Lösung 25

(77) $T(x,t) = \sum_n \frac{T_0 \alpha \sinh(\alpha)}{(\pi n)^2 + \alpha^2} e^{-tD\left(\frac{2\pi}{L}n\right)^2} e^{in\frac{2\pi}{L}x}$ ●

$\dot{T}(0,t) = -T_0 \alpha \sinh(\alpha) D\left(\frac{2}{L}\right)^2 \sum_n \frac{(\pi n)^2}{(\pi n)^2+\alpha^2} e^{-tD\left(\frac{2\pi}{L}\right)^2 n^2}$

$t \to 0$: große n! Bruch $\to 1$ ◐

$\sum_n \to \int dn$, d.h. $\sum_n \dots \Rightarrow \int dn\, e^{-tD\left(\frac{2\pi}{L}\right)^2 n^2} = \sqrt{\frac{L^2}{(2\pi)^2 tD}}\sqrt{\pi}$ ◐

also $\dot{T}(0, t\to 0) \to -T_0 \alpha \sinh(\alpha) D \frac{2}{L}\sqrt{\frac{1}{\pi D t}}$ ◐

Diff Gl. oben $\rightsquigarrow$ $T''(0,t) = \frac{1}{D}\dot{T}(0,t)$ ◐

(78) (a) $\tilde{f}(k) = \int dx\, e^{-ikx} e^{-\gamma|x|} = \int dx \cos(kx)\, e^{-\gamma|x|}$, $\int = 2\int_0^\infty$

$= \int_0^\infty dx\, e^{ikx-\gamma x} + \text{dito}_{-k} = \frac{1}{\gamma - ik} + \frac{1}{\gamma+ik} = \frac{2\gamma}{\gamma^2+k^2}$ ● ●

(b) also ist $e^{-\gamma|x|} = \frac{1}{2\pi}\int dk\, e^{ikx}\frac{2\gamma}{\gamma^2+k^2} = \frac{1}{2\pi}\int dk \cos(kx)\frac{2\gamma}{\gamma^2+k^2}$, qed. ●

(c) ∂_x auf (b) $\rightsquigarrow$ $-\int dk \frac{k\sin(kx)}{\gamma^2+k^2} = -\pi\, \text{sign}(x)\, e^{-\gamma|x|}$

$\gamma \to 0 \Rightarrow \int_0^\infty dk \frac{\sin(kx)}{k} = \frac{\pi}{2}\text{sign}(x)$ ●

(79) (a) $\tilde{T}(\vec{k},0) = \int d^3r\, e^{-i\vec{k}\vec{r}} T_0\, \Theta(R-r) = 2\pi\int_0^R dr\, r^2\, T_0 \underbrace{\int_{-1}^1 du\, e^{-ikru}}_{=\frac{2\sin(kr)}{kr}}$

$r \to \frac{1}{k}r$: $= T_0 \frac{4\pi}{k^3}\int_0^{kR} dr \left(r\sin(r) = \partial_r[\sin(r) - r\cos(r)]\right)$

$= 4\pi T_0 \frac{\sin(kR) - kR\cos(kR)}{k^3}$ ●

$T(\vec{r},t) = \left(\frac{1}{2\pi}\right)^3 \int d^3k\, e^{i\vec{k}\vec{r}} e^{-tDk^2}\, 4\pi T_0 \frac{s - kRc}{k^3}$

$= \left(\frac{1}{2\pi}\right)^2 \int_0^\infty dk\, k^2 \frac{2\sin(kr)}{kr} e^{-tDk^2}\, 4\pi T_0 \frac{s-kRc}{k^3}$ ◐

$r \to 0$: $\frac{\sin(kr)}{kr} \to 1$ ◐ $\Rightarrow$ das angegebene $T(\vec{0},t)$

(b) $t\to\infty$: kleine k, $(s - kRc) \to -\frac{1}{6}(kR)^3 + \frac{1}{2}(kR)^3 = \frac{1}{3}k^3R^3$, $k \to \frac{k}{\sqrt{tD}}$

$T(\vec{0}, t\to\infty) = \frac{2T_0}{\pi}\cdot\frac{1}{3}R^3\cdot\left(\frac{1}{\sqrt{Dt}}\right)^3 \cdot \underbrace{\int_0^\infty dk\, k^2 e^{-k^2}}_{=\frac{1}{4}\sqrt{\pi}} = \frac{T_0}{6\sqrt{\pi}}\left(\frac{R}{\sqrt{Dt}}\right)^3$ ●

(c) $\partial_r \frac{\sin(kr)}{kr} = \frac{-(\sin(kr) - kr\cos(kr))}{kr^2}$

$T'(R,t) = -\frac{2T_0}{\pi}\int_0^\infty dk\, e^{-tDk^2} \underbrace{\frac{[\sin(kR) - kR\cos(kR)]^2}{(kR)^2}}$ ● ●

$t\to 0$: führ. Term v. großen k, d.h.: $\to \cos^2(kR) \to \frac{1}{2}$

und $T'(R, t\to 0) \to -\frac{T_0}{\pi}\frac{1}{\sqrt{Dt}}\frac{\sqrt{\pi}}{2} = -\frac{T_0}{\sqrt{4\pi Dt}}$ } challenge; 1 Sonder-●