Symmetries in Physics - Fall 2018/19
Bielefeld University

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Exercise Nr. 5

Discussion on November 12, 14:15-15:45, Room U2-135

Exercises 12) should be handed in **before** the tutorial.

12) Permutation Groups *(2+2+4=8 points)*

a) Show that the alternating group A_n is a normal subgroup of S_n.

b) What is the order of A_n? Which group is S_n/A_n ?

c) How many conjugacy classes does S_6 have? Compute the table of classes.

13) Cyclic and Dihedral Groups *(2+2+4+4=12 points)*

Show that

a) a finite cyclic group is isomorphic to the quotient group $(\mathbb{Z}/n\mathbb{Z}, +_n)$ with $+_n$ addition modulo n,

b) an infinite cyclic group is isomorphic to $(\mathbb{Z}, +)$,

c) the dihedral group D_3 is isomorphic to $\mathrm{GL}(2, (\mathbb{Z}/2\mathbb{Z}, +_2))$ (invertible 2×2 matrices with entries 0 or 1).

d) the number of conjugacy classes of D_n for n even is $(n+6)/2$ and for n odd is $(n+3)/2$

Évariste Galois
(25 October 1811 - 31 May 1832)

[...] His mother served as Galois' sole teacher until he was 12 years old. She taught him Greek, Latin and religion where she imparted her own scepticism to her son. Galois' father was an important man in the community and in 1815 he was elected mayor of Bourg-la-Reine. [...] Galois was by this time [aftermath of the french revolution] at school. He had enrolled at the Lycée of Louis-le-Grand as a boarder in the 4 th class on 6 October 1823. Even during his first term there was a minor rebellion and 40 pupils were expelled from the school. Galois was not involved and during 1824-25 his school record is good and he received several prizes. However in 1826 Galois was asked to repeat the year because his work in rhetoric was not up to the required standard.

February 1827 was a turning point in Galois' life. He enrolled in his first mathematics class, the class of M. Vernier. He quickly became absorbed in mathematics [...] In 1828 Galois took the examination of the École Polytechnique but failed. It was the leading University of Paris and Galois must have wished to enter it for academic reasons. However, he also wished to enter this school because of the strong political movements that existed among its students, since Galois followed his parents example in being an ardent republican. Back at Louis-le-Grand, Galois enrolled in the mathematics class of Louis Richard. However he worked more and more on his own researches and less and less on his schoolwork. He studied Legendre's Géométrie and the treatises of Lagrange. As Richard was to report *This student works only in the highest realms of mathematics.* In April 1829 Galois had his first mathematics paper published on continued fractions in the Annales de mathématiques. On 25 May and 1 June he submitted articles on the algebraic solution of equations to the Académie des Sciences. Cauchy was appointed as referee of Galois' paper. Tragedy was to strike Galois for on 2 July 1829 his father committed suicide. The priest of Bourg-la-Reine forged Mayor Galois' name on malicious forged epigrams directed at Galois' own relatives. Galois' father was a good natured man and the scandal that ensued was more than he could stand. He hanged himself in his Paris apartment only a few steps from Louis-le-Grand where his son was studying. Galois was deeply affected by his father's death and it greatly influenced the direction his life was to take.

A few weeks after his father's death, Galois presented himself for examination for entry to the École Polytechnique for the second time. For the second time he failed, perhaps partly because he took it under the worst possible circumstances so soon after his father's death, partly because he was never good at communicating his deep mathematical ideas. Galois therefore resigned himself to enter the École Normale [..] Galois then took Cauchy's advice and submitted a new article "On the condition that an equation be soluble by radicals" in February 1830. [...] In December 1830 M. Guigniault wrote newspaper articles attacking the students and Galois wrote a reply in the Gazette des Écoles, attacking M. Guigniault for his actions in locking the students into the school. For this letter Galois was expelled and he joined the Artillery of the National Guard, a Republican branch of the militia. [...] During the dinner [of 200 republicants] Galois raised his glass and with an open dagger in his hand appeared to make threats against the King, Louis-Phillipe. After the dinner Galois was arrested and held in Sainte-Pélagie prison. [...] While in Sainte-Pélagie prison Galois attempted to commit suicide by stabbing himself with a dagger but the other prisoners prevented him. [...] In March 1832 a cholera epidemic swept Paris and prisoners, including Galois, were transferred to the pension Sieur Faultrier. There he apparently fell in love with Stephanie-Felice du Motel, the daughter of the resident physician. [...] Galois fought a duel with Perscheux d'Herbinville on 30 May, the reason for the duel not being clear but certainly linked with Stephanie. [...] Galois was wounded in the duel and was abandoned by d'Herbinville and his own seconds and found by a peasant. He died in Cochin hospital on 31 May and his funeral was held on 2 June. [...] Galois' brother and his friend Chevalier copied his mathematical papers and sent them to Gauss, Jacobi and others. [..] However the papers reached Liouville who, in September 1843, announced to the Academy that he had found in Galois' papers a concise solution *…as correct as it is deep of this lovely problem: Given an irreducible equation of prime degree, decide whether or not it is soluble by radicals.* Liouville published these papers of Galois in his Journal in 1846. The theory that Galois outlined in these papers is now called Galois theory.

[From www-history.mcs.st-andrews.ac.uk/Biographies/Galois.html [J. J. O'Connor and E. F. Robertson]