# Symmetries in Physics - Fall 2018/19

**Bielefeld University**

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# Exercise Nr. 3

Discussion on October 29, 14:15-15:45, Room U2-135

Exercises 8) should be handed in **before** the tutorial.

### 7) Group Presentation *(2+3=5 points)*

Which two groups are defined by the following presentations? Write down the Cayley table.

a) $\langle a, b | a^2 = e, b^2 = e, (ab)^3 = e \rangle$

b) $\langle a, b | a^5 = e, b^2 = e, abab^{-1} = e \rangle$

### 8) Subgroups, Conjugate Subgroups and Normal Subgroups *(3+3+3=9 points)*

a) Find all the proper subgroups of the group of permutation of four objects (which has 4! elements). Are there also normal subgroups?

b) Let $\mathcal{H}$ be a subgroup of $\mathcal{G}$. Show that for all $g \in \mathcal{G}$ the set $g\mathcal{H}g^{-1}$ is also a subgroup of $\mathcal{G}$, and that it is isomorphic to $\mathcal{H}$. That is show that the conjugate subgroup is indeed a group.

c) Let $\mathcal{N}$ be a normal subgroup of $\mathcal{G}$ and $\phi \in \mathrm{Hom}(\mathcal{G}, \mathcal{G}')$. Show that $\phi(\mathcal{N})$ is a normal subgroup of $\mathcal{G}'$

### 9) Cosets *(3+3=6 points)*

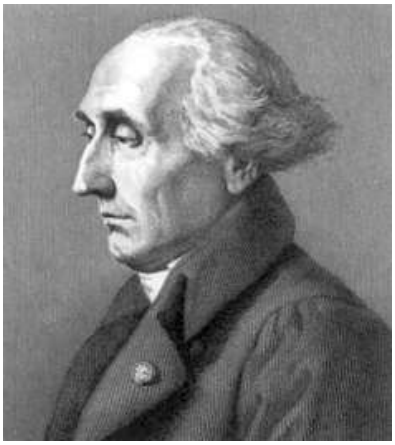
a) Determine all left cosets and right cosets of the permutation group of 3 objects.

b) Show that the relation on the left cosets of the subgroup $\mathcal{H} \leq \mathcal{G}$ defined by

$$a \sim_l b \quad \Leftrightarrow \quad b \in a\mathcal{H}$$

is ineeed an equivalence relation (check that it is reflexive, symmmetric and transitive).

**Joseph-Louis Lagrange**
(25 January 1736 - 10 April 1813)



Joseph-Louis Lagrange is usually considered to be a French mathematician [...] He studied at the College of Turin and his favourite subject was classical Latin. At first he had no great enthusiasm for mathematics, finding Greek geometry rather dull. Lagrange's interest in mathematics began when he read a copy of Halley's 1693 work on the use of algebra in optics. [...] He certainly did devote himself to mathematics, but largely he was self taught and did not have the benefit of studying with leading mathematicians. [...] He began working on the tautochrone, the curve on which a weighted particle will always arrive at a fixed point in the same time independent of its initial position. By the end of 1754 he had made some important discoveries on the tautochrone which would contribute substantially to the new subject of the calculus of variations (which mathematicians were beginning to study but which did not receive the name 'calculus of variations' before Euler called it that in 1766).

[...] Although he was still only 19 years old, Lagrange was appointed professor of mathematics at the Royal Artillery School in Turin on 28 September 1755. [...] The papers by Lagrange which appear in these transactions cover a variety of topics. He published his beautiful results on the calculus of variations, and a short work on the calculus of probabilities. In a work on the foundations of dynamics, Lagrange based his development on the principle of least action and on kinetic energy. [...] Lagrange studied the integration of differential equations and made various applications to topics such as fluid mechanics (where he introduced the Lagrangian function). Also contained are methods to solve systems of linear differential equations which used the characteristic value of a linear substitution for the first time. Another problem to which he applied his methods was the study the orbits of Jupiter and Saturn.

[...] Lagrange succeeded Euler as Director of Mathematics at the Berlin Academy on 6 November 1766. [...] His work in Berlin covered many topics: astronomy, the stability of the solar system, mechanics, dynamics, fluid mechanics, probability, and the foundations of the calculus. He also worked on number theory proving in 1770 that every positive integer is the sum of four squares. In 1771 he proved Wilson's theorem that $n$ is prime if and only if $(n-1)!+1$ is divisible by $n$. In 1770 he also presented his important work "Réflexions sur la résolution algébrique des équations" which made a fundamental investigation of why equations of degrees up to 4 could be solved by radicals. The paper is the first to consider the roots of an equation as abstract quantities rather than having numerical values. He studied permutations of the roots and, although he does not compose permutations in the paper, it can be considered as a first step in the development of group theory continued by Ruffini, Galois and Cauchy.

[...] During his years in Berlin his health was rather poor on many occasions, and that of his wife was even worse. She died in 1783 after years of illness and Lagrange was very depressed. [...] On 18 May 1787 he left Berlin to become a member of the Académie des Sciences in Paris, where he remained for the rest of his career. [...] The "Mécanique analytique" which Lagrange had written in Berlin, was published in 1788. [..] With this work Lagrange transformed mechanics into a branch of mathematical analysis. [...] Lagrange was its first professor [of the École Polytechnique] of analysis, appointed for the opening in 1794. [...] Napoleon named Lagrange to the Legion of Honour and Count of the Empire in 1808. On 3 April 1813 he was awarded the "Grand Croix of the Ordre Impérial de la Réunion". He died a week later.

[From www-groups.dcs.st-and.ac.uk/history/Biographies/Lagrange.html
(J J O'Connor and E F Robertson)]