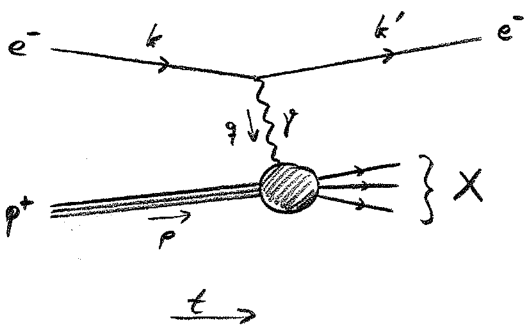
5. Deep inelastic scattering (DIS)

We now (temporarily) go back to tree-level phenomenology.

Q.: are quarks real physical constituents of hadrons, or just a mathematical convenience for describing the hadron's wavefct?

$\rightsquigarrow$ DIS gives information on internal proton structure

$\rightsquigarrow$ structure functions, Bjorken scaling, parton distribution fcts



<u>photon virtuality</u> $Q^2 = -q^2$ (($Q^2 \sim \frac{1}{\lambda^2}$))

controls resolving power of photon

$Q^2 \ll \frac{1}{R^2}$ $\to$ elastic e-p scattering

R ← proton radius

$Q^2 \gg \frac{1}{R^2}$ $\to$ resolve p constituents, elastic e-q scattering

we are interested in <u>deep</u> ($Q^2 \gg M_p^2$) <u>inelastic</u> ($(p+q)^2 \gg M_p^2$) scattering

5.1 Structure functions

- want to describe process with Lorentz-invariant variables

  center-of-mass energy $s \equiv (k+p)^2$

  at fixed s, scattered e^- has 2 non-trivial variables (E, θ);

  use $Q^2 \equiv -q^2$, $x \equiv \frac{Q^2}{2p\cdot q}$

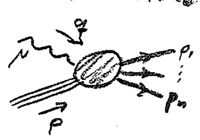
  ((other choices: $W^2 \equiv (p+q)^2 = Q^2 \frac{1-x}{x}$ (invariant mass of γp-system)

  $y \equiv \frac{p\cdot q}{p\cdot k} = \frac{Q^2}{xs}$))

  kinematic limits: $Q^2 < s$, $x > Q^2/s$

  ((of course $\underbrace{k^2 = m_e^2}_{\approx 0}$, $\underbrace{p^2 = M_p^2}_{\approx 0}$; for $Q^2 \gtrsim M_Z^2$, also Z^0 exchange))

- know nothing about detailed structure of proton

$\rightarrow$ parametrize $\mathcal{M} =$  $= e\, T_\mu(p, q; \{p_X\})$

$$\Rightarrow \frac{1}{4}\left|\right.$$  $$\left.\right|^2 = \frac{1}{4}\frac{e^4}{Q^4}\, \underbrace{tr(\not k \gamma^\mu \not k' \gamma^\nu)}_{\equiv L^{\mu\nu} = 4(k^\mu k'^\nu + k^\nu k'^\mu - k\cdot k' g^{\mu\nu})}\, T_\mu(p,q,\{p_X\})\, T_\nu^*(p,q,\{p_X\})$$

(see pg. 49)

in complete analogy to $e^+e^- \to \mu^+\mu^-, q\bar{q}$ etc.

- for total cross section, need to integrate over phase space

$$\int d\Phi_{X+1} = \underbrace{\int dQ^2\, dx\, \frac{Q^2}{16\pi^2 s x^2}}_{\text{electron kinematics}} \ \underbrace{\int d\Phi_X}_{n\text{-body phase space for } X}$$

for an inclusive process (don't measure X $\rightarrow$ sum over them),

$$\frac{d^2\sigma}{dQ^2\,dx} = \frac{1}{2s}\,\frac{Q^2}{16\pi^2 s x^2}\,\underbrace{\sum_X \int d\Phi_X\ \frac{1}{4}|\mathcal{M}|^2}$$

$$\equiv \frac{1}{4}\frac{e^4}{Q^4}\, L^{\mu\nu}\ \underbrace{\sum_X \int d\Phi_X\ T_\mu(p,q,\{p_X\})\, T_\nu^*(p,q,\{p_X\})}_{\equiv H_{\mu\nu}}$$

- consider $H_{\mu\nu}$: have summed and integrated all X dependence, so $H_{\mu\nu}(p,q)$, must be symm. in μ,ν ((parity cons. in QED, QCD))

$$\Rightarrow H_{\mu\nu} = -H_1 g_{\mu\nu} + H_2 \frac{p_\mu p_\nu}{Q^2} + H_3 \frac{q_\mu q_\nu}{Q^2} + H_4 \frac{p_\mu q_\nu + q_\mu p_\nu}{Q^2}$$

where H_i are scalar fcts, $H_i(q^2 = -Q^2,\ p\cdot q = \frac{Q^2}{2x},\ \cancel{p^2 = M_p^2})$

neglect in DIS

((including Z^0 exchange, ... $+ H_5\, \varepsilon_{\mu\nu\rho\sigma} \frac{p_\rho q_\sigma}{Q^2}$))

(check ?!)

$$\Rightarrow L^{\mu\nu}H_{\mu\nu} = 8(kk')H_1 + 8\frac{(pk)(pk')}{Q^2}H_2 + 0\cdot H_3 + 0\cdot H_4$$

used $d=4$, neglected M_p^2

now $Q^2 = -q^2 = -(k-k')^2 = 2kk' - \cancel{2m_e^2}$

$s = (p+k)^2 = 2pk + \cancel{M_p^2} + \cancel{m_e^2}$

$pk' = p(k-q) = pk\left(1 - \frac{pq}{pk}\right) = pk(1-y)$

$$= 4Q^2 H_1 + 2\frac{s^2}{Q^2}(1-y)H_2$$

- $$\frac{d^2\sigma}{dQ^2\,dx} = \frac{1}{2s}\,\frac{Q^2}{16\pi^2 s x^2}\,\frac{1}{4}\,\frac{\alpha^2 16\pi^2}{Q^4}\left(4Q^2 H_1 + 2\,\frac{s^2}{Q^2}(1-y)H_2\right) \quad ; \text{ def } \quad \begin{matrix} H_1 = 8\pi F_1 \\ H_2 = 16\pi x F_2 \end{matrix}$$
$$\equiv \frac{4\pi\alpha^2}{xQ^4}\left\{ xy^2 F_1(x,Q^2) + (1-y)\,F_2(x,Q^2)\right\}$$

((recall $y = Q^2/xs$))

→ amazing: w/o knowing ep interaction, derived s-dependence of σ !

→ the F's are called "~~structure functions~~" of the proton

sometimes, see $\quad F_T(x,Q^2) = 2x F_1(x,Q^2)$ "transverse"
$\quad F_L(x,Q^2) = F_2(x,Q^2) - 2xF_1(x,Q^2)$ "longitudinal"

so $$\frac{d^2\sigma}{d^2Q\,dx} = \frac{2\pi\alpha^2}{xQ^4}\left\{\left(1+(1-y)^2\right)F_T(x,Q^2) + 2(1-y)F_L(x,Q^2)\right\}$$
$$= \frac{2\pi\alpha^2}{xQ^4}\left\{\left(1+(1-y)^2\right)F_2(x,Q^2) - y^2 F_L(x,Q^2)\right\}$$

useful since (for most current data) $y^2 \ll 1$

- have isolated all non-trivial x, Q^2 dependence into F_2, F_L ;
but still don't know anything about these fcts.
Assumption: interaction of γ with innards of proton does not involve any dimensionful scale
$\Rightarrow$ dim-less F's can not depend on dimensionful parameter Q^2

$$\Rightarrow \frac{d^2\sigma}{dQ^2\,dx} = \frac{2\pi\alpha^2}{xQ^4}\left\{\left(1+(1-y)^2\right)F_2(x) - y^2 F_L(x)\right\} \qquad \text{Bjorken scaling}$$

→ experimentally, this is true (to a pretty good approximation);
but proton consists of quarks, bound at distance scale $\sim 1/M_p$,
so how can the interaction possibly be M_p-indep. ?!
⇝ answer via parton model

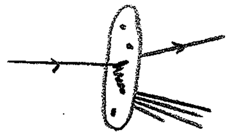
5.2 Parton distribution functions

- description of process in Lorentz-invariant;
  <u>parton model</u> most easily formulated in "<u>Breit-frame</u>": $E_\gamma = 0$, $E_p = \frac{Q}{2x}$

  proton in its rest frame: $\bigcirc \updownarrow 2R$ $\rightsquigarrow$ Breit frame: $\updownarrow 2R$ L-contraction

  $\leftrightarrow \frac{4Rx M_p}{Q} \ll 2R$

  DIS in Breit frame:



transverse size of photon $\sim \frac{1}{Q} \ll 2R$
$\Rightarrow$ photon interacts with tiny fraction of disk
$\to$ if quarks sufficiently dilute in q, photon does <u>not</u> resolve q interactions; incoherent γq collisions!

$\to$ since quarks act as if they don't interact, their interaction does <u>not</u> introduce a dimensionful scale $\Rightarrow$ Bjorken scaling

- more precisely:
  proton $\equiv$ bundle of comoving partons, carrying the proton's momentum p.
  parton of type q carries fraction between $(\eta, \eta + d\eta)$ of p
  during a fraction $d\eta \cdot \underbrace{f_q(\eta)}$ of the time
  pdf, probability / <u>parton distrib. fct.</u>

  assume partons pointlike ($r^2 \ll 1/Q^2$) and dilute ($f_q(\eta) \ll Q^2 R^2$)
  $\to$ incoherent γ-parton-scattering
  with
  $$\frac{d^2\sigma(e+p(p))}{dQ^2\,dx} = \sum_q \int_0^1 d\eta\; f_q(\eta)\; \underbrace{\frac{d^2\sigma(e+q(\eta p))}{dQ^2\,dx}}_{\text{partonic cross section}}$$

note: In partonic cross section, elastic scattering
$\to$ outgoing parton is on mass-shell.
$2\to 2$ scattering $\to$ only 1 non-triv. kinematical variable (see pg. 45),
so $\frac{d^2\sigma}{dQ^2 dx} \sim \delta$, where δ fct fixes one of the variables x, Q^2.
For massless partons, $(q+\eta p)^2 = 2\eta(pq) - Q^2 \overset{!}{=} 0 \Leftrightarrow \eta = x$

(partons = scalars $\Rightarrow F_T = 0$)

note: partons = quarks = fermions $\Rightarrow$ (helicity cons.) $F_L = 0$ <u>Callan-Gross relation</u>