EMTP II - Klausur 9.10.07 — Lösung

① $\{0,\ 0,\ s^2 \to \tfrac{1}{2} \rightsquigarrow \pi,\ 0,\ s^4 c = \tfrac{1}{5}\partial_\varphi s^5 \rightsquigarrow 0\}$

② $x = 1/u,\ dx = -du/u^2,\quad I = \int_0^1 du\, \frac{1}{u^2}\cdot u^3 \sqrt{\frac{1}{u^2}-1} = \int_0^1 du\, \sqrt{1-u^2}$

$u = \sin(\varphi),\ du = \cos(\varphi)\,d\varphi \quad \to \int_0^{\pi/2} d\varphi\, c\sqrt{1-s^2} = \int_0^{\pi/2} d\varphi\, (c^2 \to \tfrac{1}{2}) = \pi/4$

③ $m\ddot{x} = -m\omega^2 x = -\partial_x\left(\frac{m}{2}\omega^2 x^2\right) \Rightarrow V(x) = \frac{m}{2}\omega^2 x^2\ ;\quad E = V(a) = \frac{m}{2}\omega^2 a^2$

$T = 4\sqrt{\frac{m}{2}}\sqrt{\frac{2}{m\omega^2}}\int_0^a dx\,\frac{1}{\sqrt{a^2-x^2}} \overset{x\to ax}{=} \frac{4}{\omega}\int_0^1 dx\,\frac{1}{\sqrt{1-x^2}} \overset{x=\sin(\varphi)}{=} \frac{4}{\omega}\int_0^{\pi/2} d\varphi\,\frac{c}{\sqrt{1-s^2}} = \frac{4}{\omega}\int_0^{\pi/2} d\varphi = \frac{2\pi}{\omega}$

④ (a) $V(\vec{r}) = \gamma m\sigma\left[\ln(\sqrt{\ }+z) + \ln(\sqrt{\ }-z)\right] = \gamma m\sigma \ln(x^2+y^2)$

(b) alle dx' am Stab $\rightsquigarrow$ $\underline{d\sigma\cdot h} \doteq \underline{\varrho_0\, dx' h}$

$V(\vec{r}) = \int_{-a}^{a} \gamma m\,(d\sigma = \varrho_0 dx')\,\ln\left((x-x')^2+y^2\right)\ ;\quad V(0,y,0) = \gamma m\varrho_0\int_{-a}^{a} dx' \ln(x'^2+y^2)$

$K_3 = -\partial_y V(0,y,0) = -\gamma m\varrho_0 \int_{-a}^{a} dx'\,\frac{2y}{x'^2+y^2} \overset{x'\to|y|x'}{=} -\gamma m\varrho_0\,\frac{2y}{|y|}\int_{-a/|y|}^{a/|y|} dx'\,\frac{1}{1+x'^2} \xrightarrow{|y|\to 0} -\gamma m\varrho_0\, 2\,\mathrm{sign}(y)\cdot\pi$

⑤ (a) $1 \doteq \beta\int_0^\infty dx\, e^{-x/\varepsilon},\quad x\to\varepsilon x,\quad 1 = \beta\varepsilon \Rightarrow \beta = 1/\varepsilon$

(b) $\int da\, f(a)\,[lhs] = \int da\, f(a)\int dx\,\delta(x-a)\delta(x-b) = \int dx\, f(x)\delta(x-b) = f(b)$

$\int da\, f(a)\,[rhs] = \int da\, f(a)\delta(a-b) = f(b)$ ✓

(c) $\int dx\,\frac{1}{\varepsilon}\theta(x-a)e^{-(x-a)/\varepsilon}\,\frac{1}{\varepsilon}\theta(x-b)e^{-(x-b)/\varepsilon} = \frac{e^{(a+b)/\varepsilon}}{\varepsilon^2}\int_{\max(a,b)}^{\infty} dx\, e^{-2x/\varepsilon} = \frac{1}{2\varepsilon}e^{-|a-b|/\varepsilon} \overset{\text{z.B. Ü5b}}{\doteq} \delta(a-b)$ ✓

(d) $\delta(f(x)) = \sum_{NS}\frac{\delta(x-x_{NS})}{|f'(x_{NS})|},\quad \delta(x^3) = \frac{1}{3x^2}\delta(x) \Rightarrow n=2,\ c=3$

⑥ kein x (z.B.) $\Rightarrow$ "Fall ⑦b", $\dot{x} =: v(t) \Rightarrow \boxed{v' = -v^2,\ v(0)=1}$, Lsg $v(t) = \frac{1}{t+C}$, $C=1$, $v(t) = \frac{1}{1+t}$;

$\boxed{\dot{x} = \frac{1}{1+t},\ x(0)=1}$, Lsg $x(t) = A + \ln(1+t)$, $A=1 \Rightarrow x(t) = 1+\ln(1+t)$

⑦ $(\partial_t + 2t)G = \delta(t-a)$, $G_{hom} = e^{-t^2}$; VdK: $G = e^{-t^2}u$, $\dot{u} = e^{t^2}\delta(t-a) = e^{a^2}\delta(t-a)$, $u = C + e^{a^2}\theta(t-a)$

$\Rightarrow G = e^{-t^2}\left(C + e^{a^2}\theta(t-a)\right)$

⑧ $\vec{\nabla}\times\vec{e}_\vartheta = \left(\vec{e}_r\partial_r + \vec{e}_\vartheta\frac{1}{r}\partial_\vartheta + \vec{e}_\varphi\frac{1}{rs}\partial_\varphi\right)\times\vec{e}_\vartheta = 0 - \frac{1}{r}\vec{e}_\vartheta\times\vec{e}_r + \frac{c}{rs}\vec{e}_\varphi\times\vec{e}_\varphi = \frac{1}{r}\vec{e}_\varphi$

⑨ Ans. $\vec{v}(\vec{r}) = -v(y)\vec{e}_1$; $\mathrm{rot}\,\vec{v} = \vec{e}_3\partial_y v(y)$, $v' = \omega e^{-y/a}$, Lsg $v = v_0 - a\omega e^{-y/a}$, $v(0)=0$, $\vec{v} = a\omega(e^{-y/a}-1)\vec{e}_1$

⑩ wende $\int_V d^3r$ drüber : $\lambda = \int_V d^3r\,\vec{\nabla}\cdot\underbrace{\vec{\nabla}\frac{1}{r}}_{=-\vec{r}/r^3} = \oint_S d\vec{f}\cdot\left(-\frac{\vec{e}_r}{r^2}\right) \overset{\uparrow}{=} -\frac{1}{R^2}\cdot 4\pi R^2 = -4\pi$

wähle V=Kugel(R) ; S=Kugeloberfl.(F)

⑪ $\Delta e^{ikx} = \partial_x^2 e^{ikx} = -k^2 e^{ikx}$; $\mathrm{atn}(a^2\Delta)e^{ikx} = \mathrm{atn}(-a^2k^2)e^{ikx}$; $\Delta e^{i\vec{k}\vec{r}} = (-k_1^2-k_2^2-k_3^2)e^{i\vec{k}\vec{r}} = -k^2 e^{i\vec{k}\vec{r}}$

$\Delta\frac{\cos(kr-\omega t)}{r} = $ .. Laplace in Kugelkoord .. $= \frac{1}{r}\partial_r^2\, r\,\frac{\cos(kr-\omega t)}{r} = \frac{1}{r}\partial_r(-k)\sin(kr-\omega t) = (-k^2)\frac{\cos(kr-\omega t)}{r}$

⑫ $T(\vec{r},t) = e^{tD\Delta}T(\vec{r},0) = e^{tD\vec{\nabla}\cdot\vec{\nabla}}\int\frac{d^3k}{(2\pi)^3}e^{i\vec{k}\vec{r}}\underbrace{\tilde{T}(\vec{k},0)}_{=\alpha} \overset{\vec{\nabla}\to i\vec{k}}{=} \alpha\int\frac{d^3k}{(2\pi)^3}e^{-tD\vec{k}^2}e^{i\vec{k}\vec{r}}$

$= \alpha\left(\int\frac{dk_1}{2\pi}e^{ik_1x}e^{-tDk_1^2}\right)(2,y)(3,z)$ $\quad FT\{\text{Gauß}\} = \text{Gauß}$, z.B. Vorl S. 105

$= \alpha\left(\frac{1}{\sqrt{\pi}\sqrt{4tD}}e^{-x^2/4tD}\right)(y)(z) = \frac{\alpha}{\sqrt{4tD\pi}^{\,3}}e^{-r^2/4tD}$ $\quad$ (check: δ-Darst. für $t\to 0$ ✓)

⑬ (a) $\tilde{f}(k) = \int dx\, e^{-ikx}e^{-\gamma|x|} = \int_0^\infty dx\,(e^{ikx}+e^{-ikx})e^{-\gamma x} = \frac{1}{\gamma-ik} + \frac{1}{\gamma+ik} = \frac{2\gamma}{\gamma^2+k^2}$

(b) $f(x) = \int\frac{dk}{2\pi}e^{ikx}\frac{2\gamma}{\gamma^2+k^2} = \int\frac{dk}{2\pi}\left[\cos(kx) + \overset{\text{odd}}{\cancel{i\sin(kx)}}\right]\frac{2\gamma}{\gamma^2+k^2} \Rightarrow \int dk\,\frac{\cos(kx)}{\gamma^2+k^2} = \frac{\pi}{\gamma}e^{-\gamma|x|}$

Ergebnisse: ab 12. Oktober ; online + Aushang (E6)

Einsicht im Prüfungsamt (D3-150)