3.3 one-loop counterterms in QCD

$\rightsquigarrow$ now, renormalize the theory.

use freedom of redefining fields, parameters/couplings

schematically: $\phi_B = \sqrt{Z_\phi}\,\phi_R \quad , \quad \phi \in \{\psi, A, c\}$ $\qquad B \equiv$ "bare"

$\lambda_B = Z_\lambda \lambda_R \quad , \quad \lambda \in \{m, g, \xi\}$ $\qquad R \equiv$ "renormalized"

where the multiplicative renormalization factors Z_i depend on the renormalized parameters (and the dimension d), and are taken to be dimension<u>less</u>, $Z_i = 1 + \delta Z_i$, $\delta Z_i \sim g^2$ (see below)

- recall (pg. 28)

$$\mathcal{L}_B = \bar\psi_B\,(i\gamma^\mu \underbrace{D_\mu}_{\partial_\mu - i g_B A^a_{B\mu} T^a} - m_B)\,\psi_B - \frac14 \underbrace{F^a_{\mu\nu}}_{\partial_\mu A^a_\nu - \partial_\nu A^a_\mu + g f^{abc} A^b_\mu A^c_\nu} F^{a\mu\nu} - \frac{1}{2\xi}(\partial^\mu A^a_\mu)^2 + \bar c^a(-\partial^\mu \underbrace{D^{ac}_\mu}_{\partial_\mu \delta^{ac} + g f^{abc} A^b_\mu})c^c$$

(in this line, all ψ, A, c, m, g, ξ should have an index $\underline{B}$)

$$= \overbrace{\bar\psi i\not\partial\psi}^{Z_\psi} - \overbrace{m\bar\psi\psi}^{Z_m Z_\psi} + \overbrace{g\bar\psi\gamma^\mu A^a_\mu T^a\psi}^{Z_g Z_\psi Z_A^{1/2}}$$

$$- \overbrace{\tfrac14(\partial_\mu A^a_\nu - \partial_\nu A^a_\mu)^2}^{Z_A} - \overbrace{\tfrac{1}{2\xi}(\partial^\mu A^a_\mu)^2}^{Z_A Z_\xi^{-1}} - \overbrace{\tfrac12 g f^{abc} A^b_\mu A^c_\nu(\partial^\mu A^{a\nu} - \partial^\nu A^{a\mu})}^{Z_g Z_A^{3/2}}$$

$$- \overbrace{\tfrac14 g^2 f^{abc} A^b_\mu A^c_\nu f^{ade} A^{d\mu} A^{e\nu}}^{Z_g^2 Z_A^2} - \overbrace{\bar c^a \partial^\mu\partial_\mu c^a}^{Z_c} - \overbrace{g f^{abc}\bar c^a \partial^\mu A^b_\mu c^c}^{Z_g Z_c Z_A^{1/2}}$$

(here, without Z's : index B ; <u>with</u> Z's : index R for all ψ, A, c, m, g, ξ)

$$= \mathcal{L}_R + \underbrace{\mathcal{L}^{c.t.}}_{} \longleftarrow \text{counterterms}$$

$$\mathcal{L}^{c.t.} = (Z_\psi - 1)\bar\psi\left(i\not\partial - \frac{Z_m Z_\psi - 1}{Z_\psi - 1}m\right)\psi + (Z_g Z_\psi Z_A^{1/2} - 1)$$

$$- (Z_A - 1)\left[\tfrac14(\partial_\mu A^a_\nu - \partial_\nu A^a_\mu)^2 + \frac{Z_A Z_\xi^{-1} - 1}{Z_A - 1}\,\frac{1}{2\xi}(\partial^\mu A^a_\mu)^2\right] + (Z_g Z_A^{3/2} - 1)$$

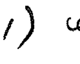
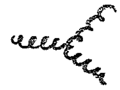
$$+ (Z_g^2 Z_A^2 - 1) \quad - (Z_c - 1)\,\bar c\,\partial^\mu\partial_\mu c \quad + (Z_g Z_c Z_A^{1/2} - 1)$$

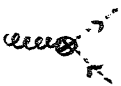
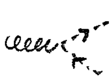
(now, all indices are $\underline{R}$, and omitted)

→ treat $\mathcal{L}^{c.t.}$ as additional interactions.

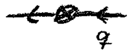
→ get additional Feynman rules

Vertices are easy (have the same form as before),

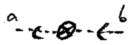
 $= \left(Z_g Z_\psi Z_A^{\frac{1}{2}} - 1\right)$  ,  $= \left(Z_g Z_A^{\frac{3}{2}} - 1\right)$ 

 $= \left(Z_g^2 Z_A^2 - 1\right)$  ,  $= \left(Z_g Z_c Z_A^{\frac{1}{2}} - 1\right)$ 

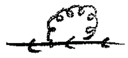
and there are also "two-point-vertices" now:

 $= i\left[(Z_\psi - 1)\not{q} - (Z_m Z_\psi - 1)m\right]$

 $= -i\delta^{ab}\left[(Z_A - 1)(g^{\mu\nu}q^2 - q^\mu q^\nu) + \underbrace{(Z_A Z_\xi^{-1} - 1)}_{=0,\text{ see pg. 39}}\frac{1}{\xi}q^\mu q^\nu\right]$

 $= i\delta^{ab}(Z_c - 1)q^2$

- from our explicit results for 1-loop divergences in §3.1, §3.2, we can now fix the yet-unknown constants Z!

- finite $\overset{!}{=}$  +  $+ \mathcal{O}(g^4)$

(pg. 35) $= i\frac{g^2}{16\pi^2}\frac{1}{\varepsilon}\frac{N_c^2-1}{2N_c}\left(\not{q}\,\xi - (\xi+3)m\right) + \mathcal{O}(\varepsilon^0) + i\left[(Z_\psi - 1)\not{q} - (Z_m Z_\psi - 1)m\right] + \mathcal{O}(g^4)$

$$\Rightarrow \underline{\underline{Z_\psi \overset{!}{=} 1 - \frac{g^2}{16\pi^2}\left(\frac{1}{\varepsilon}\frac{N_c^2-1}{2N_c}\xi\right.}} \left. + \mathcal{O}(\varepsilon^0)\right) + \mathcal{O}(g^4)$$

what one puts here is a matter of choice.

often used: "minimal subtraction" (MS) scheme : put 0 here

many other schemes possible,

e.g. modified MS ($\hat{=}\ \overline{MS}$), see below

$$\Rightarrow Z_m Z_\psi \overset{!}{=} 1 - \frac{g^2}{16\pi^2}\frac{1}{\varepsilon}\frac{N_c^2-1}{2N_c}(3+\xi) + \mathcal{O}(g^4)$$

$$\Leftrightarrow \underline{\underline{Z_m = 1 - \frac{g^2}{16\pi^2}\frac{1}{\varepsilon}\frac{N_c^2-1}{2N_c}\,3}} + \mathcal{O}(g^4) \quad \text{in MS scheme}$$

note: ξ-independent

- finite $\overset{!}{=}$  $+\ \mathcal{O}(g^4)$

(pg. 34)

$$= i\,\frac{g^2}{16\pi^2}\,\frac{1}{\varepsilon}\,\delta^{ab}\left(g^{\mu\nu}q^2 - q^\mu q^\nu\right)\left(\left(\tfrac{13}{6}-\tfrac{\xi}{2}\right)N_c - \tfrac{2}{3}N_f\right) + \mathcal{O}(\varepsilon^0)$$

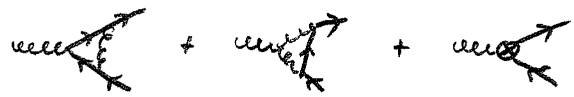
$$-i\,\delta^{ab}\left[(Z_A-1)\left(g^{\mu\nu}q^2 - q^\mu q^\nu\right) + \left(Z_A Z_\xi^{-1}-1\right)\frac{1}{\xi}q^\mu q^\nu\right] + \mathcal{O}(g^4)$$

$$\Rightarrow \underline{Z_A \overset{!}{=} 1 + \frac{g^2}{16\pi^2}\,\frac{1}{\varepsilon}\left(\left(\tfrac{13}{6}-\tfrac{\xi}{2}\right)N_c - \tfrac{2}{3}N_f\right)} + \mathcal{O}(g^4)$$

$$\Rightarrow Z_A Z_\xi^{-1} \overset{!}{=} 1 + 0\cdot g^2 + \mathcal{O}(g^4)$$

$$\Leftrightarrow \underline{Z_\xi = Z_A} + \mathcal{O}(g^4)$$

$\rightarrow$ <u>note</u> actually, one can show that $Z_\xi = Z_A$ <u>exactly</u>, to all orders of g^2, due to gauge invariance: the BRST symmetry gives rise to the so-called Ward/Takahashi/Slavnov/Taylor-identities, one of which guarantees that the longitudinal ($q^\mu q^\nu$ piece) part of the gluon propagator does not get radiative corrections, $q^\mu\,\Pi^{ab}_{\mu\nu}(q) = 0$

- finite $\overset{!}{=}$  $+\ \mathcal{O}(g^5)$

(pg. 36)

$$= -ig\,T^a\gamma^\mu\,\frac{g^2}{16\pi^2}\,\frac{1}{\varepsilon}\,\frac{\xi - 3N_c^2\frac{\xi+1}{2}}{2N_c} + \mathcal{O}(\varepsilon^0) + \left(Z_g Z_\psi Z_A^{\frac{1}{2}}-1\right) ig\,T^a\gamma^\mu + \mathcal{O}(g^5)$$

$$\Rightarrow Z_g Z_\psi Z_A^{\frac{1}{2}} \overset{!}{=} 1 + \frac{g^2}{16\pi^2}\,\frac{1}{\varepsilon}\,\frac{\xi - 3N_c^2\frac{\xi+1}{2}}{2N_c} + \mathcal{O}(g^4)$$

$$\Leftrightarrow \underline{Z_g = 1 - \frac{g^2}{16\pi^2}\,\frac{1}{\varepsilon}\left(\tfrac{11}{6}N_c - \tfrac{1}{3}N_f\right)} + \mathcal{O}(g^4)$$

note: ξ-independent

- finite $\overset{!}{=}$ 

(pg. 36)

$$= -i\,\delta^{ab}q^2\,\frac{g^2}{16\pi^2}\,\frac{1}{\varepsilon}\,(3-\xi)\frac{N_c}{4} + \mathcal{O}(\varepsilon^0) + i\delta^{ab}q^2\,(Z_c - 1)$$

$$\Rightarrow \underline{Z_c \overset{!}{=} 1 - \frac{g^2}{16\pi^2}\,\frac{1}{\varepsilon}\,\frac{N_c}{4}(\xi-3)} + \mathcal{O}(g^4)$$

$\rightarrow$ get finite (one-loop) results after fixing Z's as above!