right. \right. \\ &\quad \left. \left. + \frac{1}{2} \textcircled{2} \textcircled{2} + \frac{1}{2} \textcircled{2} \textcircled{2} + \textcircled{1} \textcircled{3} + \frac{1}{2} \textcircled{1} \textcircled{3} + \frac{1}{3} \textcircled{1} \textcircled{3} \right) \right) \end{aligned}$$

get skeletons from

$$\Phi_n[\Delta] = \frac{1}{n-1} \left\{ \frac{1}{12} \textcircled{\bullet} + \frac{1}{8} \textcircled{\circ} \textcircled{\circ} + \frac{1}{8} \textcircled{\bullet} \textcircled{\bullet} + \frac{1}{24} \textcircled{\circ} \textcircled{\circ} \right\}_n$$

and SD eqs $\Gamma_n^{1PI} = \delta_\phi^{n-1} S'[\phi + D[\phi]\delta_\phi] \Big|_{\phi=0}$

Methods I: Diagram generation

generic $\phi^3 + \phi^4$ skeletons

$$\Phi_2 = \frac{1}{12} \text{ (circle)} + \frac{1}{8} \text{ (two circles)}$$

$$\Phi_3 = \frac{1}{24} \text{ (triangle)} + \frac{1}{8} \text{ (V-shape)} + \frac{1}{48} \text{ (double circle)}$$

$$\Phi_4 = \frac{1}{72} \text{ (H-shape)} + \frac{1}{12} \text{ (square)} + \frac{1}{8} \text{ (circle with diagonal)} + \frac{1}{4} \text{ (circle with vertical line)} + \frac{1}{8} \text{ (double circle with vertical line)} + \frac{1}{8} \text{ (circle with horizontal line)} + \frac{1}{16} \text{ (circle with diagonal line)} + \frac{1}{48} \text{ (triangle with diagonal line)}$$

$$\Phi_5 = \frac{1}{4} \text{ (H-shape with vertical line)} + \frac{1}{48} \text{ (square with vertical line)} + \frac{1}{16} \text{ (circle with diagonal line and vertical line)} + \frac{1}{12} \text{ (circle with horizontal line and vertical line)} + \frac{1}{4} \text{ (circle with diagonal line and horizontal line)} + \frac{1}{2} \text{ (circle with diagonal line and vertical line)} + \frac{1}{2} \text{ (circle with horizontal line and vertical line)}$$

$$+ \frac{1}{8} \text{ (circle with diagonal line and vertical line)} + \frac{1}{4} \text{ (circle with diagonal line and horizontal line)} + \frac{1}{4} \text{ (circle with vertical line and horizontal line)}$$

$$+ \frac{1}{8} \text{ (double circle with vertical line)} + \frac{1}{2} \text{ (circle with diagonal line and triangle)} + \frac{1}{8} \text{ (circle with vertical line and circle)} + \frac{1}{4} \text{ (circle with horizontal line and circle)} + \frac{1}{4} \text{ (circle with diagonal line and circle)}$$

$$+ \frac{1}{2} \text{ (circle with diagonal line and triangle)} + \frac{1}{16} \text{ (double circle with horizontal line)} + \frac{1}{12} \text{ (circle with diagonal line and triangle)} + \frac{1}{16} \text{ (circle with vertical line and triangle)} + \frac{1}{32} \text{ (circle with horizontal line and triangle)} + \frac{1}{16} \text{ (circle with diagonal line and circle)} + \frac{1}{16} \text{ (circle with horizontal line and circle)} + \frac{1}{8} \text{ (circle with diagonal line and circle)}$$

$$+ \frac{1}{4} \text{ (circle with diagonal line and triangle)} + \frac{1}{8} \text{ (circle with vertical line and triangle)} + \frac{1}{4} \text{ (circle with horizontal line and triangle)} + \frac{1}{8} \text{ (circle with diagonal line and circle)} + \frac{1}{12} \text{ (circle with vertical line and circle)} + \frac{1}{128} \text{ (circle with horizontal line and circle)} + \frac{1}{32} \text{ (circle with diagonal line and circle)}$$

LAT: additional skeletons $\dots + \phi^5 + \dots + \phi^8 + \dots$

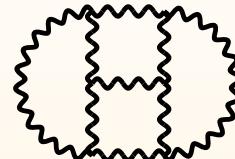
$$\Phi_3 \Big|_{\text{lat}} = \frac{1}{12} \text{ (two circles)} + \frac{1}{48} \text{ (triforce)}$$

$$\begin{aligned} \Phi_4 \Big|_{\text{lat}} = & \frac{1}{8} \text{ (triangle with V-shape)} + \frac{1}{12} \text{ (V-shape with V-shape)} + \frac{1}{240} \text{ (double circle)} + \frac{1}{12} \text{ (circle with circle)} + \frac{1}{8} \text{ (circle with V-shape)} + \frac{1}{16} \text{ (circle with circle)} \\ & + \frac{1}{48} \text{ (double circle with circle)} + \frac{1}{72} \text{ (circle with circle with circle)} + \frac{1}{48} \text{ (circle with circle with circle)} + \frac{1}{48} \text{ (circle with circle with circle)} + \frac{1}{384} \text{ (circle with circle with circle)} \end{aligned}$$

Methods II: reduction, IBP

Laporta.. no need to detail it here in KA..

in a nutshell, reduces e.g.

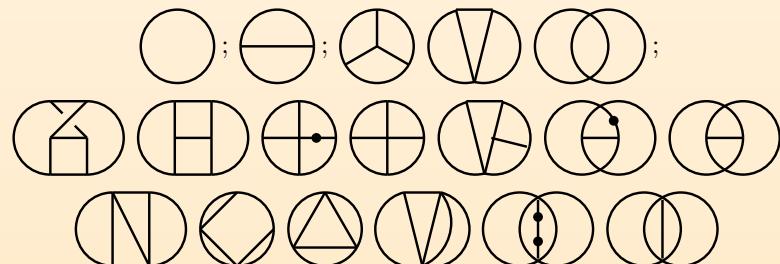


25M integrals ($2^9 6^6$)

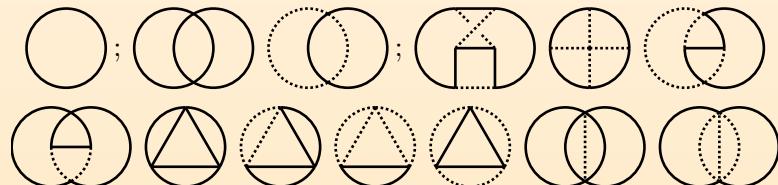
to

$$d_A C_A^3 \frac{g^6}{(4\pi)^4} \sum_i \frac{\text{poly}_i(d, \xi)}{\text{poly}_i(d)} \text{Master}_i(d)$$

18 fully massive master ints



13 ‘‘QED’’ type master ints



Methods III a: analytic Integration

3d, Euclidean, massive, dim. reg., $\overline{\text{MS}}$, x-space, ...

one 3d example[YS,AV]:

$$\left(\begin{array}{c} 3 \\ 1 \ 6 \end{array} \right) \left(\begin{array}{c} 9 \\ 8 \end{array} \right)_2 = \left(\frac{\bar{\mu}}{m_{316}} \right)^{8\epsilon} \frac{1}{32} \left[\frac{1}{\epsilon^2} + \frac{8}{\epsilon} + 4S \left(\frac{m_{316}}{m_{16289}}, \frac{2m_1}{m_{316}}, \frac{2m_3}{m_{316}} - 1 \right) + \mathcal{O}(\epsilon) \right]$$

$$\begin{aligned} \text{where } S(x, y, z) = & 13 + \frac{7}{12}\pi^2 + 2\text{Li}_2(1-y) + 2\text{Li}_2(y+z) + 2\text{Li}_2(-z) \\ & - 4(\ln x)^2 + 8 \frac{1-x}{x(1+z)} \text{Li}_2(1-x) \\ & + 8 \left(1 + \frac{1-x}{x(1+z)} \right) \left(\text{Li}_2(-xz) + \ln(x) \ln(1+xz) - \frac{\pi^2}{6} \right) \end{aligned}$$

a semi-analytic approach (need a difference equation, see next slides):

Harmonic Sums $S_{\bar{m}}(x)$ [Vermaseren]

Methods III b: numeric Integration, Deqs

very general setup [Laporta]

derive difference equation for generalized master $U(x) \equiv \int \frac{1}{D_1^x D_2 \dots D_N}$

$$\sum_{j=0}^R p_j(x) U(x+j) = F(x)$$

solve via factorial series $U(x) = U_0(x) + \sum_{j=1}^R U_j(x)$, where

$$U_j(x) = \mu_j^x \sum_{s=0}^{\infty} a_j(s) \frac{\Gamma(x+1)}{\Gamma(x+1+s-K_j)}$$

plug in, get μ , $K_j(d)$, and recursion rels for $a_j(s)$.

need bc for fixing, say, $a_j(0)$

Methods III b: numeric Integration, Deqs

particularly simple bc at large x :

$$U(x) = \int \frac{1}{(p_1^2 + 1)^x} g(p_1)$$
$$\lim_{x \rightarrow \infty} U(x) = \left[\int \frac{1}{(p_1^2 + 1)^x} \right] \times [g(0)] \sim (1)^x x^{-d/2} g(0)$$

while factorial series behaves as $\sum_j \mu_j^x x^{K_j} a_j(0)$

numerics: truncate sum. example:

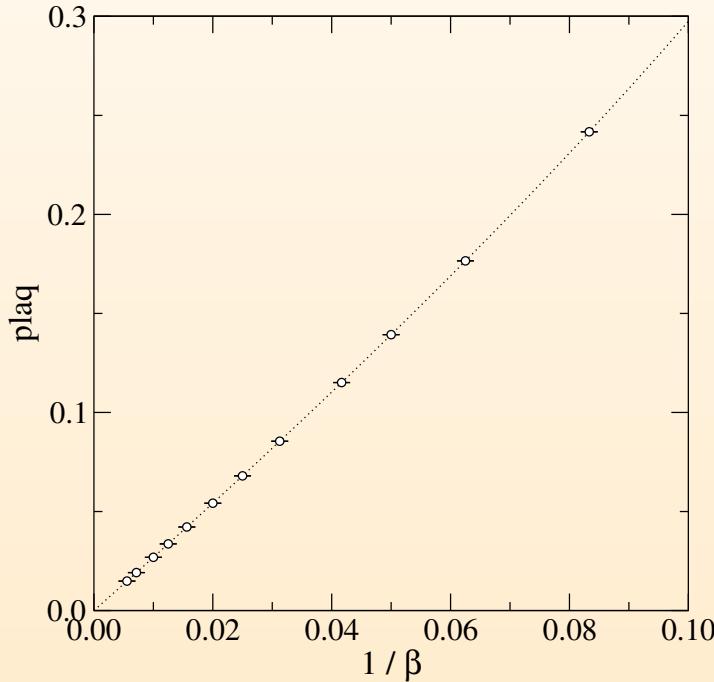
$$\begin{aligned} \textcircled{+} &= + 1.27227054184989419939788 - 5.67991293994853579036683\epsilon \\ &\quad + 17.6797238948173732343788\epsilon^2 - 46.5721846649543261864019\epsilon^3 \\ &\quad + 111.658522176214385363568\epsilon^4 - 252.46396390100217743236\epsilon^5 \\ &\quad + 549.30166596161426941705\epsilon^6 - 1164.5120588971521623546\epsilon^7 + \mathcal{O}(\epsilon^8) \end{aligned}$$

Methods IV: Lattice simulation

3d, finite box (aL)³. infinite-volume ($L \rightarrow \infty$) and continuum ($\frac{1}{\beta} \equiv \frac{g_{\mathbb{M}}^2 a}{2N_c} \rightarrow 0$) limits

$$\frac{1}{2g_{\mathbb{M}}^2} \left\langle \text{Tr} [F_{kl}^2] \right\rangle_{\overline{\text{MS}}} \equiv g_{\mathbb{M}}^2 \frac{\partial}{\partial g_{\mathbb{M}}^2} p_{G,\overline{\text{MS}}} = 3g_{\mathbb{M}}^6 \frac{d_A C_A^3}{(4\pi)^4} \left[\alpha_G \left(\ln \frac{\bar{\mu}}{2C_A g_{\mathbb{M}}^2} - \frac{1}{3} \right) + B_G + \mathcal{O}(\epsilon) \right]$$

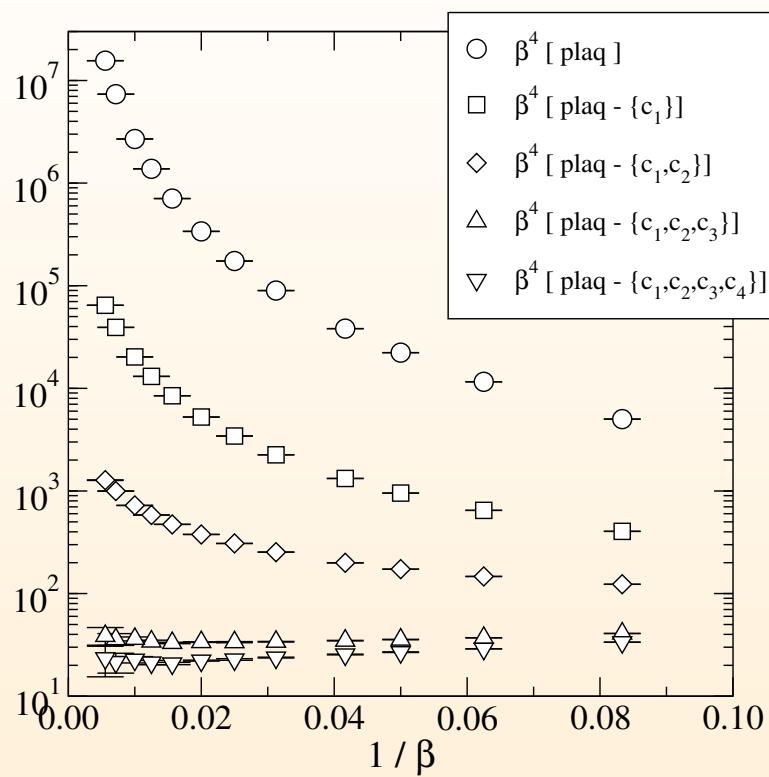
$$8 \frac{d_A C_A^6}{(4\pi)^4} B_G = \lim_{\beta \rightarrow \infty} \beta^4 \left\{ \left\langle 1 - \frac{1}{C_A} \text{Tr} [P_{12}] \right\rangle_a - \left[\frac{c_1}{\beta} + \frac{c_2}{\beta^2} + \frac{c_3}{\beta^3} + \frac{c_4}{\beta^4} \left(\ln \beta + c'_4 \right) \right] \right\}$$



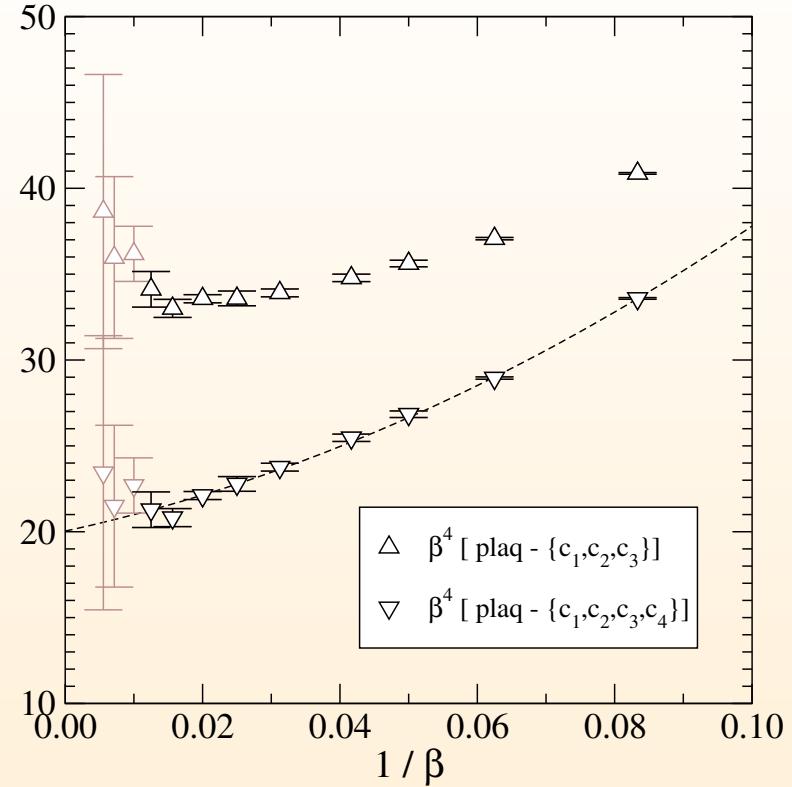
statistical errors are (much) smaller than the symbol sizes

Fit: $c_1/\beta + c_2/\beta^2 + c_3/\beta^3 + c_4 \ln \beta/\beta^4 + c'_4/\beta^4 + c_5/\beta^5 + c_6/\beta^6$

Methods IV: Lattice simulation



significance loss due to
the UV subtractions



continuum limit of infinite-
volume extrapolated data

$$B_G + \left(\frac{43}{12} - \frac{157}{768}\pi^2 \right) c'_4 = 10.7 \pm 0.4 \quad (N_c = 3)$$

Methods V: Lattice perturbation theory

amusing: 1loop tadpole has elliptic integral in 3d [M.Shaposhnikov]

$$a^{2-d} \int_{-\pi}^{\pi} \frac{d^d \hat{k}}{(2\pi)^d} \frac{1}{\sum_{\mu=0}^{d-1} 4 \sin^2(\hat{k}_\mu/2) + \hat{m}^2} = \frac{1}{a} \sum_{n \geq 0} \hat{m}^{2n} (\{1, \Sigma, \xi\} + \{1\} \hat{m})$$

where $\Sigma = 4\pi G(0) = \frac{8}{\pi}(18 + 2\sqrt{2} - 10\sqrt{3} - 7\sqrt{6})K^2((2 - \sqrt{3})^2(\sqrt{3} - \sqrt{2})^2)$

2loop example:

$$\kappa_5 = \frac{1}{\pi^4} \int_{-\pi/2}^{\pi/2} d^3x d^3y \frac{\sum_i \sin^2 x_i \sin^2(x_i + y_i) \sin^2 y_i}{\sum_i \sin^2 x_i \sum_i \sin^2(x_i + y_i) \sum_i \sin^2 y_i} = 1.013041(1)$$

→ classification? *very little is known systematically.*

1loop IBP + coordinate-space method [Lüscher/Weisz] [Becher/Melnikov]

or

Numerical Stochastic Perturbation Theory [with F. Di Renzo, V. Miccio]

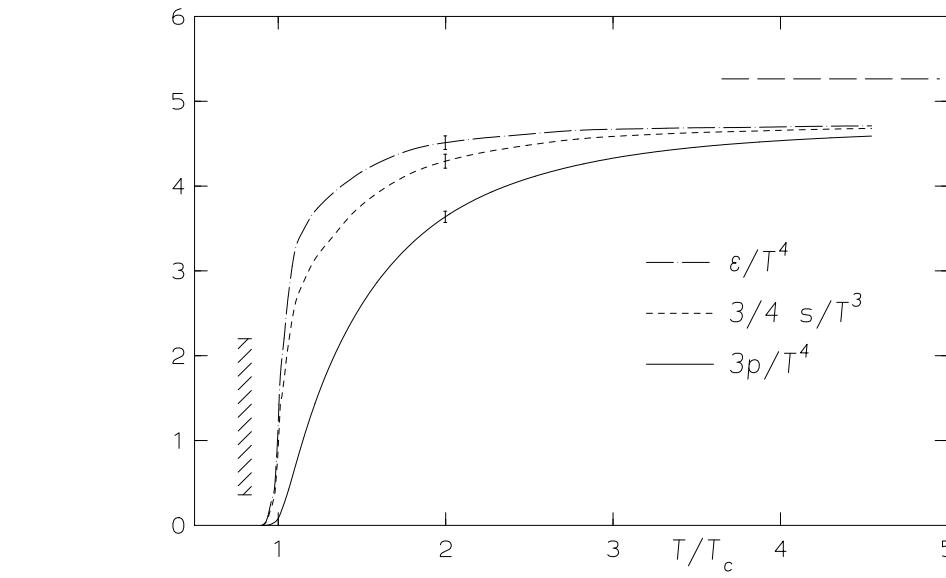
no diagrams!

Conclusions

- $p(T, \mu_B)$ is plagued by serious IR sensitivity, even at (asymptotically) high T.
- effective theory methods reduce the problem to a perturbative + 3d lattice MC setup for $T \gtrsim 2T_c$, $\mu_B \lesssim T$.
- 3d eff. th. brings major benefits:
superren., cost, universality
 $(N_f \dots \mu \dots i\mu N_f A_0^3)$
- a few well-defined (but hard) calculations left.
Manpower and/or Brainpower (new methods)?
- on the way, tested a few nice techniques..

Backup slides

energy density in pure YM



[pure YM, Boyd/Engels/Karsch/Laermann/Legeland/Lütgemeier/Petersson 1995]

continuum extrapolated, $T_0 \approx 270\text{MeV}$

more on diagram generation

Schwinger-Dyson (SD) eqs

$$\begin{aligned}
 \text{-} \bullet \text{-} &= \frac{1}{2} \cdot \text{---} + \frac{1}{6} \cdot \text{---} \\
 \text{-} \bullet \text{-} &= \text{-} \cdots \bullet \text{-} + \frac{1}{2} \cdot \text{---} + \frac{1}{2} \cdot \text{---} + \frac{1}{2} \cdot \text{---} + \frac{1}{6} \cdot \text{---} \\
 &= \text{-} \cdots \bullet \text{-} + \text{-} \text{---} \\
 \text{---}^3_{1,2} &= \text{---} + \text{---} + \frac{1}{2} \left(\text{---} + \text{---} + \text{---} + \text{cyclic}(2,3) \right) \\
 &\quad + \text{---} + \frac{1}{2} \cdot \text{---} + \frac{1}{6} \cdot \text{---} \\
 \text{---}^4_{1,2}^3 &= \text{---} + \left(\text{---} + \text{---} + \text{---} + \frac{1}{2} \cdot \text{---} + \text{cyclic}(2,3,4) \right) + \frac{1}{2} \cdot \text{---} \\
 &\quad + \{2\text{-loop terms}\}
 \end{aligned}$$

(2PI) skeletons generate self-energies. for bosonic particles ($c_i = \frac{1}{2}$):

$$\begin{aligned}
 \Pi_1^{\text{irr}} &= \text{-} \bullet \text{-} = \frac{1}{2} \cdot \text{---} + \frac{1}{2} \cdot \text{---} \\
 \Pi_2^{\text{irr}} &= \text{-} \bullet \text{-} = \frac{1}{2} \cdot \text{---} + \frac{1}{2} \cdot \text{---} + \frac{1}{2} \cdot \text{---} + \frac{1}{4} \cdot \text{---} + \frac{1}{6} \cdot \text{---} \\
 \Pi_2^{\text{red}(1)} &= \text{-} \bullet \text{-} = \text{---} + \frac{1}{2} \cdot \text{---}
 \end{aligned}$$

LAT: additional irreducible self-energy

$$\Pi_2^{\text{irr}} \Big|_{\text{lat}} = \text{-} \bullet \text{-} \Big|_{\text{lat}} = +\frac{1}{4} \cdot \text{---} + \frac{1}{4} \cdot \text{---} + \frac{1}{6} \cdot \text{---} + \frac{1}{8} \cdot \text{---}$$

more on diagram generation

Now, consider $SU(N)$ gauge theory with fermions and a scalar field.
This class includes QCD, QED, EW, SQED



more on diagram generation

$$\begin{aligned}
 \Phi_2 &= \frac{1}{8} \text{Diagram A} + \frac{1}{12} \text{Diagram B} - \frac{1}{2} \text{Diagram C} + \frac{1}{4} \text{Diagram D} + \frac{1}{4} \text{Diagram E} + \frac{1}{8} \text{Diagram F} \\
 \Phi_3 &= \frac{1}{24} \text{Diagram G} - \frac{1}{3} \text{Diagram H} - \frac{1}{4} \text{Diagram I} + \frac{1}{8} \text{Diagram J} + \frac{1}{48} \text{Diagram K} + \frac{1}{6} \text{Diagram L} + \frac{1}{8} \text{Diagram M} \\
 &\quad + \frac{1}{2} \text{Diagram N} + \frac{1}{4} \text{Diagram O} + \frac{1}{8} \text{Diagram P} + \frac{1}{8} \text{Diagram Q} + \frac{1}{48} \text{Diagram R} \\
 \Phi_4 &= \frac{1}{72} \text{Diagram S} - \frac{1}{4} \text{Diagram T} - \frac{1}{6} \text{Diagram U} + \frac{1}{12} \text{Diagram V} - \frac{1}{2} \text{Diagram W} - \frac{1}{2} \text{Diagram X} \\
 &\quad - 1 \text{Diagram Y} - \frac{1}{3} \text{Diagram Z} + \frac{1}{6} \text{Diagram AA} + \frac{1}{6} \text{Diagram BB} + \frac{1}{8} \text{Diagram CC} - \frac{1}{4} \text{Diagram DD} \\
 &\quad + \frac{1}{4} \text{Diagram EE} - \frac{1}{2} \text{Diagram FF} + \frac{1}{8} \text{Diagram GG} + \frac{1}{8} \text{Diagram HH} + \frac{1}{16} \text{Diagram II} + \frac{1}{48} \text{Diagram JJ} \\
 &\quad + \frac{1}{8} \text{Diagram KK} + \frac{1}{12} \text{Diagram LL} - \frac{1}{3} \text{Diagram MM} + \frac{1}{4} \text{Diagram NN} + \frac{1}{4} \text{Diagram OO} + \frac{1}{2} \text{Diagram PP} \\
 &\quad + \frac{1}{6} \text{Diagram QQ} + \frac{1}{12} \text{Diagram RR} + \frac{1}{2} \text{Diagram SS} + \frac{1}{2} \text{Diagram TT} + \frac{1}{2} \text{Diagram UU} + \frac{1}{8} \text{Diagram VV} + \frac{1}{4} \text{Diagram WW} \\
 &\quad + \frac{1}{4} \text{Diagram XX} - \frac{1}{2} \text{Diagram YY} + \frac{1}{4} \text{Diagram ZZ} + \frac{1}{4} \text{Diagram AAA} + \frac{1}{4} \text{Diagram BBB} + 1 \text{Diagram CCC} + 1 \text{Diagram DDD} \\
 &\quad + \frac{1}{4} \text{Diagram EEE} + \frac{1}{8} \text{Diagram FFF} + \frac{1}{2} \text{Diagram GGG} + \frac{1}{2} \text{Diagram HHH} + \frac{1}{8} \text{Diagram III} + \frac{1}{4} \text{Diagram JJJ} \\
 &\quad + \frac{1}{8} \text{Diagram KKK} + \frac{1}{2} \text{Diagram LLL} + \frac{1}{2} \text{Diagram MMM} + \frac{1}{8} \text{Diagram NNN} + \frac{1}{16} \text{Diagram OOO} + \frac{1}{2} \text{Diagram PPP} + \frac{1}{16} \text{Diagram RRR} \\
 &\quad + \frac{1}{16} \text{Diagram SSS} + \frac{1}{6} \text{Diagram TTT} + \frac{1}{4} \text{Diagram UUU} + \frac{1}{4} \text{Diagram VVV} + \frac{1}{4} \text{Diagram WWW} + \frac{1}{4} \text{Diagram XXX} + \frac{1}{2} \text{Diagram YYY} \\
 &\quad + \frac{1}{8} \text{Diagram ZZZ} + \frac{1}{16} \text{Diagram AAAA} + \frac{1}{8} \text{Diagram BBBB} + \frac{1}{16} \text{Diagram CCCC} + \frac{1}{48} \text{Diagram DDDD}
 \end{aligned}$$

more on diagram generation

the skeletons immediately produce the self-energies of the model

more on diagram generation

ring diagrams for the model

$$\begin{aligned} (-F_{(\text{rings})})_3 &= \frac{1}{4} \text{ (wavy loop with two nodes)} - \frac{1}{2} \text{ (dotted loop with one node)} + \frac{1}{4} \text{ (solid loop with two nodes)} \\ (-F_{(\text{rings})})_4 &= \frac{1}{6} \text{ (wavy loop with three nodes)} + \frac{1}{2} \text{ (dotted loop with two nodes)} + \frac{1}{4} \text{ (solid loop with three nodes)} - \frac{1}{3} \text{ (dotted loop with one node)} \\ &\quad + \frac{1}{6} \text{ (solid loop with one node)} + \frac{1}{2} \text{ (solid loop with two nodes)} + \frac{1}{4} \text{ (solid loop with three nodes)} \end{aligned}$$

extremely economic structure of the skeleton expansion:

the few ring diagrams above summarize

22 (276) 3-loop (4-loop) diagrams