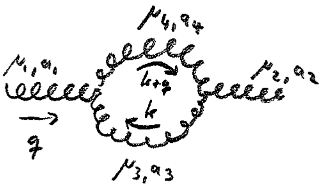
• 2^{nd} diagram : calculation parallels the one above !



symmetry factor — Feynman gauge — color indices a_i

$$= \frac{1}{2}\int\frac{d^dk}{(2\pi)^d}\left(\frac{-i}{k^2}\right)\left(\frac{-i}{(k+q)^2}\right)g^2 f^{134}f^{243}\times$$

Lorentz indices μ_i

$$\times\left[(q-k)^4 g^{13}+(2k+q)^1 g^{34}+(-k-2q)^3 g^{41}\right]\times$$

$$\times\left[(-k-2q)_3\, g^2{}_4+(2k+q)^2 g_{34}+(-k+q)_4\, g_3{}^2\right]$$

Feynman parameters ; shift $k\to k-xq$; $\Delta\equiv -x(1-x)q^2$;
in numerator, linear $k\to 0$, $k^\mu k^\nu \to \frac{g^{\mu\nu}k^2}{d}$

$$= \frac{1}{2}g^2 f^{134}f^{234}\int_0^1 dx\int\frac{d^dk}{(2\pi)^d}\frac{1}{(k^2-\Delta)^2}\times$$

$$\times\Big\{g^{12}\Big[k\cdot k\; 6\,\frac{d-1}{d}+q\cdot q\left((2-x)^2+(1+x)^2\right)\Big]$$

$$-q^1q^2\left[(2-d)(1-2x)^2+2(1+x)(2-x)\right]\Big\}$$

Wick rotate, use basic 1-loop tadpole integrals, $I_2^1=I_2^0\,\frac{d}{d-2}\,\Delta$

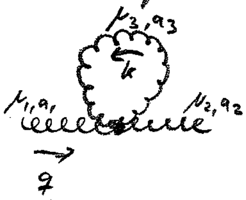
$$= i g^2 f^{a_1a_3a_4}f^{a_2a_3a_4}\int_0^1 dx\, I_2^0(-x(1-x)q^2)\;\frac{1}{2}\Big\{g^{\mu_1\mu_2}q^2\Big[6\,\frac{d-1}{2-d}\,x(1-x)+5-2x+2x^2\Big]$$

$$-q^{\mu_1}q^{\mu_2}\left[(2-d)(1-2x)^2+2(1+x)(2-x)\right]\Big\}$$

• 3^{rd} diagram: template for 

: $f^{12\bullet}f^{\bullet 34}(g_{13}g_{24}-g_{14}g_{23})+1324+1423$

$12\quad 33 \qquad\qquad 1323\quad 1323$ ← here



$$= \frac{1}{2}\int\frac{d^dk}{(2\pi)^d}\left(\frac{-i}{k^2}\right)(-ig^2)\Big\{f^{12\bullet}\cancel{f^{\bullet 33}}^{\,0}(..)+2f^{13\bullet}f^{\bullet 23}\underbrace{\left(g^{12}g^3{}_3-g^{13}g_3{}^2\right)}_{\overset{!}{=}\,g^{12}(d-1)}\Big\}$$

↑ symm. factor

$$= -g^2 f^{134}f^{234}g^{12}(d-1)\underbrace{\int\frac{d^dk}{(2\pi)^d}\frac{1}{k^2}}_{\text{would be zero}}\times\underbrace{\frac{k^2+2kq+q^2}{(k+q)^2}}_{\text{try to make it look like } 2^{\text{nd}} \text{ diag}}$$

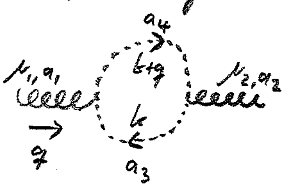
Fy par, shift $k\to k-xq$, $\Delta\equiv -x(1-x)q^2$, numerator $k=0$

$$= -g^2 f^{134}f^{234}g^{12}(d-1)\int_0^1 dx\int\frac{d^dk}{(2\pi)^d}\frac{k^2+(1-x)^2q^2}{(k^2-\Delta)^2}$$

Wick rotate , use basic 1-loop tadpole ints

$$= i g^2 f^{a_1a_3a_4}f^{a_2a_3a_4}\int_0^1 dx\, I_2^0(-x(1-x)q^2)\; g^{\mu_1\mu_2}q^2(1-d)\Big[\frac{d}{2-d}\,x(1-x)+(1-x)^2\Big]$$

• 4^{th} diagram



closed loop of anticomm. fields — color — Lorentz

$$= -\int \frac{d^dk}{(2\pi)^d}\,\frac{i}{k^2}\,\frac{i}{(k+q)^2}\,(-g)^2\, f^{413}\,(k+q)^1\, f^{324}\, k^2$$

$$= -g^2 f^{134} f^{234} \int \frac{d^dk}{(2\pi)^d}\, \frac{(k+q)^1 k^2}{k^2 (k+q)^2}$$

Fy par, shift $k \to k - xq$, $\Delta \equiv -x(1-x)q^2$, numerator $k=0$, $k^\mu k^\nu \to \frac{g^{\mu\nu}k^2}{d}$

$$= -g^2 f^{134} f^{234} \int_0^1 dx \int \frac{d^dk}{(2\pi)^d}\, \frac{1}{(k^2-\Delta)^2} \left\{ \frac{g^{12}k^2}{d} - x(1-x)\, q^1 q^2 \right\}$$

Wick rotate, use 1-loop tadpole integrals

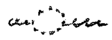
$$= i g^2 f^{a_1 a_3 a_4} f^{a_2 a_3 a_4} \int_0^1 dx\, I_2^0(-x(1-x)q^2)\; x(1-x) \left\{ -g^{\mu_1\mu_2} q^2 \frac{1}{2-d} + q^{\mu_1} q^{\mu_2} \right\}$$

• Sum diagrams 2+3+4

$$2+3+4 = i g^2 f^{134} f^{234} \int_0^1 dx\, I_2^0(-x(1-x)q^2)\, \times$$

$$\times \Big\{ g^{\mu_1\mu_2} q^2 \Big[3\frac{d-1}{2-d}x(1-x) + \frac{1}{2}(5-2x+2x^2) + (1-d)\frac{d}{2-d}x(1-x) + (1-d)(1-x)^2 - \frac{1}{2-d}x(1-x) \Big]$$

$$- q^{\mu_1} q^{\mu_2} \Big[\underbrace{\frac{2-d}{2}(1-2x)^2 + (1+x)(2-x)} \qquad\qquad\qquad \underbrace{- x(1-x)} \Big] \Big\}$$

from   

note that first line is invariant under $x \to 1-x$

$\{\dots\}$ is polynomial $\sim 1 + x + x^2$

re-express this in terms of $a \equiv 1-2x$ which is odd under $x \to 1-x$

((i.e. $x^2 = \frac{1}{4}(a^2-2a+1)$, $x = \frac{1}{2}(1-a)$))

$$= i g^2 f^{134} f^{234} \int_0^1 dx\, I_2^0(-x(1-x)q^2) \left\{ g^{12} q^2 \Big[(1-\tfrac{d}{2})a^2 + \underbrace{\cancel{\tfrac{1-d}{2}a}}_{\text{integrates to 0 (odd under } x\to 1-x)} + 2 \Big] - q^1 q^2 \Big[(1-\tfrac{d}{2})a^2 + 2 \Big] \right\}$$

$$= i g^2 \underbrace{f^{a_1 a_3 a_4} f^{a_2 a_3 a_4}}_{= N_c \delta^{a_1 a_2} \text{ for } SU(N_c)} \left(g^{\mu_1\mu_2} q^2 - q^{\mu_1} q^{\mu_2} \right) \int_0^1 dx\, I_2^0(-x(1-x)q^2) \Big[(1-\tfrac{d}{2})(1-2x)^2 + 2 \Big]$$

((wow! same Lorentz-structure as  !))

<u>derivation</u>: from Lie algebra (pg. 9) $[T^a, T^b] = i f^{abc} T^c$

$$\Rightarrow f^{abc} = -2i\, \mathrm{tr}\left([T^a, T^b] T^c \right) = -2i \left\{ (T^a)_{ij} (T^b)_{jk} (T^c)_{ki} - (T^b)_{ij} (T^a)_{jk} (T^c)_{ki} \right\}$$

then use Fierz identity (pg. 9) $(T^a)_{ij}(T^a)_{kl} = \frac{1}{2}\left(\delta_{il}\delta_{kj} - \frac{1}{N_c}\delta_{ij}\delta_{kl} \right)$,

normalization (pg. 9) $\mathrm{tr}(T^a T^b) = \frac{1}{2}\delta^{ab}$, and $\delta_{ii} = N_c$