# Symmetries in Physics - Fall 2018/19

**Bielefeld University**

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# Exercise Nr. 1

Discussion on October 15, 14:00-16:00, Room U2-135

Exercises 1) and 2) should be handed in **before** the tutorial.

### 1) Review of Classical Mechanics *(2+2+2=6 points)*

A system has the following Lagrange function:

$$L = \frac{1}{2}m(\dot{r}^2 + r^2\dot{\theta}^2) + \frac{k}{r}$$

a) Determine the equations of motion via the Euler-Lagrange equation.

b) Compute the conjugate momentum $p_r$ and $p_\theta$.

c) Are there conservation laws? If so, which are there? Can you identify symmetry transformations which do not change the Lagrangian?

### 2) Review of Quantum Mechanics *(2+2+1=5 points)*

A particle moves in a spherically symmetric potential $V(r)$.

a) Show that

$$[\hat{L}_i, \hat{x}_j] = i\hbar \sum_k \epsilon_{ijk} x_k, \quad \text{where} \quad \hat{L}_i = \frac{\hbar}{i} \sum_{j,k} \epsilon_{ijk} x_j \frac{\partial}{\partial x_k}.$$

b) Show that

$$[L_i, V(r)] = 0.$$

c) Show by using b) that also

$$[\vec{L}^2, V(r)] = 0.$$

### 3) Symmetry Transformation *(2+2+2+3=9 points)*

What are the symmetry transformations (such as reflections, rotations) of ...

a) ... a snowflake

b) ... the letters A, F, S, O and X?

c) ... a donut (without topping)

d*) ... a dice (not taking into account the numbers)

Explain this by descibing the symmetry geometrically.

**Amalie Emmy Noether**
(23 March 1882 - 14 April 1935)

German mathematician whose innovations in higher algebra gained her recognition as the most creative abstract algebraist of modern times. Noether received a Ph.D. degree from the University of Erlangen in 1907, with a dissertation on algebraic invariants. From 1913 she lectured occasionally at Erlangen, substituting for her father, Max Noether (1844-1921). In 1915 she went to the University of Göttingen and was persuaded by the eminent mathematicians David Hilbert and Felix Klein to remain there over the objections of some faculty members; she won formal admission as an academic lecturer in 1919. The appearance of "Moduln in nichtkommutativen Bereichen, insbesondere aus Differential- und Differenzen-Ausdrücken" (1920; "Concerning Moduli in Noncommutative Fields, Particularly in Differential and Difference Terms"), written in collaboration with a Göttingen colleague, Werner Schmeidler, and published in Mathematische Zeitschrift, marked the first notice of Noether as an extraordinary mathematician. For the next six years her investigations centred on the general theory of ideals (special subsets of rings), for which her residual theorem is an important part. On an axiomatic basis she developed a general theory of ideals for all cases. Her abstract theory helped draw together many important mathematical developments.

From 1927 Noether concentrated on noncommutative algebras (algebras in which the order in which numbers are multiplied affects the answer), their linear transformations, and their application to commutative number fields. She built up the theory of noncommutative algebras in a newly unified and purely conceptual way. In collaboration with Helmut Hasse and Richard Brauer, she investigated the structure of noncommutative algebras and their application to commutative fields by means of cross product (a form of multiplication used between two vectors). Important papers from this period are "Hyperkomplexe Grössen und Darstellungstheorie" (1929; "Hypercomplex Number Systems and Their Representation") and "Nichtkommutative Algebra" (1933; "Noncommutative Algebra"). In addition to research and teaching, Noether helped edit the Mathematische Annalen. From 1930 to 1933 she was the centre of the strongest mathematical activity at Göttingen. The extent and significance of her work cannot be accurately judged from her papers. Much of her work appeared in the publications of students and colleagues; many times a suggestion or even a casual remark revealed her great insight and stimulated another to complete and perfect some idea. When the Nazis came to power in Germany in 1933, Noether and many other Jewish professors at Göttingen were dismissed. In October she left for the United States to become visiting professor of mathematics at Bryn Mawr College and to lecture and conduct research at the Institute for Advanced Study in Princeton, New Jersey.

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