Symmetries in Physics - Fall 2018/19

Bielefeld University

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Exercise Nr. 11

Discussion on January 8, 14:15-15:45, Room U2-135

Exercises 25) should be handed in **before** the tutorial.

25) Conserved Quantities *(3+5=8 points)*

A continuous symmetry of a physical system implies a conserved quantity due to the Noether theorem. Determine all symmetries and conserved quantities of the following systems:

a) A free particle in d dimensions

b) The motion of a planet in the Newtonian gravitational potential of a point source (black hole).

26) Energy-Momentum Tensor *(3+3+6=12 points)*

a) Show that the energy-momentum tensor is covariantly conserved: $\partial_\mu T^{\mu\nu} = 0$. You may need to use the Euler-Langrange equation.

b) Determine the energy momentum tensor for the real-valued Klein gordon field

$$\mathcal{L} = \frac{1}{2}(\partial_\mu \phi)(\partial^\mu \phi) - \frac{m^2}{2}\phi^2$$

and show explicity that $T^{\mu\nu}$ is a symmetric tensor.

c) Consider an infinitesimal proper orthochronous Lorentz transformation:

$$x^\mu \xrightarrow{\mathrm{SO}^\uparrow(1,3)} x'^\mu = x^\mu + \omega^{\mu\nu} x_\nu$$

with $\omega^{\mu\nu} = -\omega^{\nu\mu}$ and $|\omega| \ll 1$. Show that the conserved current associated with this symmetry transformation is

$$J^{\mu\nu\lambda} = -\frac{1}{2}(T^{\mu\nu} x^\lambda - T^{\mu\lambda} x^\nu).$$

This current is connected to angular momentum of the field: Show that the conservation of angular momentum $\partial_\mu J^{\mu\nu\lambda} = 0$ implies a symmetric energy momentum tensor.

Henri Poincaré
(29 April 1854 - 17 July 1912)

French mathematician who did important work in many different branches of mathematics. However, he did not stay in any one field long enough to round out his work. He had an amazing memory and could state the page and line of any item in a text he had read. He retained this memory all his life. He also remembered verbatim by ear. His normal work habit was to solve a problem completely in his head, then commit the completed problem to paper. Despite his keen mathematical ability, he was physically clumsy and artistically inept. In fact, he received a score of 0 on his Polytechnique entrance exam. He was always in a rush and disliked going back for changes or corrections. He was also a popularizer of mathematics. Poincaré's brother Raymond was president of the French Republic during World War I.

Poincaré is quoted as saying, "It is the simple hypotheses of which one must be most wary; because these are the ones that have the most chances of passing unnoticed" (Boyer and Merzbach 1991, p. 599). In 1880, he created generalized elliptic functions called automorphic functions. He discovered that automorphic functions invariant under the same group are connected by an algebraic equation. Conversely, he found that the coordinates of a point on any algebraic curve can be expressed in terms of automorphic functions. He showed they could be used to solve second order linear differential equation with algebraic coefficients.

Poincaré did fundamental work in celestial mechanics in his treatises "Les Méthodes Nouvelles de la Mécanique Céleste" (1892, 1893, 1899), in which he used variational equations and integral invariants, and "Leçons de Mécanique Céleste" (3 volumes, 1905-1910). In these works, he attacked the three-body problem. In "Sur les Figures d'équilibre d'une Masse Fluide", he treated tides and rotating fluid spheres. The latter was extended by George Darwin. Poincaré found that a rotating fluid having a pear shape (piriform) would be stable. Bell (1986) states that this conclusion is incorrect, but Boyer (1991) does not contradict it.

Poincaré also did work in partial differential equations and complex analysis. Poincaré also introduced modern methods of topology in "Analysis Situs" (1895), set forth the fundamentals of homology, used asymptotic series to solve differential equations, and extended the polyhedral formula for spaces of higher dimensionality using Betti numbers.

[From http://scienceworld.wolfram.com/biography/Poincare.html]