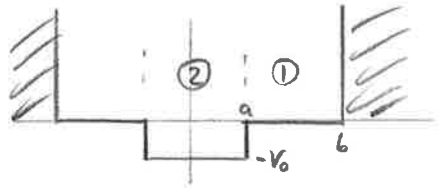
① 

$E = 0$

Bereich ①: SG $\partial_x^2 \psi_1 = 0 \Rightarrow \psi_1(x) = A + Bx$; $\psi_1(b) = 0 \Rightarrow \psi_1(x) = B(x-b)$

②: SG $(\partial_x^2 + \kappa^2)\psi_2 = 0 \Rightarrow \psi_2 = C\sin(\kappa x) + D\cos(\kappa x)$

GZ hat keine Knoten $\Rightarrow \psi_2(x) = D\cos(\kappa x)$

Anschluss: ψ stetig @ a : $B(a-b) = D\cos(\kappa a)$

ψ' stetig @ a : $B = -D\kappa\sin(\kappa a)$

Glgn teilen $\Rightarrow a - b = -\frac{\cos(\kappa a)}{\kappa\sin(\kappa a)} \Rightarrow \underline{\underline{b = a + \frac{1}{\kappa}\cot(\kappa a)}}$

② SG $i\hbar\partial_t \chi = H\chi \Rightarrow \underline{\chi(t)} = e^{-\frac{i}{\hbar}Ht}\chi(0) = \underline{e^{-i\omega t\sigma_x}\begin{pmatrix}0\\1\end{pmatrix}}$

$\sigma_x^2 = \mathbb{1}$, e-Reihe $\Rightarrow e^{-i\omega t\sigma_x} = c\mathbb{1} - is\sigma_x$ mit $c \equiv \cos(\omega t)$, $s \equiv \sin(\omega t)$

$\Rightarrow \underline{\underline{\chi(t)}} = \begin{pmatrix} c & -is \\ -is & c\end{pmatrix}\begin{pmatrix}0\\1\end{pmatrix} = \underline{\underline{\begin{pmatrix}-is\\c\end{pmatrix}}}$

$\underline{\underline{P_+}} = |(1\ 0)\chi(t)|^2 = \underline{\underline{s^2}}$, $\underline{\underline{P_-}} = |(0\ 1)\chi|^2 = \underline{\underline{c^2}} = 1 - P_+$ ✓

$\underline{\underline{\langle\sigma_z\rangle}} = s^2 - c^2 = \underline{\underline{1 - 2c^2}}$, $\underline{\underline{\langle\sigma_x\rangle}} = isc - isc = \underline{\underline{0}}$

③ ⓐ $H = H_0 + \lambda H_1$, $H_0\varphi_n = E_{0n}\varphi_n$ gelöst, $H\psi_n = E_n\psi_n$ gesucht

$E_n \approx E_{0n} + \lambda\langle\varphi_n|H_1|\varphi_n\rangle + O(\lambda^2)$

$= E_{0n} + \lambda V_0 \underbrace{\int_0^a dx\, \varphi_n^*\varphi_n}_{=1\ (\text{Norm})} + O(\lambda^2) = E_{0n} + \underline{\underline{\lambda V_0}} + O(\lambda^2)$

ⓑ $\psi_n(x) \approx \varphi_n(x) + \sum_{p\neq n}\frac{\boxed{\langle\varphi_p|\lambda H_1|\varphi_n\rangle}}{E_{0n} - E_{0p}}\varphi_p(x) + O(\lambda^2)$

$= \lambda V_0 \int_0^a dy\, \frac{2}{a}\sin\left(\frac{p\pi y}{a}\right)\sin\left(\frac{n\pi y}{a}\right) = \frac{2\lambda V_0}{\pi}\int_0^\pi dy\, \sin(py)\sin(ny)$

$= \frac{2\lambda V_0}{\pi}\frac{1}{2}\left[\frac{\sin((p-n)\pi)}{p-n} - \frac{\sin((p+n)\pi)}{p+n}\right] = 0 \quad \forall p,n \in \mathbb{Z}$

$= \varphi_n(x) + \underline{\underline{0\cdot\lambda}} + O(\lambda^2)$

<u>keine</u> Korrektur, weil φ_n bereits <u>exakte</u> Lsg: $(H_0 + \lambda H_1)\varphi_n = (E_{0n} + \lambda V_0)\varphi_n$

ⓒ $E_n \approx E_{0n} + \lambda V_0 \int_0^{a/2} dx\, \frac{2}{a}\sin^2\left(\frac{n\pi x}{a}\right) + O(\lambda^2)$

$= E_{0n} + \frac{2\lambda V_0}{n\pi}\int_0^{n\pi/2} dx\, \sin^2 x + O(\lambda^2) = E_{0n} + \underline{\underline{\frac{\lambda V_0}{2}}} + O(\lambda^2)$

④ $H = -\frac{\hbar^2}{2m}\partial_x^2 - \alpha\delta(x)$, $\psi = C e^{-bx^2}$

Norm : $1 \overset{!}{=} |C|^2 \int_{-\infty}^{\infty} dx\, e^{-2bx^2} = |C|^2 \sqrt{\frac{\pi}{2b}} \Rightarrow \underline{\underline{|C|^2 = \sqrt{\frac{2b}{\pi}}}}$

Vari: $E_0 \le \langle\psi|H|\psi\rangle$

$$= -\frac{\hbar^2}{2m}|C|^2 \boxed{\int_{-\infty}^{\infty} dx\, e^{-bx^2}\partial_x^2 e^{-bx^2}} - \alpha |C|^2 \int_{-\infty}^{\infty} dx\, e^{-2bx^2}\delta(x)$$

$$= \int_{-\infty}^{\infty} dx\, e^{-bx^2}(-2b+4b^2x^2)e^{-bx^2} = -2b(1+b\partial_b)\int_{-\infty}^{\infty} dx\, e^{-2bx^2} = -\sqrt{\frac{b\pi}{2}}$$

$$= |C|^2\left(\frac{\hbar^2}{2m}\sqrt{\frac{b\pi}{2}} - \alpha\right) = \frac{\hbar^2 b}{2m} - \alpha\sqrt{\frac{2b}{\pi}}$$

Min? $\partial_b \langle H\rangle = \frac{\hbar^2}{2m} - \frac{\alpha}{\sqrt{2\pi b}} \overset{!}{=} 0 \Rightarrow \underline{\underline{b_{min} = \frac{2m^2\alpha^2}{\pi\hbar^4}}}$

Min or Max? $\partial_b^2\langle H\rangle = +\frac{\alpha}{\sqrt{8\pi b^3}} > 0$ ✓ Min

$\Rightarrow \underline{\underline{E_0 \le -\frac{m\alpha^2}{\pi\hbar^2}}}$. Vergleich: $-\frac{m\alpha^2}{2\hbar^2} \le -\frac{m\alpha^2}{\pi\hbar^2}$ <u>OK, da $\pi > 2$.</u>

⑤ SG $\left(-\frac{\hbar^2}{2m}\vec\nabla^2 + V(\vec r) - E\right)\psi(\vec r) = 0$

Separationsansatz $\psi(\vec r) = \frac{u(r)}{r} Y_{lm}(\vartheta,\varphi) \Rightarrow \left(\partial_r^2 - \frac{l(l+1)}{r^2} + \frac{2m}{\hbar^2}(E - V(r))\right)u(r) = 0$

$\underline{l=0}$: innen $(r<a)$ $u = A\sin(kr) + B\cos(kr)$, $k = \frac{\sqrt{2mE}}{\hbar}$

$\psi(0)$ endlich $\Rightarrow B = 0$

aussen ist $\psi = 0 \Rightarrow u(a) = 0 = A\sin(ka)$

$\Rightarrow k = \frac{n\pi}{a}$, $n \in \mathbb{N}$ $\Rightarrow \underline{\underline{E = \frac{\hbar^2\pi^2 n^2}{2ma^2}}}$

$\psi(\vec r) = A\sin\left(\frac{n\pi r}{a}\right)\frac{1}{r} Y_{00}$

Norm: $1 \overset{!}{=} \int d^3r\, \psi^*\psi = |A|^2 \frac{a}{n\pi}\int_0^{n\pi} dx\, \sin^2 x = |A|^2\frac{a}{2}$

$\underline{\underline{\psi(\vec r) = \frac{1}{\sqrt{2\pi a}\, r}\sin\left(\frac{n\pi r}{a}\right)}}$

⑥ ⓐ Quantenoptik

ⓑ Manipulation von individuellen Teilchen, ohne deren QM Natur zu zerstören;
Ionenfalle mit Laserkühlung / Seitenband-Kühlung;
Spiegel / Resonator hoher Güte → Einzelphotonenfalle , $\tau_\gamma \sim \frac{1}{10}$ s

ⓒ Ionen in der Falle als Qubits → Quantencomputer
Übergangsfrequenz der Ionen in Falle höher als Cs-Atomuhr → Zeitmessung
QM-Grundlagen: Dekohärenz / Verschränkung expt. testen
reine → gemischte Zustände