- short summary:

  from pg. 18, 26, 27 we have now

$$\boxed{\mathcal{L}_{QCD} = \sum_f \bar{\psi}_f \left(i\not{D} - m_f\right)\psi_f - \frac{1}{4} F^a_{\mu\nu} F^{a\mu\nu} - \frac{1}{2\xi}\left(\partial^\mu A^a_\mu\right)^2 + \bar{c}^a\left(-\partial^\mu D^{ac}_\mu\right)c^c}$$

  where $D^{ac}_\mu = \partial_\mu \delta^{ac} + g f^{abc} A^b_\mu$ is the "covariant derivative in the adjoint representation"

  → this expression is still invariant, but not under a local gauge transformation as in § 2.2; the relevant transformation now includes the ghost fields $\bar{c}, c$ in an essential way and is called "BRST" transformation

  [Becchi/Rouet/Stora, Ann. Phys. 98 (1976) 287;
  Tyutin, Theor. Math. Phys. 27 (1976) 316]

  (a symmetry with continuous but anticommuting parameters)
  ((more: QFT lecture; e.g. Peskin/Schroeder § ))

  → the formal treatment of pg. 26 had implicitly assumed that the gauge condition $f(\phi)$ (($f^a(A^a_\mu)$)) selects (via the Delta-fct) one unique representative $\phi$ (($A^a_\mu$)) for each "gauge orbit" $\phi_\Omega$.
  However, Gribov [Gribov, Nucl. Phys. B 139 (1978) 1] has demonstrated that for non-Abelian theories, this cannot always be guaranteed.
  In practice, this fact has little relevance.

- set of "Feynman rules" as usual

  → see QFT lecture; Particle physics lecture; ...

  draw diagrams – fix symmetry factors – insert Feynman rules for propagators + vertices – perform traces and Lorentz algebra – regularize divergent integrals – Wick rotation – evaluate loop integrals – ...

# 3. Fundamentals

→ our $\mathcal{L}_{QCD}$ contains operators of dimension $\leq 4$

⇒ theory is renormalizable : all divergences can be removed by a finite number of counterterms

here: illustrate some divergences in QCD

important physical consequence: asymptotic freedom !

- mini-review: renormalization [see e.g. Peskin / Schroeder §10]

loops → $\int d^4k$ → ultraviolet (UV) divergences (from large $k$ ⟺ small $x$)

common and natural in QFT.

→ counting of UV div's (in 1PI diags) : superficial degree of divergence

→ idea: "hide" div's in $\mathcal{L}$ by rescaling fields + couplings

$$\underbrace{\phi_B}_{\text{bare}} \to Z_\phi \underbrace{\phi_R}_{\text{renormalized}} \quad \text{etc} \quad , \quad Z = 1 + \mathcal{O}(g_R^2)$$

→ need intermediate regularization of divergent loop integrals

many possibilities ( discretize space-time ; cutoff large momenta ; Pauli-Villars; ...)

most elegant for us: dimensional regularization $\int \frac{d^4k}{(2\pi)^4} \to \int \frac{d^dk}{(2\pi)^d}$ ,

$\left(\begin{matrix} d = 4-2\varepsilon \\ \varepsilon \to 0 \end{matrix}\right)$

$d \in \mathbb{C}$ ; analytic cont. $d \to 4$ ; div's will be poles $\sim \frac{1}{d-4}$

introduce artificial mass-scale $\mu$

$$\text{e.g.} \quad (m^2)^{-\varepsilon} = \mu^{-2\varepsilon} \left(\frac{\mu^2}{m^2}\right)^{\varepsilon} \overset{x^\varepsilon = e^{\varepsilon \ln x} \approx \text{ Taylor} \dots}{=} \mu^{-2\varepsilon} \left(1 + \varepsilon \ln \frac{\mu^2}{m^2} + \mathcal{O}(\varepsilon^2)\right)$$

→ the renormalization group (RG) equations

describe the $\mu$-dependence of parameters / Greens functions

- mini-review: dimensional regularization

$$\int d^dk \, f(k+q) = \int d^dk \, f(k)$$

$$\int d^dk \, f(\lambda k) = |\lambda|^{-d} \int d^dk \, f(k) \quad \Rightarrow \quad \int d^dk = 0 = \int \frac{d^dk}{(k^2)^{\alpha \neq d/2}} \quad (!)$$

$$\int d^dk \, e^{-\alpha k^2} = \left[\int_{-\infty}^{\infty} dk_1 \, e^{-\alpha k_1^2}\right]^d = \left(\frac{\pi}{\alpha}\right)^{d/2}$$

$$\int d^dk \, f(|k|) = \frac{2\pi^{d/2}}{\Gamma(d/2)} \int_0^\infty dk \, k^{d-1} f(k) \qquad \text{d-dim. spherical coordinates}$$

$$\int d^dk \, k^\mu f(|k|) = 0 \qquad (\text{( odd in } k \text{ )})$$

$$\int d^dk \, k^\mu k^\nu f(|k|) \overset{!}{=} g^{\mu\nu} I = \int d^dk \, \frac{g^{\mu\nu} k^2}{d} f(|k|) \qquad \left(\left(\text{since } \int d^dk \, k^2 f = g^\mu{}_\mu I = d I\right)\right)$$