# 6. "Anomalies"

[ Peskin/Schroeder §19 ; Zee § IV.7 ]

$\rightsquigarrow$ Q.: symmetry of classical physics $\hat{=}$ symmetry of quantum physics ?

(action $S[\phi]$ invariant under $\phi \to \phi + \delta\phi$) (path integral $\int \mathcal{D}\varphi\, e^{iS[\phi]}$ invariant)

$\to$ A.: not always ; measure $\mathcal{D}\varphi$ may or may not be invariant.

example: rotational invariance of QFT

would be strange if quantum fluctuations preferred a specific direction!

but: quantum fluctuation can break (some) classical symmetries ;

this phenomenon is called "anomaly" ;

conceptually clear: change of integration variables $\to$ don't forget Jacobian

important subject in QFT $\to$ many ways of looking at it

here: see a class. symmetry vanishing, by explicit Feynman diag calc.

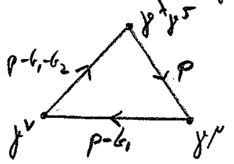
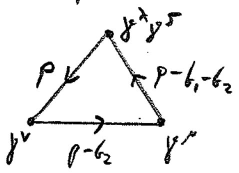
- theory of one massless fermion : $\boxed{\mathcal{L}_1 \equiv \bar\psi\, i\gamma^\mu \partial_\mu \psi}$

  invariant under $\psi \to e^{i\theta}\psi$ and $\psi \to e^{i\theta\gamma^5}\psi$

  conserved current $J^\mu \equiv \bar\psi\gamma^\mu\psi$ (vector) and $J_5^\mu \equiv \bar\psi\gamma^\mu\gamma^5\psi$ (axial)

  (( check : $\partial_\mu J^\mu = 0 = \partial_\mu J_5^\mu$ via class. eqn. of motion $i\gamma^\mu\partial_\mu\psi = 0$ ))

- calculate (Fourier transform of) amplitude $\langle 0 | T\, J_5^\lambda(0)\, J^\mu(x_1)\, J^\nu(x_2) | 0 \rangle$

$\Delta^{\lambda\mu\nu}(k_1, k_2) =$  +  , $q \equiv k_1 + k_2$





$$= (-1)\int \frac{d^4p}{(2\pi)^4}\, \mathrm{tr}\left(\gamma^\lambda\gamma^5 \frac{i}{\not p - \not q}\gamma^\nu \frac{i}{\not p - \not k_1}\gamma^\mu \frac{i}{\not p} + \gamma^\lambda\gamma^5 \frac{i}{\not p - \not q}\gamma^\mu \frac{i}{\not p - \not k_2}\gamma^\nu \frac{i}{\not p}\right)$$

if (in QFT) $\partial_\mu J^\mu = 0$ holds, then $k_{1\mu}\Delta^{\lambda\mu\nu} = 0$ and $k_{2\nu}\Delta^{\lambda\mu\nu} = 0$

〃 $\partial_\mu J_5^\mu = 0$ holds, then $q_\lambda \Delta^{\lambda\mu\nu} = 0$

$\to$ can check this by explicit computation.

## 6.1 Vector current conservation

- would non-conservation of either current be a disaster?
  - $J^\mu$: charge $Q = \int d^3x\, J^0$ counts # of fermions
    $\rightarrow$ would be difficult to interpret if not conserved!
    couple photon to $\psi$, photon line coming into vertex $\gamma^\mu$
    would have propag $\frac{-i}{k_1^2}\left(g_{\mu\varrho} - (1-\xi)\frac{k_{1\mu}k_{1\varrho}}{k_1^2}\right)$
    $\rightarrow$ gauge dependence falls out if $k_{1\mu}\Delta^{\lambda\mu\nu} = 0$
  - $J_5^\mu$: who care if axial charge $Q^5 = \int d^3x\, J_5^0$ changes in time?

- naive calculation

$$k_{1\mu}\Delta^{\lambda\mu\nu}(k_1,k_2) = i\int\frac{d^4p}{(2\pi)^4}\, tr\left(\gamma^\lambda\gamma^5\frac{1}{\not p-\not q}\gamma^\nu\frac{1}{\not p-\not k_1}\overset{=\not p-(\not p-\not k_1)}{\not k_1}\frac{1}{\not p} + \gamma^\lambda\gamma^5\frac{1}{\not p-\not q}\overset{=(\not p-\not k_2)-(\not p-\not q)}{\not k_1}\frac{1}{\not p-\not k_2}\gamma^\nu\frac{1}{\not p}\right)$$

$$= i\int\frac{d^4p}{(2\pi)^4}\, tr\left(\gamma^\lambda\gamma^5\frac{1}{\not p-\not q}\gamma^\nu\frac{1}{\not p-\not k_1} - \underbrace{\gamma^\lambda\gamma^5\frac{1}{\not p-\not k_2}\gamma^\nu\frac{1}{\not p}}_{\text{shift } p\to p-k_1}\right)$$

$$= 0$$

- more careful calculation
  when is it OK to shift integration variables?

1 dim: $\int_{-\infty}^{\infty}dp\,[f(p+a)-f(p)] = \int_{-\infty}^{\infty}dp\,[a\,\partial_p f(p) + \mathcal{O}(a^2)] = a\left(f(\infty)-f(-\infty)\right) + \mathcal{O}(a^2)$

d dim: $\int d^dp_E\,[f(p+a)-f(p)] = \int d^dp_E\,[a^\mu\partial_{p_\mu}f(p) + \dots] \overset{\text{Gauss}}{=} \lim_{P\to\infty}\left\langle a^\mu\left(\frac{P_\mu}{P}\right)f(P)\,S_{d-1}(P)\right\rangle$

$\left(\left(\frac{2\pi^{d/2}}{\Gamma(d/2)}P^{d-1}\right)\right)$ "surface" of d dim sphere, radius $P$

for our 4 dim Minkowskian integral, ← from Wick rot.

$$\int d^4p\,[f(p+a)-f(p)] = \lim_{P\to\infty}\left\langle i\,a^\mu\left(\frac{P_\mu}{P}\right)f(P)\,(2\pi^2P^3)\right\rangle \leftarrow \text{angular average}$$

$\left(\left( tr\left(\gamma^\lambda\gamma^\nu\gamma^\rho\gamma^\sigma\gamma^5\right) = -4i\,\varepsilon^{\lambda\nu\rho\sigma}\right)\right)$

use this for $a = -k_1$

and $f(p) = tr\left(\gamma^\lambda\gamma^5\frac{1}{\not p-\not k_2}\gamma^\nu\frac{1}{\not p}\right) = \frac{tr\left(\gamma^5(\not p-\not k_2)\gamma^\nu\not p\gamma^\lambda\right)}{(p-k_2)^2p^2} \overset{\text{(check ?!)}}{=} \frac{4i\,\varepsilon^{\tau\nu\sigma\lambda}k_{2\tau}p_\sigma}{(p-k_2)^2p^2}$

$$\Rightarrow k_{1\mu}\Delta^{\lambda\mu\nu}(k_1,k_2) = \frac{i}{(2\pi)^4}\lim_{P\to\infty}\left\langle i(-k_1)^\mu\frac{P_\mu}{P}\,\frac{4i\,\varepsilon^{\tau\nu\sigma\lambda}k_{2\tau}P_\sigma}{(P-k_2)^2P^2}\,2\pi^2P^3\right\rangle$$

$\left(\left\langle P_\mu P_\sigma\right\rangle = \frac{g_{\mu\sigma}}{4}P^2\right)$

$$= \frac{i}{8\pi^2}k_1^\mu\,\varepsilon^{\tau\nu\sigma\lambda}g_{\mu\sigma}k_{2\tau} = \frac{i}{8\pi^2}\varepsilon^{\lambda\nu\tau\sigma}k_{1\tau}k_{2\sigma}$$

$\rightsquigarrow k_{1\mu}\Delta^{\lambda\mu\nu} \neq 0$ ?! ↯ fermion # not conserved, we disintegrate