Symmetries in Physics - Fall 2018/19

Bielefeld University

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Exercise Nr. 4

Discussion on November 5, 14:15-15:45, Room U2-135

Exercises 10) should be handed in **before** the tutorial.

10) Conjugacy Classes *(3+2=5 points)*

a) Show that conjugation is indeed an equivalence relation that gives rise to a partition of $\mathcal{G}$.

b) Determine all the conjugacy classes of the permutation group of 3 objects, S_3.

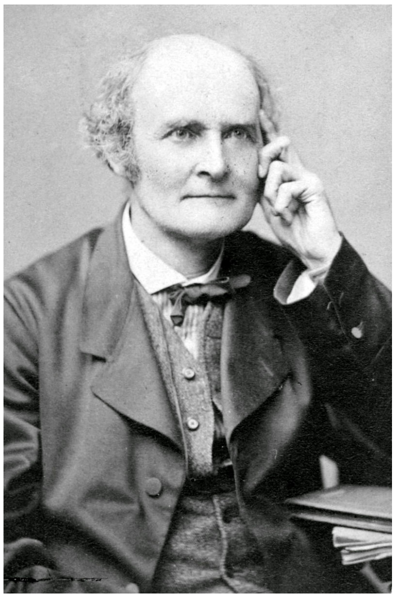
11) Direct Products *(3+2=5 points)*

a) Determine the Cayley table of the direct product $Z_2 \times Z_3$ and show that it is isomorphic to Z_6, with Z_n defined as the quotient group Z/nZ, i.e. integer addition modulo n.

b) Determine the neutral element and the inverse of any element of the group of rigid movements, defined as the semi-direct product

$$\mathcal{E}_3 = \mathbb{R}^3 \rtimes O(3) : (\vec{a}, R) * (\vec{a}', R') = (\vec{a} + R'\vec{a}', RR').$$

Arthur Cayley
(16 August 1821 - 26 January 1895)



Brilliant English mathematician who primarily worked in algebra. Even as a third year student at Cambridge, the examiner put him in a class by himself–above the first. He had an uncanny memory. He also was an avid novel reader and mountaineer. He had difficulty, however, obtaining a job after graduation, so became a lawyer for 14 years. During his free time, he published more than 200 mathematical papers. He initiated analytic geometry of n-dimensional spaces and was one of the first to study matrices in “On the Theory of Linear Transformations” (1845). He also developed the theory of invariants, and studied the geometry of plane curves. He showed, for instance, that two circles intersect in four points, two of them being imaginary. He unified metric and projective geometry.

[From http://scienceworld.wolfram.com/biography/Cayley.html [Eric. W. Weisstein]]