→ for the propagators, need to look at

$$S_0 = \int d^4x\, \mathcal{L}_0 = \int d^4x \left\{ \frac{1}{2} A_\mu^a \delta^{ab} (\partial^2 g^{\mu\nu} - \partial^\mu \partial^\nu) A_\nu^b + \sum_{\text{flavor}} \bar\psi_f (i\not\partial - m_f)\psi_f \right\}$$

- mini-review: anticommuting (Grassmann) numbers

$\{\theta,\eta\}=0 \Rightarrow \theta^2=0$, Taylor $f(\theta)=a+b\theta$ terminates!

integrals: $\int d\theta = 0$, $\int d\theta\,\theta \equiv 1$

complex Grassmann #s: $\theta=\theta_1+i\theta_2$, $\theta^*=\theta_1-i\theta_2$, $(\theta\eta)^* = \eta^*\theta^* = -\theta^*\eta^*$

complex Gauss int: $\int d\theta^* d\theta\, e^{-\theta^* b\theta} = \int d\theta^* d\theta\, (1-\theta^* b\theta) = \int d\theta^* d\theta\, (1+\theta\theta^* b) = b$

another one: $\int d\theta^* d\theta\, \theta\theta^* e^{-\theta^* b \theta} = \int d\theta^* d\theta\, \theta\theta^* = 1$

higher dim Gauss int: $\left(\prod_i \int d\theta_i^* d\theta_i\right) e^{-\theta_i^* B_{ij}\theta_j} = (\quad)\, e^{-\sum_i \theta_i^* b_i \theta_i} = \prod_i b_i = \det B$

↑ hermitean; diagonalize by unitary trafo.

derivatives: $\partial_\theta \theta = 1$; e.g. $\partial_\theta \eta\theta = -\partial_\theta \theta\eta = -\eta$ etc.

- quark propagator: consider one quark flavor, $\mathcal{L}_0 \ni \bar\psi(i\not\partial - m)\psi$

Grassmann-valued source fields

$$Z^q_{free}[\bar\eta,\eta] = \int D\bar\psi D\psi\, e^{i\int d^4x [\bar\psi(i\not\partial - m + i\varepsilon)\psi + \bar\eta\psi + \bar\psi\eta]}$$

shift ψ to complete square (see pg. 22)

$$= Z^q_{free}[0,0]\, e^{-\int d^4x \int d^4y\, \bar\eta(x) S_F(x-y)\eta(y)}$$

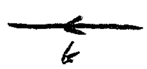
where $S_F(x-y) = \int \frac{d^4k}{(2\pi)^4} e^{-ik(x-y)} \frac{i}{\not k - m + i\varepsilon}$

((is again Greens fct: $(i\not\partial - m + i\varepsilon) S_F(x-y) = i\delta^{(4)}(x-y)$))

now e.g. $\langle 0|T\psi(x_1)\bar\psi(x_2)|0\rangle = \frac{\int D\bar\psi D\psi\, \psi(x_1)\bar\psi(x_2)\, e^{i\int d^4x \mathcal{L}}}{\int D\bar\psi D\psi\, e^{i\int d^4x\mathcal{L}}}$

$$= \frac{1}{Z^q_{free}[0,0]} \left(-i\delta_{\bar\eta(x_1)}\right)\left(+i\delta_{\eta(x_2)}\right) Z^q_{free}[\bar\eta,\eta]\Big|_{\bar\eta=\eta=0}$$

$$= S_F(x_1-x_2)$$ Feynman propagator ✓



$$\boxed{\;\hat{=}\; \frac{i}{\not k - m + i\varepsilon} = \frac{i(\not k + m)}{k^2-m^2+i\varepsilon}}$$

(($(\not k - m)(\not k + m) = \not k\not k - m^2$, $\not k \not k = \frac{1}{2}\{\gamma^\mu,\gamma^\nu\}k_\mu k_\nu = k^2$))

→ for gluon propagator, will have same problem as in QED:
$(\partial^2 g^{\mu\nu} - \partial^\mu \partial^\nu)$ has no inverse (see pg 21)
need Faddeev-Popov gauge fixing

- mini-review: defining the functional integral of a gauge theory

$Z \equiv \int \mathcal{D}\phi\, G(\phi)$; ϕ some gauge-fields (A^a_μ) $\left(\!\left(G(A) = e^{i\int d^4x \left(-\frac{1}{4} F^a_{\mu\nu} F^{a\mu\nu}\right)} \right)\!\right)$

gauge invariance: $G(\phi) = G(\phi_\alpha)$, $\int \mathcal{D}\phi = \int \mathcal{D}\phi_\alpha$ $\left(\!\left((A_\mu)_\alpha = V(A_\mu + \frac{i}{g}\partial_\mu)V^\dagger,\ V = e^{iT^a\alpha^a} \right)\!\right)$ see §2.3

$= \int \mathcal{D}\phi\, G(\phi) \underbrace{\Delta(\phi,b) \int \mathcal{D}\alpha\, \delta(f(\phi_\alpha) - b)}_{\equiv 1 \text{ (defines } \Delta)}$ ← arbitrary field b

"gauge condition" $\left(\!\left(f^a(A) \overset{\text{covariant gauge}}{=} \partial^\mu A^a_\mu \right)\!\right)$

note: $\Delta(\phi,b) = \Delta(\phi_\alpha, b)$ owing to $\int \mathcal{D}\alpha$ in its definition

$= \int \mathcal{D}\alpha \int \mathcal{D}\phi_\alpha\, G(\phi_\alpha)\, \Delta(\phi_\alpha, b)\, \delta(f(\phi_\alpha) - b)$ used gauge invariance of $\int \mathcal{D}\phi, G, \Delta$ (at each α)

$= \left(\int \mathcal{D}\alpha\right) \int \mathcal{D}\phi\, G(\phi)\, \Delta(\phi,b)\, \delta(f(\phi) - b)$ renamed int variable $\phi_\alpha \to \phi$

↳ "volume" of gauge orbit factors out; cancels in expectation values!

now average over b, with weight $B(b)$ $\left(\!\left(B(b) = e^{-\frac{i}{2\xi}\int d^4x\, b^a b^a} \right)\!\right)$

$= \frac{\left(\int \mathcal{D}\alpha\right)}{\left(\int \mathcal{D}b\, B(b)\right)} \int \mathcal{D}\phi\, G(\phi) \int \mathcal{D}b\, B(b)\, \Delta(\phi,b)\, \delta(f(\phi) - b)$

$= \frac{\left(\int \mathcal{D}\alpha\right)}{\left(\int \mathcal{D}b\, B(b)\right)} \int \mathcal{D}\phi\, G(\phi)\, B(f(\phi))\, \Delta(\phi, f(\phi))$ used δ-fct

now compute the "Faddeev-Popov determinant" Δ from its definition:

$\Delta(\phi, f(\phi)) = \left[\int \mathcal{D}\alpha\, \delta(f(\phi_\alpha) - f(\phi))\right]^{-1}$ cross δ-peak with infinit. gauge trafo

$= \left[\int \mathcal{D}\alpha\, \delta\left(\alpha \mathcal{F}(\phi) + \mathcal{O}(\alpha^2)\right)\right]^{-1}$ $f(\phi_\alpha) \approx f(\phi) + \alpha\, \mathcal{F}(\phi) + \mathcal{O}(\alpha^2)$

$= \left[\frac{1}{\det \mathcal{F}(\phi)} \int \mathcal{D}\alpha\, \delta(\alpha)\right]^{-1} = \det \mathcal{F}(\phi)$

$= \frac{\left(\int \mathcal{D}\alpha\right)}{\left(\int \mathcal{D}b\, B(b)\right)} \int \mathcal{D}\phi\, G(\phi)\, B(f(\phi)) \det \mathcal{F}(\phi)$, where $\mathcal{F}(\phi) = (\partial_\phi f(\phi))(\partial_\alpha \phi_\alpha)_{\alpha=0}$

$\left(\!\left(\mathcal{F}(A) = \partial^\mu\left(f^{abc} A^b_\mu + \frac{1}{g}\delta^{ac}\partial_\mu\right),\ \text{pg } 17 \right)\!\right)$

note: in QED, $(A_\mu)_\alpha = A_\mu - \frac{1}{e}\partial_\mu \alpha$ (pg 12),
so $\mathcal{F}$ does not depend on A, hence $\det \mathcal{F}$ cancels in correlators.
⇒ in QCD, $\det \mathcal{F}(A)$ remains inside the functional integral.

- <u>gluon propagator</u>

collection from above, $\quad G(\phi)\, B(f(\phi)) = e^{i\int d^4x\, \mathcal{L}_{QCD}}\, e^{i\int d^4x \left(-\frac{1}{2\xi}\right)(\partial^\mu A_\mu^a)^2}$

such that $\quad S_0 \ni \int d^4x \;\; \frac{1}{2} A_\mu^a\, \delta^{ab} \left(\partial^2 g^{\mu\nu} - \partial^\mu\partial^\nu + \frac{1}{\xi}\partial^\mu\partial^\nu\right) A_\nu^b$

$$\stackrel{!}{=} \int \frac{d^4k}{(2\pi)^4} \;\; \frac{1}{2} \tilde{A}_\mu^a(k)\, \underbrace{\delta^{ab}\left(-k^2 g^{\mu\nu} + \left(1-\frac{1}{\xi}\right)k^\mu k^\nu\right)} A_\nu^b(-k)$$

((check?!))

$$\boxed{\;\hat{=}\; \frac{-i}{k^2+i\varepsilon}\left(g_{\mu\nu} - (1-\xi)\frac{k_\mu k_\nu}{k^2}\right)\delta^{ab}}$$

((since $(\text{eeee})^{ab}_{\mu\nu} \cdot (\sim\!\sim)^{bc,\nu\rho} \stackrel{!}{=} i\,\delta^{ac} g_\mu{}^\rho$))

gauge parameter ; often, use $\xi = 1$ Feynman gauge

as in QED, physics is ξ-independent

- <u>Faddeev-Popov ghost fields</u>

have to take care of $\det \mathcal{F}(\phi)$ factor (on bottom of pg 26)

using Grassmann numbers again (see Gauss integral on pg 25), rewrite

$$\det \mathcal{F}(A) \;=\; \det\left(\frac{1}{g}\partial^\mu \left[g f^{abc} A_\mu^b + \delta^{ac}\partial_\mu\right]\right)$$

$$\stackrel{!}{=} \int \mathcal{D}c\, \mathcal{D}\bar{c}\;\; e^{i\int d^4x\, \bar{c}^a\left(-\partial^\mu\left[\partial_\mu \delta^{ac} + g f^{abc} A_\mu^b\right]\right)c^c}$$

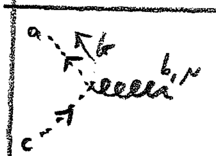
where the "FP ghosts" are anticommuting fields

(but there are no γ-matrices $\Rightarrow$ spin 0 !)

$\Rightarrow$ ghost propagator

$\hat{=} \dfrac{i}{k^2+i\varepsilon}\,\delta^{ab}$

$\Rightarrow$ ghost-gluon-vertex



$\hat{=} -g f^{abc} k_\mu$

((for physical interpretation of ghosts, see e.g. Peskin/Schroeder § 16.3))

$\longrightarrow$ now, know all propagators and vertices of QCD,

so we can again (as in ϕ^4 theory, see § 2.4) do

perturbative expansions via generating functional.