• from cross section $\frac{d^2\sigma(e+p)}{dQ^2\,dx} \overset{(pg. 62)}{=} \sum_q \int_0^1 d\eta\, f_q(\eta)\, \frac{d^2\sigma(e+q(\eta p))}{dQ^2\,dx}$

$\overset{(pg. 61)}{=} \frac{2\pi\alpha^2}{xQ^4}\left\{\left[1+(1-y)^2\right]F_2(x,Q^2) - y^2 F_L(x,Q^2)\right\}$

read off $F_2(x,Q^2) = \frac{C_F\alpha_s}{2\pi}\sum_q\int_0^1 d\eta\, f_q(\eta)\, Q_q^2\, \underbrace{\boxed{\frac{xQ^4}{2\eta x^2 s^2 y^2}}}_{=\frac{x}{2\eta}} \int_0^1 dz\, \hat f_2\left(\frac{x}{\eta},z\right)$

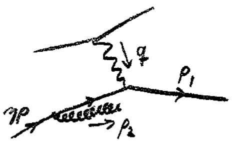
$\left(\bar x \equiv \frac{x}{\eta}\right)$ $= \frac{C_F\alpha_s}{2\pi}\sum_q \frac{1}{2}\int_x^\infty d\bar x\, f_q\left(\frac{x}{\bar x}\right)\frac{x}{\bar x}\,Q_q^2\int_0^1 dz\left(\frac{1+\bar x^2}{1-\bar x}\,\frac{1+z^2}{1-z}+3-z-\bar x+11\bar x z\right)$

$\left(\frac{d\eta}{\eta} = -\frac{d\bar x}{\bar x}\right)$

• consider the divergence in F_2

comes from $z \to 1$, where $z = \frac{p_1\cdot p}{q\cdot p}$

→ outgoing gluon collinear with incoming quark :



$\eta p\cdot p_2 = \eta p\cdot(\eta p+q-p_1) = \cancel{\eta^2 p^2} + \eta p\cdot q - \eta p\cdot p_1 = \eta p\cdot q\,(1-z)$

→ internal line becomes on-shell, causing the divergence:

$(\eta p - p_2)^2 = \cancel{\eta^2 p^2} - 2\eta p\cdot p_2 + \cancel{p_2^2}^{\,0} = -2\eta p\cdot q\,(1-z)$, quark propagator $\sim \frac{1}{1-z}$

Note also: coefficient of divergence $\sim \frac{1}{1-\bar x}$, diverges at $\bar x = 1$,
when gluon is infinitely soft

• regulate divergence

consider transverse momentum $k_\perp$ of outgoing quark in CMS system of $(\eta p + q)$;
((it turns out that $k_\perp^2 = Q^2\left(\frac{\eta}{x}-1\right)z(1-z)$))

$z\to 1$ means $k_\perp^2 \to 0$, so restricting $k_\perp^2 > \mu^2$ (with $\mu^2 \ll Q^2$)

regulates the divergence at $z \to 1$ ($\mu \to 0$ gives full result)

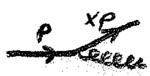
⇝ $\int_0^1 dz \to \int_{z_-}^{z_+} dz$, where $z_{+/-} = \frac{1}{2}\left(1 \pm \sqrt{1-4\frac{\mu^2}{Q^2\left(\frac{\eta}{x}-1\right)}}\right)$; $\int_0^1 dz \approx \int_0^{1-\mu^2/Q^2} dz$

⇒ $F_2(x,Q^2) = \frac{\alpha_s}{2\pi}\sum_q \frac{1}{2}\int_x^\infty d\bar x\, f_q\left(\frac{x}{\bar x}\right)\frac{x}{\bar x}\,Q_q^2\left(2\hat P(\bar x)\ln\frac{Q^2}{\mu^2} + R(\bar x)\right)$

↑ divergence ↑ finite; $\mu\to 0$ done

with "(unregularized) splitting function" $\hat P(x) \equiv C_F\frac{1+x^2}{1-x}$

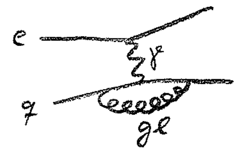
describes probability distribution of outgoing quark in



note: have not (yet) removed the divergence,
only regulated it.

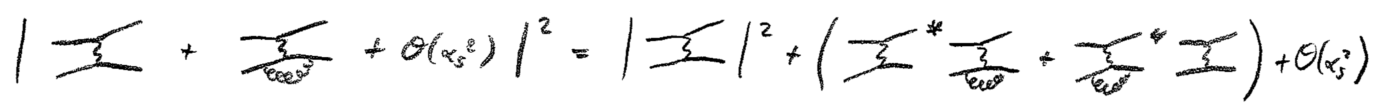
5.3.2 DIS @ NLO: 1-loop $eq \to eq$

diagram:



$\sim \alpha_s$

in $|\mathcal{M}|^2$, only need interference term with tree level diagram



LO   NLO, virtual correction

can again take everything from $e^+e^- \to q\bar{q}$ (§4.3.2) via crossing

→ here, recall only some of the features of that calculation, to illustrate the physics; interested in divergences.

- external particles same as at LO ⇒ kinematics is the same
  → $\int d\Phi \sim \delta(y-x)$ (see §5.2, pg. 63)
- as in $e^+e^- \to q\bar{q}$, interference term is divergent (and negative); divergence comes from same kinematic region as in §5.3.1: if gluon is soft, or collinear with either of the quarks.
- result: same form as $F_2(x, Q^2)$ above, with (unregularized) splitting fct $\hat{P}(\bar{x})$ replaced by $\underbrace{\tilde{P}(\bar{x})}_{\sim\,\delta(1-\bar{x}) \text{ from } \delta(y-x)}$ ((and different $R(\bar{x})$))

⇒ real + virtual $\sim P(x) \equiv \hat{P}(x) + \tilde{P}(x)$

[mathematical trick: plus-distribution

given $f(x)$, well-defined for $0 \le x < 1$

define distribution $f(x)_+$ on $0 \le x \le 1$ as:

$$f(x)_+ \equiv f(x) - \delta(1-x)\int_0^1 dx' f(x')$$

((most useful for $f(x)$ which diverge at $x \to 1$))

⇒ for any test function $g(x)$ which is smooth at $x=1$,

$$\int_0^1 dx\, f(x)_+\, g(x) = \int_0^1 dx\, f(x)\left[g(x) - g(1)\right]$$

((the latter integral being finite if $g(x) \to g(1)$ sufficiently quickly))]

- splitting fct $P(x) = C_F \left[\frac{1+x^2}{(1-x)_+} + \frac{3}{2}\delta(1-x) \right]$

  note: – inserting $P(x)$ into $F_2(x,Q^2)$, the divergence at $\bar{x}\to 1$ cancels; but the divergence at $z\to 1$, parametrized by $\ln\frac{Q^2}{\mu^2}$, remains.

  – in fact, $P(x)$ is the first correction to a fct that describes the momentum distribution of quarks within quarks,

  $$\mathcal{P}(x) \equiv \delta(1-x) + \frac{\alpha_s}{2\pi}\ln\frac{Q^2}{Q_0^2}\,P(x) + \mathcal{O}\left(\alpha_s^2 \ln^2\left(\frac{Q^2}{Q_0^2}\right)\right)$$

  ((see pg. 66: $\xrightarrow{p} \xrightarrow{xp}$ etc ; "pure quark" at scale Q_0^2))

5.3.3 Factorization, evolution

→ understand why results are still divergent!

compare to $e^+e^-\to q\bar{q}$ case: there, real + virtual = finite

here, main difference is: introduced pdf's f_q

- $z\to 1$ divergence comes from internal quark propagator $\sim \frac{1}{1-z}$ (see pg. 66);
  roughly, vanishingly small virtuality $\hat{=}$ arbitrarily long time scales (uncertainty princ.)
  → contradiction to assumptions of parton model: rapid snapshot of proton
- the actual problem is <u>overcounting</u>:
  pdf's $\hat{=}$ internal proton structure $\hat{=}$ result of QCD interactions
  our calculation: QCD corrections to q scattering
  integrated over all final states (hence all E scales)
  → but are these QCD corr. already somehow in the pdf's?

⇒ to resolve overcounting: separate ("factorize") different types of physics at different energy scales!

introduce <u>factorization scale</u> μ.

physics at scales $<\mu$ ≡ part of hadron wave fct ≡ pdf's

$>\mu$ ≡ part of partonic scattering cross section ≡ coefficient fcts

note: the cutoff introduced in § 5.3.2, $eq\to eqg$, was hence correct.