((• kinematic limits: note that $\int_x^\infty d\bar{x} \to \int_x^1 d\bar{x}$ (cf pg. 66);

outgoing quark (p_1) and gluon (p_2) are <u>real</u> particles

$$0 \le (p_1+p_2)^2 = (\eta p+q)^2 = \cancel{\eta^2 p^2} + 2\eta p\cdot q + q^2 = 2p\cdot q\left(\eta - \frac{Q^2}{2p\cdot q}\right) = 2p\cdot q(\eta - x) = 2p\cdot q\,\eta(1-\bar{x})$$))

• since pdf's contain physics below μ only, they <u>depend</u> on μ

$$F_2(x,Q^2) = \sum_q Q_q^2 \int_x^1 d\bar{x}\, f_q\left(\frac{x}{\bar{x}},\mu^2\right)\frac{x}{\bar{x}}\left\{\delta(1-\bar{x}) + \frac{\alpha_s}{2\pi}\left(P(\bar{x})\ln\frac{Q^2}{\mu^2} + R(\bar{x})\right) + O(\alpha_s^2)\right\}$$

$\Rightarrow$ structure fct depends on Q^2 now, <u>violates</u> Bjorken scaling!

$\rightarrow$ μ^2-dependence ?! ↯ ad-hoc theoretical construct

• physical cross sections cannot depend on μ^2

$$\Rightarrow \partial_{\mu^2} F_2(x,Q^2) \overset{!}{=} 0$$

or, at least, in our calculation $\mu^2\partial_{\mu^2} F_2(x,Q^2) = O(\alpha_s^2)$

$$\Leftrightarrow \quad \mu^2\partial_{\mu^2} f_q(x,\mu^2) = \frac{\alpha_s}{2\pi}\int_x^1 \frac{d\bar{x}}{\bar{x}} f_q\left(\frac{x}{\bar{x}},\mu^2\right)P(\bar{x}) + O(\alpha_s^2)$$

<u>Dokshitzer – Gribov – Lipatov – Altarelli – Parisi</u> eqn (DGLAP, GLAP, AP)

• try to understand <u>physical content</u> of DGLAP eqn

$$\left(\frac{1+x^2}{1-x}\right)_+ \overset{(pg\ 67)}{=} \frac{1+x^2}{1-x} - \delta(1-x)\int_0^1 dx' \frac{1+x'^2}{1-x'} \qquad 1+x'^2 = 2-(1-x')(1+x')$$

$= 1+x^2$ due to δ fct

$$= (1+x^2)\left\{\frac{1}{1-x} - \delta(1-x)\int_0^1 dx' \frac{1}{1-x'}\right\} + \delta(1-x)\underbrace{\int_0^1 dx'(1+x')}_{=3/2}$$

$$= \frac{1+x^2}{(1-x)_+} + \frac{3}{2}\delta(1-x)$$

$$\Rightarrow P(x) = C_F\left(\frac{1+x^2}{1-x}\right)_+ \quad , (\text{cf. pg } 68)$$

now,
$$\mu^2\partial_{\mu^2} f_q(x,\mu^2) = \underbrace{\frac{\alpha_s}{2\pi}\int_x^1 \frac{d\bar{x}}{\bar{x}} f_q\left(\frac{x}{\bar{x}},\mu^2\right) C_F \frac{1+\bar{x}^2}{1-\bar{x}}}_{\text{pdf increase}} \underbrace{- \frac{\alpha_s}{2\pi}\int_x^1 \frac{d\bar{x}}{\bar{x}} f_q\left(\frac{x}{\bar{x}},\mu^2\right) C_F\,\delta(1-\bar{x})\int_0^1 dx' \frac{1+x'^2}{1-x'}}_{\overset{!}{=}\, -\frac{\alpha_s}{2\pi} f_q(x,\mu^2)\, C_F \int_0^1 d\bar{x}\,\frac{1+\bar{x}^2}{1-\bar{x}}\ \text{— pdf decrease}}$$

so at given x, pdf $\begin{Bmatrix}\text{increase}\\ \text{decrease}\end{Bmatrix}$ from $\begin{Bmatrix}\text{higher-}x\text{ quarks}\\ \text{quarks at } x\end{Bmatrix}$ reducing their momentum fraction by radiating off gluons.

$\rightsquigarrow$ both pieces are <u>divergent</u> at $\bar{x} \to 1$ (due to infinitesimally soft gluon radiation); div's <u>cancel</u> because # gained quarks = # lost quarks

- <u>solve</u> DGLAP eqn ?

  in practice: done numerically

- <u>scheme / scale dependence</u>

  the above factorization (of structure fct F in $f_{\text{non-pert}} * \text{coeff. fct}_{\text{pert.}}$) may look pretty arbitrary; can be proven to all orders in pert. theory;

  $\rightarrow$ instead of transverse momentum cutoff μ^2 above, use <u>dim. reg.</u>

  $\Rightarrow$ NLO cross section in d dimensions: divergence is pole $\frac{1}{\varepsilon}$ now

  (from $\frac{g_{\text{bare}}^2}{4\pi} = Z_g^2 \alpha_s \mu^{4-d}$)

$$F_2(x, Q^2) = \sum_q Q_q^2 \int_x^1 d\bar{x}\; \bar{f}_q\left(\frac{x}{\bar{x}}\right) \frac{x}{\bar{x}} \left\{ \delta(1-\bar{x}) + \frac{\alpha_s}{2\pi} \left(P(\bar{x}) \left(\frac{4\pi\mu^2}{Q^2} \right)^{\varepsilon} \left(-\frac{1}{\varepsilon} \right) + \bar{R}(\bar{x}) + O(\varepsilon) \right) + O(\alpha_s^2) \right\}$$

"bare" pdf — <u>same</u> P as above — different from R

now (cf §3.3), since pdf's are not physical observables, define modified set of pdf's as

$$x \bar{f}_q(x) \equiv \int_x^1 d\bar{x}\; f_q\left(\frac{x}{\bar{x}}, \mu_F^2\right) \frac{x}{\bar{x}} \left\{ \delta(1-\bar{x}) - \frac{\alpha_s}{2\pi} \left(P(\bar{x}) \left(\frac{4\pi\mu^2}{\mu_F^2} \right)^{\varepsilon} \left(-\frac{1}{\varepsilon} \right) + K(\bar{x}) \right) \right\}$$

again, arbitrary "factorization" scale — arbitrary finite fct

(check?!)

$$\Rightarrow F_2(x, Q^2) = \sum_q Q_q^2 \int_x^1 d\bar{x}\; f_q\left(\frac{x}{\bar{x}}, \mu_F^2\right) \frac{x}{\bar{x}} \left\{ \delta(1-\bar{x}) + \frac{\alpha_s}{2\pi} \left(P(\bar{x}) \ln \frac{Q^2}{\mu_F^2} + \bar{R}(\bar{x}) - K(\bar{x}) \right) + O(\alpha_s^2) \right\}$$

note:
- same form as p. 69 (top), except for finite piece
- μ-dependence has cancelled
- for $\mu_{(F)}$-evolution of pdf, DGLAP eqn is valid
- f_q also depends on <u>choice</u> of K, "scheme dependence"; factorization theorem proves that
  - for any physical quantity, all K and μ_F-dependence cancels
  - scheme- and scale-dependent pdf's $f_q(x, \mu^2)$ are <u>universal</u> (i.e. process-independent)
- common choices: $\overline{MS}$ scheme ($K(\bar{x}) \equiv 0$)

  DIS scheme ($K(\bar{x}) \equiv \bar{R}(\bar{x})$)

note: • dependence on scheme and scale cancels in physical quantities after calculating infinitely many orders in pert. theory ...

→ at finite order: residual dependence

→ need a procedure to choose value for μ_F

n^{th} order of pert. theory contains terms $\sim \alpha_s^n \ln^m \frac{Q^2}{\mu_F^2}$ ($m \leq n$)

so for reasons of convergence, take μ_F "not too far from" Q^2.

5.3.4 DIS @ NLO: $eg \to eq\bar{q}$

→ have so far not looked at process (3), see pg. 65

• most of §5.3.1–3 carries over

again have a collinear singularity, coming from internal quark going "onshell".

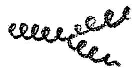
• singularity can again be absorbed into factorized universal gluon-pdf f_g

$$\Rightarrow F_2(x,Q^2) \ni \sum_q Q_q^2 \int_x^1 d\bar{x}\, f_g\left(\frac{x}{\bar{x}},\mu_F^2\right) \frac{x}{\bar{x}} \left\{ \frac{\alpha_s}{2\pi}\left(P_{qg}(\bar{x}) \ln\frac{Q^2}{\mu_F^2} + R_g(\bar{x}) - K_{qg}(\bar{x})\right) + O(\alpha_s^2)\right\}$$

with splitting fct $P_{qg}(x) = \frac{1}{2}\left[x^2 + (1-x)^2\right]$ ((so far, had $P = P_{qq}$))

• in general, DGLAP eqn is a set of coupled eqs

$$\mu^2 \partial_{\mu^2} f_a(x,\mu^2) = \sum_b \frac{\alpha_s}{2\pi} \int_x^1 \frac{d\bar{x}}{\bar{x}} f_b\left(\frac{x}{\bar{x}},\mu^2\right) P_{ab}(\bar{x}) + O(\alpha_s^2)$$

(at higher orders, also  → $P_{gg}(x)$ contributes)

((systematics of labelling the splitting fcts: 

internal line $\sim \frac{1}{1-z}$))

• Q^2-dependence of $F_2(x,Q^2)$ is entirely driven by μ_F^2-dependence of pdf's, which is predicted by DGLAP evolution eqs;

⇒ structure fct data over a wide range of Q^2 provide a stringent test of perturbative QCD → Figure 4