Lösung 20

(62) $\dot N = [G_0 - S_0 + \alpha(V-V_0)]N$, $N(0)=N_0$

$\dot V = -\beta N$, $V(0)=V_0$

neue Var, Fkn ⇝ $\dot N = N_0 u' \dot\tau$, $\dot V = (V-V_0)^{\cdot} = -\sqrt{\beta N_0/\alpha}\, v'\dot\tau$, $\dot\tau = \sqrt{\alpha\beta N_0}$

oben einsetzen ... $\boxed{}$ stimmt ✓

$u' = \boxed{v'' = v'(\eta - v)}$, $v(0)=0$, $v'(0)=1$

Besonderheit: kein τ; Fall 7a, setze $v' = p(v)$ ⇒ $\boxed{p'^{\,v} = \frac{1}{p}p(\eta-v),\ p(0)=1}$

$= p(v(0)) = v'(0) = u(0)$

Lsg: $p(v) = C + \eta v - \frac12 v^2$, $p(0) = C \overset{!}{=} 1$

löse jetzt noch $\boxed{v' = 1+\eta v - \frac12 v^2,\ v(0)=0}$

z.B. per TdV: $\frac{v'}{2+2\eta v - v^2} = \frac12$

für lhs: $\partial_v \frac{1}{2\omega}\ln\left(\frac{\omega-\eta+v}{\omega+\eta-v}\right) = \frac{1}{2\omega}\left(\frac{1}{\omega-\eta+v}+\frac{1}{\omega+\eta-v}\right) = \frac{1}{\omega^2-(v-\eta)^2} = \frac{1}{\underbrace{\omega^2-\eta^2}_{=2}+2v\eta-v^2}$

⇒ $v'\partial_v \frac{1}{2\omega}\ln(\dots) = \partial_\tau \frac{1}{2\omega}\ln(\dots) = \frac12$

$\frac{1}{2\omega}\ln(\dots) = \frac12\tau + C$, $\tau=0$: $\frac{1}{2\omega}\ln\left(\frac{\omega-\eta}{\omega+\eta}\right) = C$

muß jetzt $\tau = \frac1\omega \ln\left(\frac{\omega-\eta+v}{\omega+\eta-v}\,\frac{\omega+\eta}{\omega-\eta}\right)$ nach v auflösen: $v = \dots(\omega+\eta)\left[1 - \frac{2\omega}{\omega+\eta+(\omega-\eta)e^{\omega\tau}}\right]$

also schließlich $u(\tau) = v' = \dots = \frac{4\omega^2 e^{\omega\tau}}{[\omega+\eta+(\omega-\eta)e^{\omega\tau}]^2}$

(63) (a) $\Delta_2 G = \frac{1}{4\pi}\left(\partial_x \frac{2x}{\rho^2+\varepsilon^2} + \partial_y\frac{2y}{\rho^2+\varepsilon^2}\right) = \frac1\pi\left(\frac{1}{\rho^2+\varepsilon^2} - \frac{\rho^2}{(\rho^2+\varepsilon^2)^2}\right) = \frac1\pi \frac{\varepsilon^2}{(\rho^2+\varepsilon^2)^2}$

$2\pi\int_0^\infty d\rho\,\rho\,\frac1\pi\frac{\varepsilon^2}{(\rho^2+\varepsilon^2)^2} = \int_0^\infty d\varrho\, \frac{\varrho}{(\varrho^2+1)^2} = \left[-\frac{1}{\varrho^2+1}\right]_0^\infty = 1$

(b) $G = \Theta e^{-\gamma t}\frac{s(\Omega t)}{\Omega}$, $G' = \delta e \frac{s}{\Omega} - \gamma\Theta e\frac{s}{\Omega} + \Theta e c$, $G'' = \delta' e\frac{s}{\Omega} + \gamma^2\Theta e \frac{s}{\Omega} - \Omega^2\Theta e\frac{s}{\Omega} + 2\left(-\gamma\delta e\frac{s}{\Omega} + \delta e c - \gamma\Theta e c\right)$

⇒ $G'' + 2\gamma G' + \omega_0^2 G = \delta' e \frac{s}{\Omega} + 2\delta e c$

$= -\delta e\frac{s}{\Omega t} + 2\delta e c = \delta$

$\omega_0 = 0$, $\Omega = \pm i\gamma$, $\sin(\pm i\gamma t) = \pm\frac{e^{-\gamma}-e^{\gamma}}{2i} = \pm i\sinh(\gamma t)$, $G = \Theta(t)e^{-\gamma t}\frac{\sinh(\gamma t)}{\gamma}$

(c) $t\partial_t G(t,a) = \delta(t-a)$, $\dot G = \frac1a\delta(t-a)$, $G = \frac1a\Theta(t-a) + C$

$v = \int_0^T da\, f(a)\left(\frac1a\Theta(t-a)+C\right) = \int_0^t da\,\frac1a f(a) + C$

(64) (a) $\partial_r \underbrace{\partial_r rG(r,a)}_{\equiv u} = r\delta(r-a) = a\delta(r-a)$, $u' = a\delta(r-a)$, $u = a\Theta(r-a)$ verschwindet bei $r<a$

⇒ $rG = a(r-a)\Theta(r-a)$, $G = \frac{a}{r}(r-a)\Theta(r-a)$

(b) $V_{sp}(r) = \int_0^r da\, 4\pi\gamma m \varrho(a)\frac{a}{r}(r-a) = 4\pi\gamma m\left(-\frac1r\int_0^r da\,a^2\varrho + \int_0^r da\,a\varrho\right)$

((Bem.: vgl Ü 54(a), denn $\int_0^r da\, a\varrho = \text{const} - \int_r^\infty da\, a\varrho$))