53 (a) $V = \int_0^R dr\, r^2 \int_0^{\pi/2} d\vartheta \sin(\vartheta) \int_0^{\pi/2} d\varphi \cdot 1$

$= \left(\left[\frac13 r^3\right]_0^R\right)\cdot\left(\left[-\cos(\vartheta)\right]_0^{\pi/2}\right)\cdot\frac{\pi}{2} = \frac13 R^3\cdot 1\cdot\frac{\pi}{2} = \frac{\pi R^3}{6} \quad \Rightarrow \quad \rho = \frac{6M}{\pi R^3}$

(b) allg.: $\vec R = \int_0^R dr\, r^2 \int_0^{\pi/2} d\vartheta \sin(\vartheta)\int_0^{\pi/2} d\varphi \cdot \underbrace{\frac1M \rho(\vec r)}_{=\frac{6}{\pi R^3}} \vec r$, hängt nicht v. $\vec r$ ab.

$R_3 = \frac{6}{\pi R^3}\int_0^R dr\, r^3 \int_0^{\pi/2} d\vartheta \underbrace{\sin(\vartheta)\cos(\vartheta)}_{=\partial_\vartheta\left[\frac12 \sin^2(\vartheta)\right]} \int_0^{\pi/2} d\varphi = \frac{6}{\pi R^3}\cdot\frac14 R^4\cdot\frac12\cdot\frac{\pi}{2} = \frac38 R$

$R_1 = \frac{6}{\pi R^3}\int_0^R dr\,r^3\int_0^{\pi/2} d\vartheta \underbrace{\sin^2(\vartheta)}_{=\partial_\vartheta\left[\frac12(\vartheta-\cos(\vartheta)\sin(\vartheta))\right]}\int_0^{\pi/2} d\varphi\cos(\varphi) = \frac{6}{\pi R^3}\cdot\frac14R^4\cdot\frac{\pi}{4}\cdot 1 = \frac38 R$

$R_2 = \frac{6}{\pi R^3}\int dr\, r^3\int d\vartheta \sin^2(\vartheta)\int d\varphi \sin(\varphi) = \frac{6}{\pi R^3}\cdot\frac14R^4\cdot\frac{\pi}{4}\cdot1 = \frac38 R$

54 (a) $(\sqrt{\ } - \sqrt{\ }) = |r+r'| - |r-r'| = \begin{cases} r<r': 2r \\ r>r': 2r'\end{cases}$

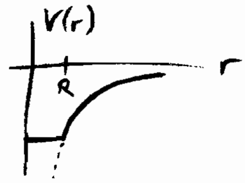
$\Rightarrow \quad V(r) = -\gamma m 4\pi\left(\frac1r\int_0^r dr'\, r'^2\rho(r') + \int_r^\infty dr'\, r'\rho(r')\right)$

(b) $V_{innen} = -\gamma m 4\pi \rho_0\left(\frac1r\int_0^r dr'\, r'^2 + \int_r^R dr'\, r'\right) = \gamma m 4\pi\rho_0\left(\frac{r^2}{6}-\frac{R^2}{2}\right)$

(c) $\frac1r\partial_r\partial_r(-\gamma m 4\pi)\left(\int_0^r dr'\, r'^2\rho + r\int_r^\infty dr'\, r'\rho\right) = -\gamma m 4\pi \frac1r \partial_r \int_r^\infty dr'\, r'\rho(r')$

$= \gamma m 4\pi\rho(r)$

55 (a)



Kugelschale, Vol. $= \frac{4\pi}{3}\left((r'+dr')^3 - r'^3\right) = 4\pi\, dr'\, r'^2 + O(dr'^2)$

Masse $= \rho\cdot$ Vol.

$dV = \{M\to\rho\cdot \text{Vol}, R\to r'\} = -\gamma m\rho 4\pi\, dr'\, r'^2 \begin{cases} 1/r' & (r<r') \\ 1/r & (r>r')\end{cases}$

$V(r>R_E) = \int_0^{R_E} dr'(-\gamma m\rho 4\pi)\, r'^2 \frac1r = -\gamma m \underline{\rho 4\pi \frac13 R_E^3}\frac1r = -\gamma m M_E/r$

$V_{innen}(r) = V(r<R_E) = -\gamma m\rho 4\pi\left(\int_0^r dr' \frac{r'^2}{r} + \int_r^{R_E} dr'\frac{r'^2}{r'}\right) = \gamma m\rho 4\pi\left(\frac{r^2}{6}-\frac{R_E^2}{2}\right)$

(b) Schicht, Vol. $= dz'\cdot\pi(R_E^2 - z'^2)$, Masse $= \rho\cdot$ Vol.

$dV = \{M\to\rho\cdot \text{Vol}, R\to\sqrt{R_E^2-z'^2}, z\to z-z'\} = -\gamma m\rho 2\pi\, dz'\left(\sqrt{R_E^2+z^2-2zz'} - |z-z'|\right)$

$V(z>R_E) = -\gamma m\rho 2\pi\int_{-R_E}^{R_E} dz'\left(\sqrt{R_E^2+z^2-2zz'} - z + z'\right)$, $\quad = \partial_{z'}\left[-\frac{1}{3z}(R_E^2+z^2-2zz')^{3/2} - zz' + \frac{z'^2}{2}\right]$

$= -\gamma m\rho 2\pi\left\{-\frac{1}{3z}\left[\left((z-R_E)^2\right)^{3/2} - \left((z+R_E)^2\right)^{3/2}\right] - 2R_E z\right\} = -\gamma m \underbrace{\rho 2\pi\frac23 R_E^3}_{M_E}\frac1z$

$V(-R_E<z<R_E) = -\gamma m\rho 2\pi\left\{\int_{-R_E}^z dz'\left(\sqrt{\ } - z + z'\right) + \int_z^{R_E} dz'\left(\sqrt{\ } - z' + z\right)\right\}$

$= -\gamma m\rho 2\pi\left\{-\frac{1}{3z}\left[\left((R_E-z)^2\right)^{3/2} - \left((R_E+z)^2\right)^{3/2}\right] - z^2 - R_E^2\right\} = \gamma m\rho 4\pi\left(\frac{z^2}{6}-\frac{R^2}{2}\right)$