- now, consider an interacting theory, $\mathcal{L} = \mathcal{L}_0 + \mathcal{L}_I$ (e.g. $-\frac{\lambda}{4!}\phi^4$)

look at generating functional again

$$Z[J] = \int \mathcal{D}\phi \; e^{i\int d^4x \,[\mathcal{L}_0 + \mathcal{L}_I + J\phi]}$$
$$= \int \mathcal{D}\phi \; \underbrace{e^{i\int d^4x \,\mathcal{L}_I(\phi \to -i\delta_J)}}_{\phi\text{-independent!}} \; \underbrace{e^{i\int d^4x \,[\mathcal{L}_0 + J\phi]}}_{\text{as in free theory} \Rightarrow \text{ shift as above}}$$
$$= e^{i\int d^4x \,\mathcal{L}_I(\phi \to -i\delta_J)} \cdot e^{-\frac{1}{2}\int d^4x \int d^4y \, J(x) D_F(x-y) J(y)} \cdot \underbrace{\int \mathcal{D}\phi \, e^{i\int d^4x \,\mathcal{L}_0}}_{Z_{free}[0]}$$

such that the correlation functions follow from

$$\langle 0 | T\, \mathcal{O}(\phi) | 0 \rangle = \frac{1}{Z[0]} \mathcal{O}(\phi \to -i\delta_J)\, Z[J]\Big|_{J=0}$$

$$= \frac{\mathcal{O}(\phi \to -i\delta_J)\, e^{i\int d^4x \,\mathcal{L}_I(\phi \to -i\delta_J)} \, e^{-\frac{1}{2}\int d^4x \int d^4y \, J(x) D_F(x-y) J(y)}\Big|_{J=0} \cdot \cancel{Z_{free}[0]}}{e^{i\int d^4x \,\mathcal{L}_I(\phi \to -i\delta_J)} \, e^{-\frac{1}{2}\int d^4x \int d^4y \, J(x) D_F(x-y) J(y)}\Big|_{J=0} \cdot \cancel{Z_{free}[0]}}$$

((note: in denominator, sum of "vacuum diagrams"))

→ perturbative expansion (Feynman diagrams) follow from expanding $e^{i\int \mathcal{L}_I}$ in terms of (small) coupling constants (here: λ)

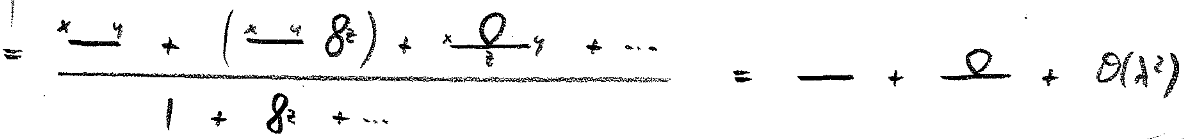
→ all combinatorics for evaluating correlation fcts is just in exponentials!

⇒ two-point function $\langle 0 | T\, \phi(x_1)\phi(x_2) | 0 \rangle$

$$= \frac{\delta_{iJ(x_1)}\,\delta_{iJ(x_2)} \left\{ 1 + \int d^4z \left(-\frac{i\lambda}{4!}\right) \delta^4_{iJ(z)} + \mathcal{O}(\lambda^2) \right\} e^{-\frac{1}{2}\int d^4x \int d^4y \, J(x) D_F(x-y) J(y)}\Big|_{J=0}}{\{ \quad \dots \quad \} \, e^{-\frac{1}{2}\,\dots}\Big|_{J=0}}$$

((check!)) $(D_{ij} \equiv D_F(x_i - x_j))$

$$= \frac{D_{12} + \left(-\frac{i\lambda}{4!}\right) \int d^4z \left(3 D_{12} D_{zz} D_{zz} + 12\, D_{1z} D_{zz} D_{z2} \right) + \mathcal{O}(\lambda^2)}{1 + \left(-\frac{i\lambda}{4!}\right) \int d^4z \; 3 D_{zz} D_{zz} + \mathcal{O}(\lambda^2)}$$



→ this cancellation is actually generic: $\frac{(\text{connected pieces}) \cdot \cancel{e^{(\text{disconnected pieces})}}}{\cancel{e^{(\text{disconnected pieces})}}}$

works for all higher correlation fcts as well.

2.5 QCD Feynman rules

→ recall from above remarks that for a perturbative treatment, we need to read off <u>propagators</u> (cf pg 21) and <u>vertices</u> (cf pg 23) from the Lagrangian $\mathcal{L}$.

→ $\mathcal{L} = \mathcal{L}_0 + \mathcal{L}_I$

$\mathcal{L}_I$ ← interactions ⇒ vertices

$\mathcal{L}_0$ ← bilinear in fields ⇒ propagators

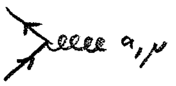
→ recall: $\mathcal{L}_{QCD} = -\frac{1}{4}(F^a_{\mu\nu})^2 + \bar\psi(i\not{D} - m)\psi$ (pg 18)

$F^a_{\mu\nu} = \partial_\mu A^a_\nu - \partial_\nu A^a_\mu + g f^{abc} A^b_\mu A^c_\nu$ (pg 17)

$D_\mu = \partial_\mu - ig A^a_\mu T^a$

$= \mathcal{L}_0 + \bar\psi A^a_\mu \underline{g\gamma^\mu T^a}\psi \; \underline{- g f^{abc} g^{\nu\rho}(\partial^\mu A^a_\nu)} A^b_\mu A^c_\rho \underline{- \frac{1}{4}g^2 f^{eab} f^{ecd} g^{\mu\rho} g^{\nu\sigma}} A^a_\mu A^b_\nu A^c_\rho A^d_\sigma$

($A(x) = \int \frac{d^4k}{(2\pi)^4} e^{-ikx} \tilde{A}(k)$)

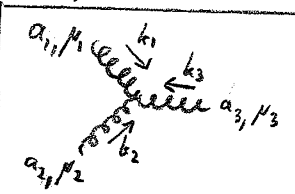
• quark-gluon vertex: $\boxed{i \cdot g\gamma^\mu T^a \;\hat{=}}$ 

from $e^{i\int \mathcal{L}_I}$

• 3-gluon-vertex: need to fix conventions

in Fourier space, $\partial_\mu \to -ik_\mu \;\Rightarrow\; i\cdot(-g f^{abc} g^{\nu\rho})(-ik^\mu)$

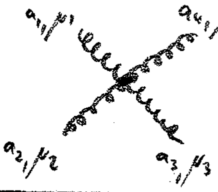
symmetrize this wrt A : 3! possible permutations



$$\hat{=} \; g f^{123}\left\{ (k_1 - k_2)_3 \, g_{12} + (k_2 - k_3)_1 \, g_{23} + (k_3 - k_1)_2 \, g_{31} \right\}$$

f^{123} ← color indices $a_1, \ldots$; g_{12} ← Lorentz indices $\mu_1, \ldots$

• 4-gluon-vertex: $i\cdot\left(-\frac{1}{4}g^2 f^{e12} f^{e34} g^{13} g^{24}\right)$, 4! possible permutations (sets of 4 are equal)



$$\hat{=} -ig^2 \left\{ f^{12e} f^{e34} (g_{13}g_{24} - g_{14}g_{23}) + (1324) + (1423) \right\}$$