EMTP II – Klausur 17.7.07

Lösung

① $9y^2 + 5a^2 + x^2 = \text{const.}$, $(x/3)^2 + y^2 = C^2$, Ellipsen

② $I = -\partial_\alpha J|_{\alpha=1}$ mit $J = \int_0^\infty du\, e^{-\alpha u^2} = \frac{\sqrt{\pi}}{2}\alpha^{-1/2}$ $\Rightarrow$ $I = \frac{\sqrt{\pi}}{4}$

③ $A = \int_0^\pi d\alpha\, (\partial_\alpha \vec r)\cdot(-\vec K(\vec r)) = \int_0^\pi d\alpha\; R\alpha\,(s,c)\cdot\left(\frac{\gamma m M}{r^3}\vec r\right)$ mit $s \equiv \sin(\alpha)$, $c \equiv \cos(\alpha)$

$r^2 = R^2(1+\alpha^2)$, $(s,c)\cdot\vec r = R$

$\frac{\gamma m M}{R}\int_0^\pi d\alpha \left(\frac{\alpha}{(1+\alpha^2)^{3/2}} = \partial_\alpha\left[-(1+\alpha^2)^{-1/2}\right]\right) = \frac{\gamma m M}{R}\left(-\frac{1}{\sqrt{1+\pi^2}}+1\right)$

oder $\vec K = \vec\nabla \frac{\gamma m M}{r} = -\vec\nabla V$, $A = V(r(\pi)) - V(r(0)) = V(R\sqrt{1+\pi^2}) - V(R) = -\frac{\gamma m M}{R}\left(\frac{1}{\sqrt{1+\pi^2}}+1\right)$

④ $M = \varrho_0 V_{HK} = \varrho_0 \frac{2\pi R^3}{3}$, $\vec R = (0,0,R_3)$, $M R_3 = \varrho_0 \int_{(HK)} d^3r\; z$

Zylinderko.: $M R_3 = \varrho_0 2\pi \int_0^R d\varrho\,\varrho \int_0^{\sqrt{R^2-\varrho^2}} dz\, z = \varrho_0 \pi \int_0^R d\varrho\,\varrho(R^2-\varrho^2) = \varrho_0\pi\left(\frac{R^4}{2}-\frac{R^4}{4}\right)$

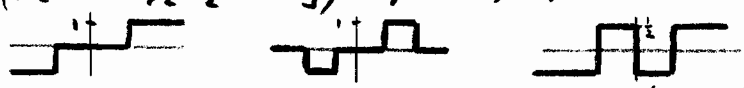
oder Ku.: $M R_3 = \varrho_0 \int_0^R dr\, r^2 \int_{(2\pi)} d\varphi \int_0^{\pi/2} d\vartheta\, \sin(\vartheta)\, r\cos(\vartheta) = \varrho_0 \frac{R^4}{4} 2\pi \int_0^{\pi/2} d\vartheta \left(sc = \partial_\vartheta \tfrac{1}{2}s^2\right) = \varrho_0 \pi \frac{R^4}{4}$

$\Rightarrow R_3 = \frac{3}{8}R$

⑤ (a) $1 \overset{!}{=} \alpha \int_0^\infty dx\, e^{-x^2/\varepsilon^2} = \alpha\varepsilon\frac{\sqrt{\pi}}{2}$, $\alpha = \frac{2}{\varepsilon\sqrt{\pi}}$

(b) $1 \overset{!}{=} \eta\, 2\pi \int_0^\infty dr \left(r e^{-r^2/\varepsilon^2} = \partial_r\left[-\frac{\varepsilon^2}{2} e^{-r^2/\varepsilon^2}\right]\right) = \eta\pi\varepsilon^2$, $\eta = \frac{1}{\pi\varepsilon^2}$

(c) $\partial_x \theta = \delta$ $\Rightarrow$



⑥ $\dot v + P v = Q$ mit $P = \alpha$, $Q(t) = k_0 e^{\beta t}$. PQ $\Rightarrow$ $v_{allg}(t) = e^{-\int_{t_0}^t dt' P}\left(C + \int_{t_1}^t dt'\, Q(t')\, e^{\int_{t_0}^{t'} dt'' P}\right)$

$\Rightarrow v_{allg}(t) = e^{-\alpha(t-t_0)}\left(C + \int_{t_1}^t dt'\, k_0 e^{\beta t'} e^{\alpha(t'-t_0)}\right) = \tilde C e^{-\alpha t} + \frac{k_0}{\alpha+\beta} e^{\beta t}$

⑦ $\sin(\varphi) = b/r$ $\Rightarrow$ $r(\varphi) = b/\sin(\varphi)$

⑧ (a) $\vec\nabla\vec a\cdot(\vec b\times\vec r) = \partial_i a_j \varepsilon_{jkl} b_k x_l = a_j\varepsilon_{jki} b_k = \varepsilon_{ijk} a_j b_k = \vec a\times\vec b$

(b) $(\vec r\cdot\vec\nabla) r^2 = \vec r\cdot 2r\,\vec\nabla r = \vec r\cdot 2r\frac{\vec r}{r} = 2r^2$, $e^{\vec r\cdot\vec\nabla} r^2 = e^2 r^2$

⑨ $(\vec r\cdot\vec\nabla)\,\vec\omega\times\vec r = \vec\omega\times(\vec r\cdot\vec\nabla)\vec r = 1\cdot\vec\omega\times\vec r$, $\vec A = -\frac{1}{3}\vec r\times(\vec\omega\times\vec r)$

$\vec\nabla\times\vec A = -\frac{1}{3}\vec\nabla\times\left(\vec\omega r^2 - \vec r(\vec\omega\cdot\vec r)\right) = -\frac{1}{3}\left(-\vec\omega\times\underbrace{\vec\nabla r^2}_{=2\vec r} - \underbrace{(\vec\nabla\times\vec r)}_{=0}(\vec\omega\cdot\vec r) + \vec r\times\underbrace{\vec\nabla(\vec\omega\cdot\vec r)}_{=\vec\omega}\right) = \vec\omega\times\vec r = \vec B$ ✓

⑩ $n = N\frac{3}{4\pi R^3}$, Conti.: $\vec\nabla\cdot\vec j = -\dot n = \frac{9Nv}{4\pi R^4}$, Ansatz: $\vec j = \vec r f(r,t)$

$3f + \vec r(\vec\nabla\vec r)\partial_r f = \boxed{3f + r f' = \frac{9Nv}{4\pi R^4}}$, $f_{hom} = \frac{c}{r^3}$ (z.B. via Potenzansatz), $f_{spez} = \frac{3Nv}{4\pi R^4}$

$c \overset{!}{=} 0$, damit $\vec j = \vec 0$ im Ursprung $\Rightarrow$ $\vec j = \vec r\,\frac{3Nv}{4\pi R^4}$

⑪ $\Delta_r \chi = \frac{1}{r}\partial_r\partial_r \frac{r}{r+\varepsilon} = \frac{1}{r}\partial_r \frac{\varepsilon}{(r+\varepsilon)^2} = \frac{-2\varepsilon}{r(r+\varepsilon)^3}$

$\int d^3r\left(\frac{-2\varepsilon}{r(\,)^3}\right) = -2\varepsilon\, 4\pi\int_0^\infty dr\,\frac{r+\varepsilon-\varepsilon}{(\,)^3} = -2\varepsilon\, 4\pi\left[-\frac{1}{(\,)} + \frac{\varepsilon}{2(\,)^2}\right]_0^\infty = -4\pi$ ✓

⑫ $c_n = \frac{1}{L}\int_0^a dx\, e^{-in\frac{2\pi}{L}x}$, $c_0 = \frac{a}{L}$, $c_{n\neq 0} = \frac{i}{2\pi n}\left(e^{-in\frac{2\pi}{L}a} - 1\right)$

⑬ $\tilde\varrho(\vec k) = \int d^3r\;\gamma\,\delta(r-R)\,e^{i\vec k\vec r} = \gamma 2\pi \int_0^\infty dr\, r^2\delta(r-R)\underbrace{\int_{-1}^1 du\, e^{ikru}}_{=\frac{2}{kr}\sin(kr)} = \gamma 4\pi R\frac{\sin(kR)}{k}$

Ergebnisse: ab 20. Juli; online + Aushang (E6)

Einsicht im Prüfungsamt (D3-150)