1 Basic massless scalar integrals

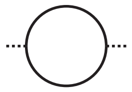
We work in $D = d - 2\epsilon$ dimensions (d being an integer), and take the integral measure to be

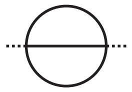
$$\int_p \equiv \left(\frac{e^\gamma}{4\pi}\right)^\epsilon \int \frac{\mathrm{d}^D p}{\pi^D} \quad .$$

For simplicity, the square of the external momentum will be set to unity. The correct prefactor for each integral can be restored by simple dimensional analysis.

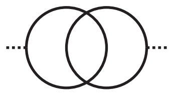
1.1 Definitions

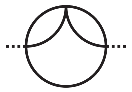
The basic scalar integrals we need are (every unlabelled line corresponds to a massless propagator with power one, e.g. $1/p^2$):

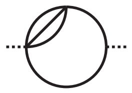
$B \;=$  $= G(1,1)$ (1)

$S \;=$  $= G(1,1)G(2-\frac{D}{2},1)$ (2)

$E \;=$  $= [G(1,1)]^2$ (3)

$Y_0 \;=$  $= G(1,1)\,O_1(2-\frac{D}{2})$ (4)

$Y_2 \;=$  $= [G(1,1)]^2\,G(4-D,1)$ (5)

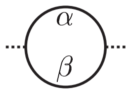
$Y_3 \;=$  $= G(1,1)\,G(2-\frac{D}{2},1)\,G(4-D,1)$ (6)

$Y_4 \;=$  $= [G(1,1)]^2\,G(2-\frac{D}{2},1)$ (7)

$Y_5 \;=$  $= [G(1,1)]^3$ (8)

Everything can be expressed in term of the massless scalar oneloop integral G,

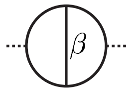
$$G(\alpha,\beta) \;\equiv\; \left(k^2\right)^{\alpha+\beta-\frac{D}{2}} \int_p \frac{1}{[p^2]^\alpha\,[(p-k)^2]^\beta} \;=$$



$$= \left(\frac{e^\gamma}{4\pi}\right)^e \frac{1}{\pi^{\frac{D}{2}}} \frac{\Gamma(\alpha+\beta-\frac{D}{2})}{\Gamma(D-\alpha-\beta)} \frac{\Gamma(\frac{D}{2}-\alpha)}{\Gamma(\alpha)} \frac{\Gamma(\frac{D}{2}-\beta)}{\Gamma(\beta)} \;, \tag{9}$$

except for Y_0, in which the more complicated integral O_1 enters:

$$O_1(\beta) \;\equiv\; \left(k^2\right)^{4+\beta-D} \int_p \int_q \frac{1}{p^2(p-k)^2\left[(p-q)^2\right]^\beta (q-k)^2 q^2} \;=\; \tag{10}$$

 ,

There is no known analytic solution. The expansion was discussed in the file 'integralO1.tex'.

1.2 Expansions

The expansions needed for $D = 3 - 2\epsilon$ read

$$G(1,1) \;\overset{3-2\epsilon}{=}\; \left(1 + \frac{5}{12}\,\pi^2\epsilon^2 + O(\epsilon^3)\right) \tag{11}$$

$$G(2-\tfrac{D}{2},1) \;\overset{3-2\epsilon}{=}\; \frac{2^{-4\epsilon}}{1-6\epsilon}\,\frac{1}{\pi^2}\,\frac{1}{\epsilon}\left(1 - \frac{19}{12}\,\pi^2\epsilon^2 + O(\epsilon^3)\right) \tag{12}$$

$$G(4-D,1) \;\overset{3-2\epsilon}{=}\; \left(1 + \frac{37}{12}\,\pi^2\epsilon^2 + O(\epsilon^3)\right) \tag{13}$$

$$O_1(2-\tfrac{D}{2}) \;\overset{3-2\epsilon}{=}\; 2^{-8\epsilon}\left(1 + \frac{84}{\pi^2}\,\zeta(3)\,\epsilon + O(\epsilon^2)\right) \tag{14}$$

Hence, we get for the basic integrals

$$B \;\overset{3-2\epsilon}{=}\; \left(1 + \frac{5}{12}\,\pi^2\epsilon^2 + O(\epsilon^3)\right) \tag{15}$$

$$S \;\overset{3-2\epsilon}{=}\; \frac{2^{-4\epsilon}}{1-6\epsilon}\,\frac{1}{\pi^2}\,\frac{1}{\epsilon}\left(1 - \frac{14}{12}\,\pi^2\epsilon^2 + O(\epsilon^3)\right) \tag{16}$$

$$E \;\overset{3-2\epsilon}{=}\; \left(1 + \frac{5}{6}\,\pi^2\epsilon^2 + O(\epsilon^3)\right) \tag{17}$$

$$Y_0 \;\overset{3-2\epsilon}{=}\; 2^{-8\epsilon}\left(1 + \frac{84}{\pi^2}\,\zeta(3)\,\epsilon + O(\epsilon^2)\right) \tag{18}$$

$$Y_2 \;\overset{3-2\epsilon}{=}\; \left(1 + \frac{47}{12}\,\pi^2\epsilon^2 + O(\epsilon^3)\right) \tag{19}$$

$$Y_3 \;\overset{3-2\epsilon}{=}\; \frac{2^{-4\epsilon}}{1-6\epsilon}\,\frac{1}{\pi^2}\,\frac{1}{\epsilon}\left(1 + \frac{23}{12}\,\pi^2\epsilon^2 + O(\epsilon^3)\right) \tag{20}$$

$$Y_4 \;\overset{3-2\epsilon}{=}\; \frac{2^{-4\epsilon}}{1-6\epsilon}\,\frac{1}{\pi^2}\,\frac{1}{\epsilon}\left(1 - \frac{3}{4}\,\pi^2\epsilon^2 + O(\epsilon^3)\right) \tag{21}$$

$$Y_5 \;\overset{3-2\epsilon}{=}\; \left(1 + \frac{5}{4}\,\pi^2\epsilon^2 + O(\epsilon^3)\right) \tag{22}$$

Only S, Y_3 and Y_4 are divergent in 3D, due to the logarithmically divergent sunset (sub)diagram.