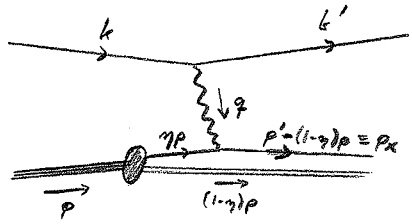
- to obtain the parton model prediction for structure fcts, need to calculate partonic cross section.
  → need matrix el. for $eq \to eq$
  get by "crossing symmetry" from $e^+e^- \to q\bar{q}$



$$\langle |\mathcal{M}|^2 \rangle = \frac{1}{4}\frac{1}{N_c} N_c \sum_s \left| \quad \right|^2$$

↑ from sum over outgoing color
from average over incoming color

(see pg. 44, $\mu \to q$, $m=0$)

$$= \frac{8 e^2 (Q_q e)^2}{(q^2)^2} \left[(\eta p \cdot k)^2 + \left((p' - (1-\eta)p) \cdot k \right)^2 \right]$$

convert to our kinematic invariants

$Q^2 = -q^2$, $\quad s = (p+k)^2 = 2p\cdot k + \cancel{m_p^2} + \cancel{m_e^2}$, $\quad p'k = (p+q)\cdot k = \frac{s}{2} + q\cdot k$

$q\cdot k = (k-k')\cdot k = \frac{1}{2}(k-k')^2 + \cancel{\frac{1}{2}m_e^2} - \cancel{\frac{1}{2}m_e^2} = -Q^2/2$

$$\Rightarrow [\ldots] = \left[\left(\frac{\eta s}{2}\right)^2 + \left(\cancel{\frac{s}{2}} - \frac{Q^2}{2} - (\cancel{1}-\eta)\frac{s}{2} \right)^2 \right] = \left(\frac{\eta s}{2}\right)^2 \left[1 + \left(1 - \frac{Q^2}{\eta s}\right)^2 \right]$$

$$= 8(4\pi\alpha)^2 Q_q^2 \; \frac{1 + \left(1 - \frac{Q^2}{\eta s}\right)^2}{4\left(\frac{Q^2}{\eta s}\right)^2}$$

→ phase space integration, see pg. 60

$$\int d\Phi_{X+1} = \underbrace{\int dQ^2 dx \frac{Q^2}{16\pi^2 s x^2}}_{\text{electron kinematics}} \int d\Phi_X \qquad \text{here: } X \text{ consists of } \underline{\text{one}} \text{ massless parton}$$

$$\int d\Phi_X = \int \frac{d^4 p_X}{(2\pi)^3} \delta(p_X^2) (2\pi)^4 \delta^4(\eta p + q - p_X)$$

$$= (2\pi)\,\delta\left((\eta p + q)^2\right) = (2\pi)\,\delta\left(\cancel{\eta^2 m_p^2} + 2\eta p q + q^2\right)$$

$$= (2\pi)\frac{1}{|2pq|}\delta\left(\eta - \frac{Q^2}{2pq}\right) = (2\pi)\frac{x}{Q^2}\delta(\eta - x)$$

→ partonic cross section

$$\sigma(e + q(\eta p)) = \frac{1}{2\eta s}\int d\Phi_{1+1} \langle |\mathcal{M}|^2 \rangle$$

$$\frac{d^2\sigma(e+q(\eta p))}{dQ^2 dx} = \frac{1}{2\eta s}\frac{Q^2}{16\pi^2 s x^2}\frac{2\pi x}{Q^2}\delta(\eta - x)\langle|\mathcal{M}|^2\rangle \overset{y = \frac{Q^2}{xs}}{=} \frac{y^2}{16\pi Q^4}\delta(\eta - x)\, 2(4\pi\alpha)^2 Q_q^2 \frac{1 + (1-y)^2}{y^2}$$

$$= \frac{2\pi\alpha^2 Q_q^2}{Q^4}\delta(\eta - x)\left[1 + (1-y)^2\right]$$

- finally get cross section for e+p (see pg. 62)

$$\frac{d\sigma(e+p(p))}{dQ^2\,dx} = \sum_q \int_0^1 dy\, f_q(y)\cdot \frac{2\pi\alpha^2 Q_q^2}{Q^4}\,\delta(y-x)\left[1+(1-y)^2\right]$$

$$= \frac{2\pi\alpha^2}{xQ^4}\left[1+(1-y)^2\right]\sum_q Q_q^2\, x\, f_q(x)$$

→ comparing with §5.1 ((result in terms of structure fcts F_2, F_L (pg. 61)))

$$\Rightarrow\quad F_2(x,Q^2) = \sum_q Q_q^2\, x\, f_q(x)\quad,\qquad F_L(x,Q^2)=0$$

note: F_2 is Q^2-independent: Bjorken scaling!
$F_L = 0$ was the Callan-Gross relation.

→ we will see that QCD corrections do violate Bjorken scaling; in experimental data, however, it is satisfied pretty well → Figure 1

- in practice, measure F_2 from different data sets and extract the f_q's.

$$F_2^{ep} = x\left[\tfrac{1}{9}(f_d+f_{\bar d}+f_s+f_{\bar s}) + \tfrac{4}{9}(f_u+f_{\bar u}+f_c+f_{\bar c})\right]$$

since the pdf's contribute differently in different experiments
(e.g. $F_2^{en}, F_2^{\nu p}, F_2^{\bar\nu p}, \ldots$)
can do global fits to extract them. Typical results → Figure 2

→ useful checks via <u>sum rules</u>: $\int_0^1 dx\, f_{u_v}(x) = 2\ ,\ \int_0^1 dx\, f_{d_v}(x) = 1$, etc.

5.3 QCD corrections in DIS

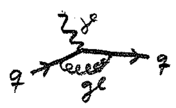
→ α_s is not small, so our above LO treatment of DIS might get important corrections.

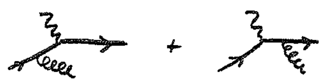
→ how does the parton model emerge from QCD?

→ structure fcts will (slowly: logarithmically) depend on Q^2, leading to violation of Bjorken scaling

→ have to compute NLO corrections to DIS;
divergences, splitting fcts, factorization, (DGLAP) evolution eqs, data

- three sources of NLO corrections

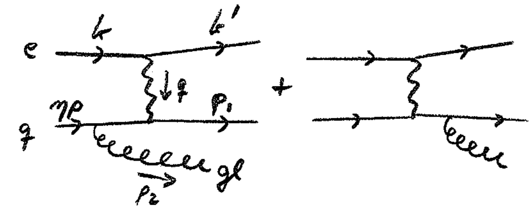
(1) $eq \to eq$ at 1-loop ("virtual corr.") 

(2) $eq \to eqg$ at tree-level 

(3) $eg \to eq\bar{q}$ at tree-level  ← completely new structure, not present in parton model

5.3.1 DIS @ NLO : $eq \to eqg$

- 2 diagrams, get $\langle |\mathcal{M}|^2 \rangle$ by "crossing" from $e^+e^- \to q\bar{q}g$ (see §4.3.1)



$$\langle |\mathcal{M}|^2 \rangle = \frac{1}{4}\frac{1}{N_c} N_c \frac{8 C_F e^4 Q_q^2 g_s^2}{(kk')(p_1 p_2)(\eta p \cdot p_2)} \left[(p_1 \cdot k)^2 + (\eta p \cdot k)^2 + (p_1 \cdot k')^2 + (\eta p \cdot k')^2 \right]$$

→ phase space $\displaystyle \int d\Phi_3 = \underbrace{\int dQ^2 dx \frac{Q^2}{16\pi^2 s x^2}}_{\text{electron}} \int d\Phi_X$ here: X consists of 2 partons → non-triv.

$$\int d\Phi_X = \int_0^{2\pi} d\varphi \int_{-1}^{1} d(\cos\theta) \frac{1}{32\pi^2} = \int_0^{2\pi} d\varphi \int_0^1 dz \frac{1}{16\pi^2}$$

where (φ, θ) refer to direction of p_1 in CMS system of $\eta p + q$

alternatively, use Lorentz-invar. variable $z \equiv \frac{p_1 \cdot p}{q \cdot p} = \frac{1}{2}(1 - \cos\theta)$

→ partonic cross section

$$\frac{d^2\sigma(e+q)}{dQ^2 dx} = \frac{1}{2\eta s} \frac{Q^2}{16\pi^2 s x^2} \int_0^{2\pi} d\varphi \int_0^1 dz \frac{1}{16\pi^2} \frac{2 C_F e^4 Q_q^2 g_s^2}{(kk')(p_1 p_2)(\eta p \cdot p_2)} \left[\text{see} \uparrow \right]$$

$$= \frac{Q^2 C_F \alpha^2 Q_q^2 \alpha_s}{2\eta x^2 s^2} \int_0^1 dz \frac{1}{2\pi} \int_0^{2\pi} d\varphi \frac{[\ \dots\ ]}{(kk')(p_1 p_2)(\eta p \cdot p_2)}$$

rewrite scalar products in terms of kinematic variables, perform φ-integration, use $\bar{x} \equiv x/\eta$

$$= \frac{Q^2 C_F \alpha^2 Q_q^2 \alpha_s}{2\eta x^2 s^2} \int_0^1 dz \frac{1}{y^2 Q^2} \Big\{ \left[1 + (1-y)^2\right] \underbrace{\left(\frac{1+\bar{x}^2}{1-\bar{x}} \frac{1+z^2}{1-z} + 3 - z - \bar{x} + 11\bar{x}z \right)}_{\equiv \hat{F}_2(\bar{x}, z)} - y^2 (8\bar{x}z) \Big\}$$

will give a non-zero F_L

will give divergence in F_2 due to $\int dz$