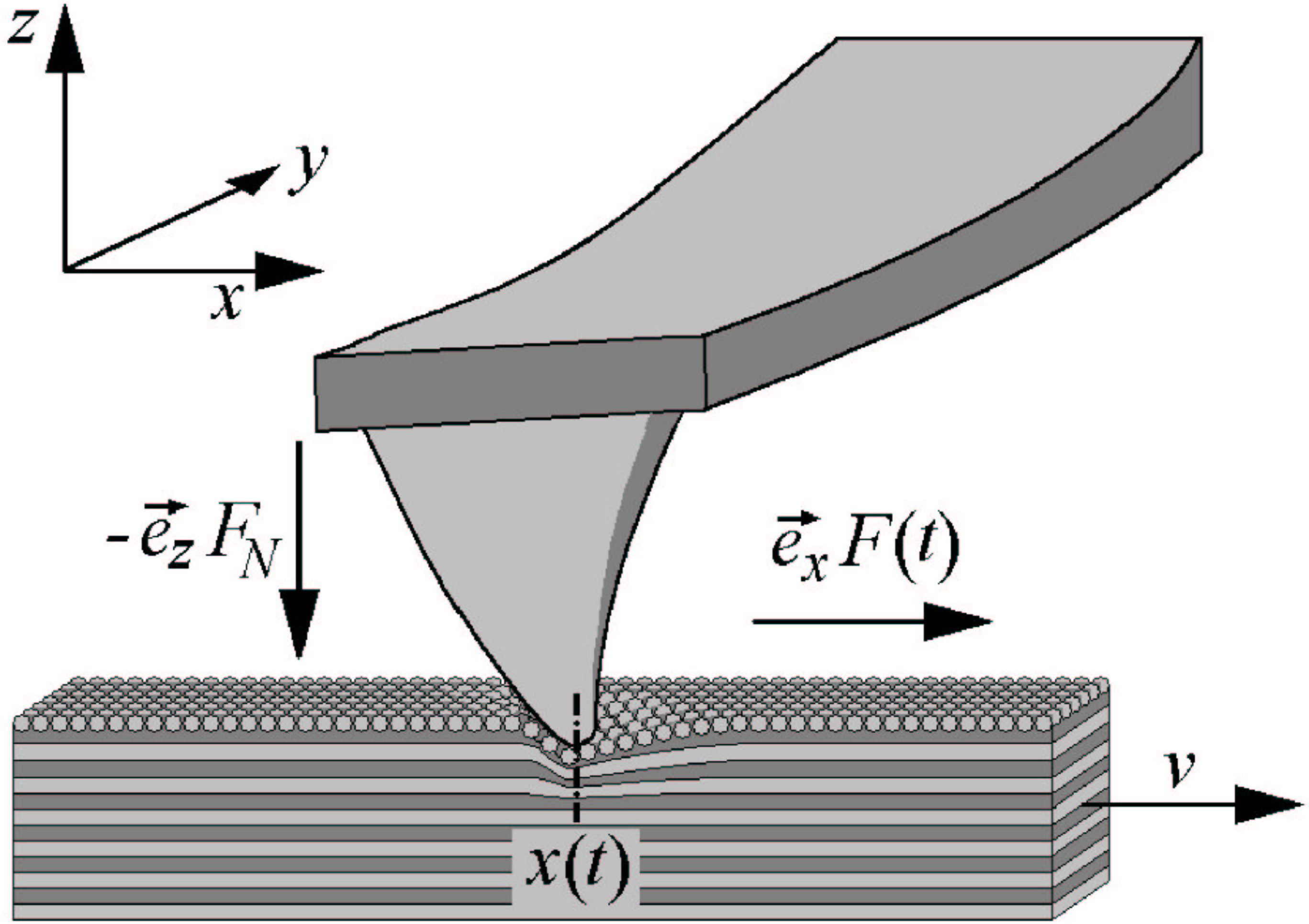
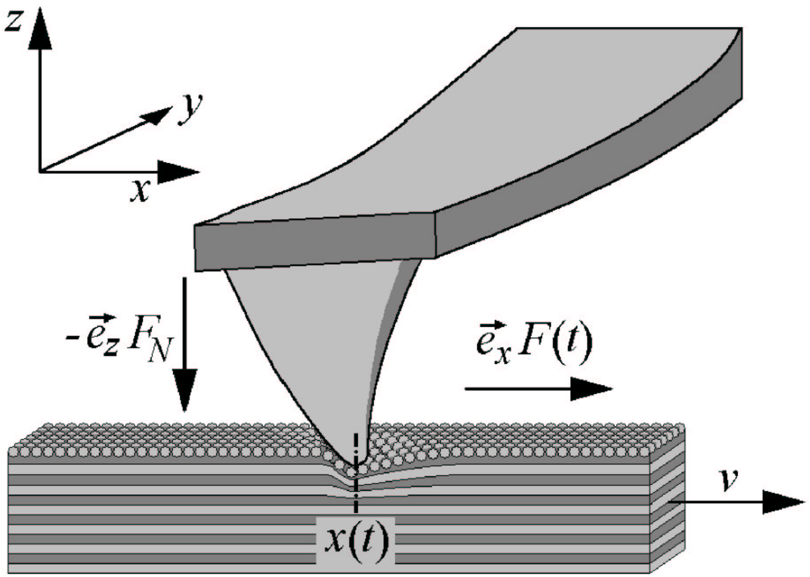
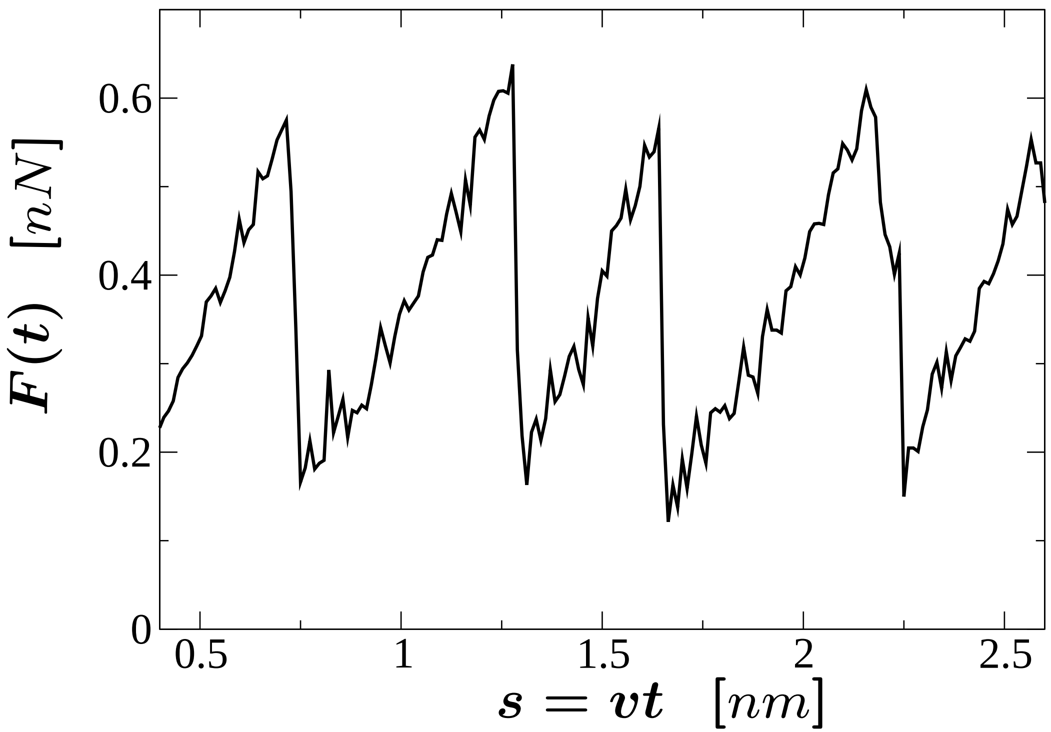


- cantilever dimensions $\approx$ 200 μm $\times$ 50 μm $\times$ 1 μm
- tip height and basis radius $\approx$ 5 μm $\times$ 1 μm
- tip apex radius $\approx$ 10 nm
- lattice constant $L \approx$ 0.5 nm

Friction Force Microscopy





- stick-slip motion
- "atomic resolution" ($L \approx 0.5$ nm)
- thermal noise effects

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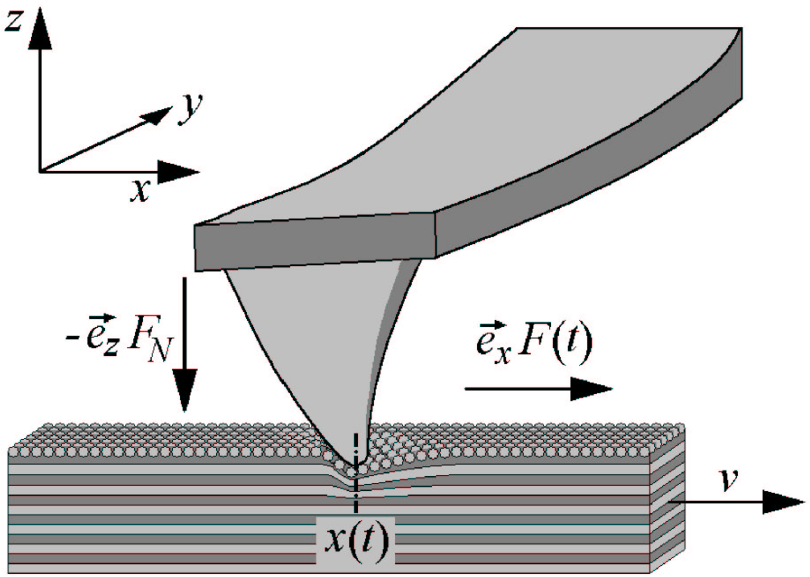

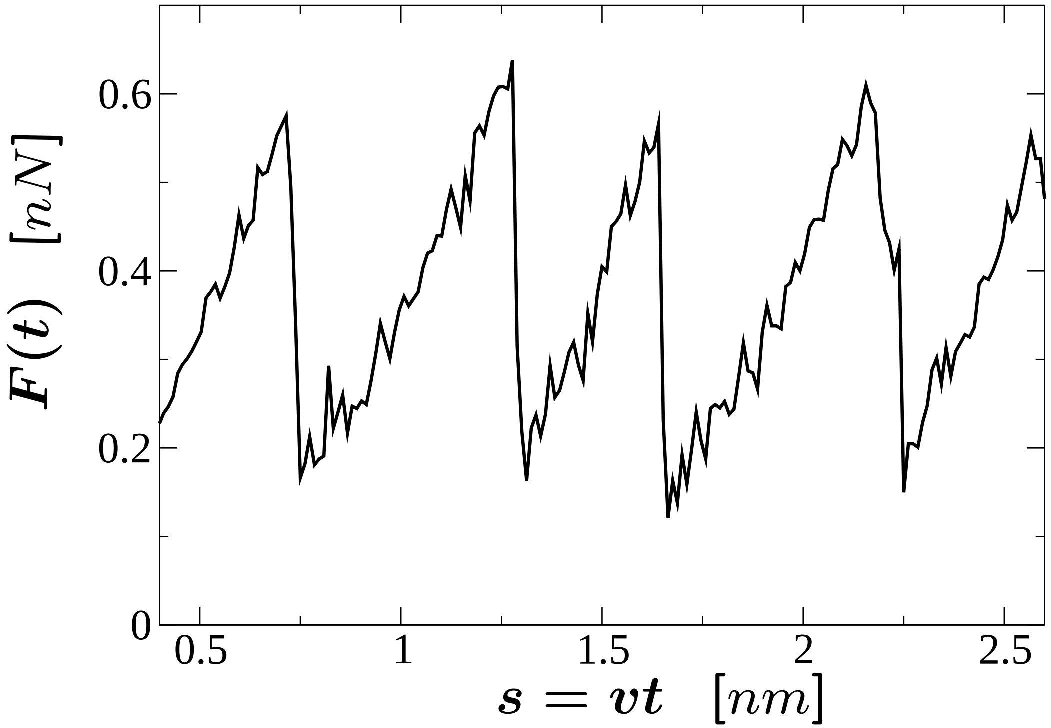

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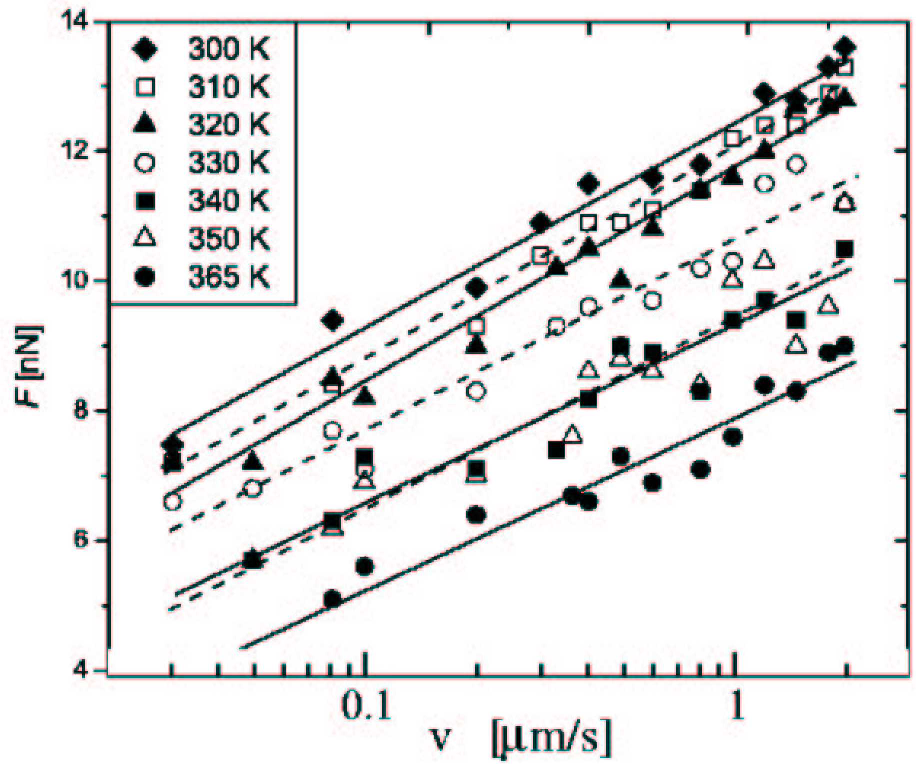
Quantity of main interest: $\bar{F} := \lim_{t\to\infty} \frac{1}{t} \int_0^t dt' \, F(t')$

$\bar{F}$ versus v



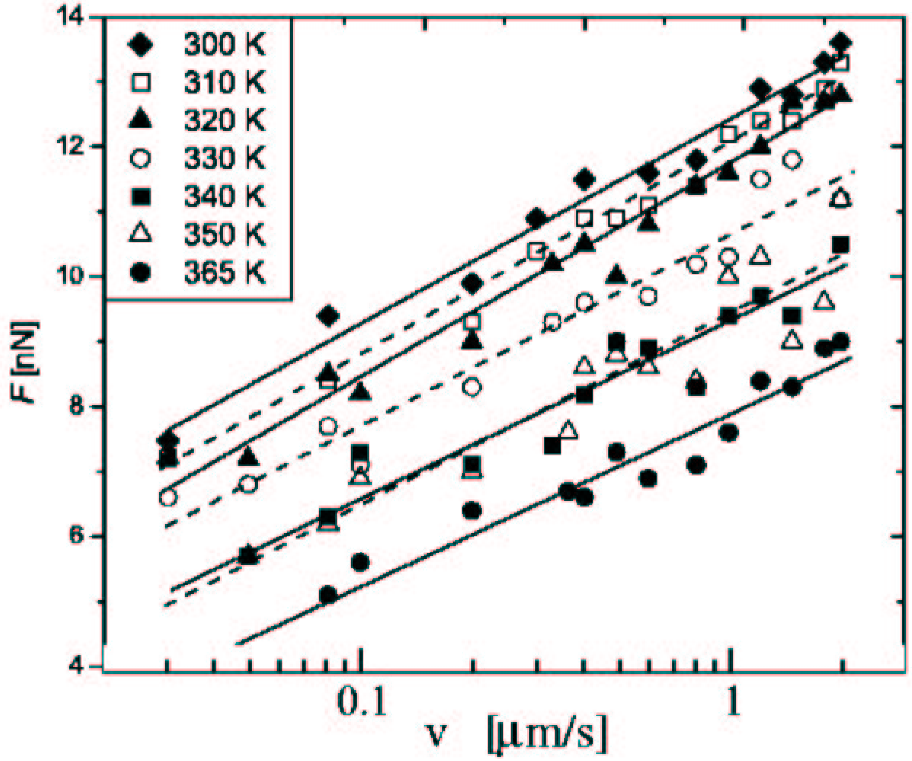
Sills and Overney, PRL 91, 095501 (2003)

glassy polysterene surface

$F_N = 15$ nN

- thermal noise matters

$\bar{F}$ versus v



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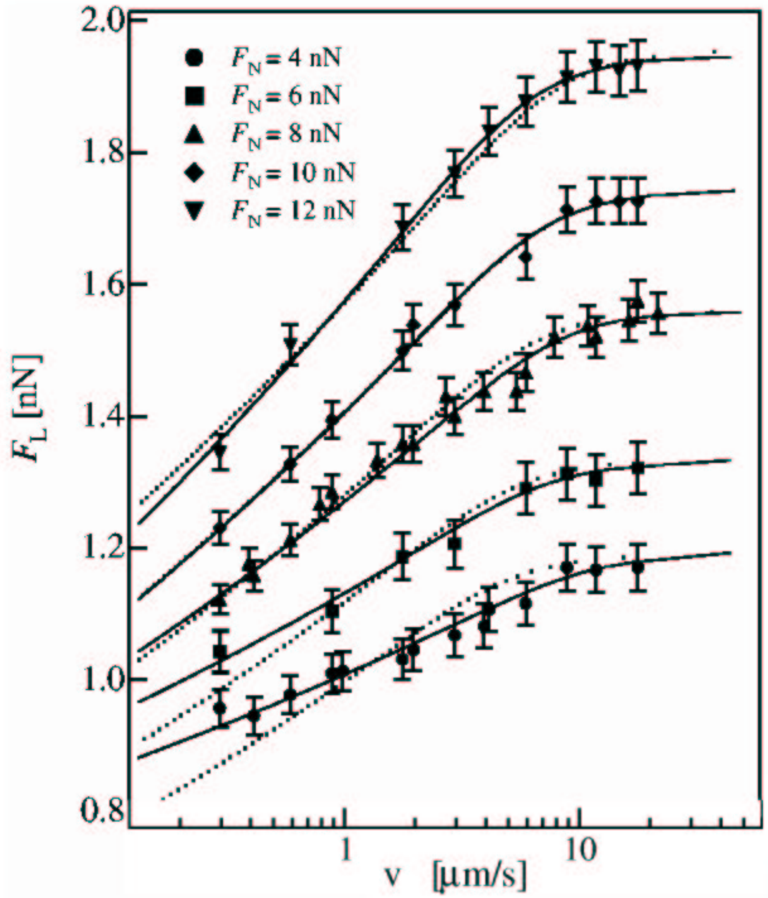


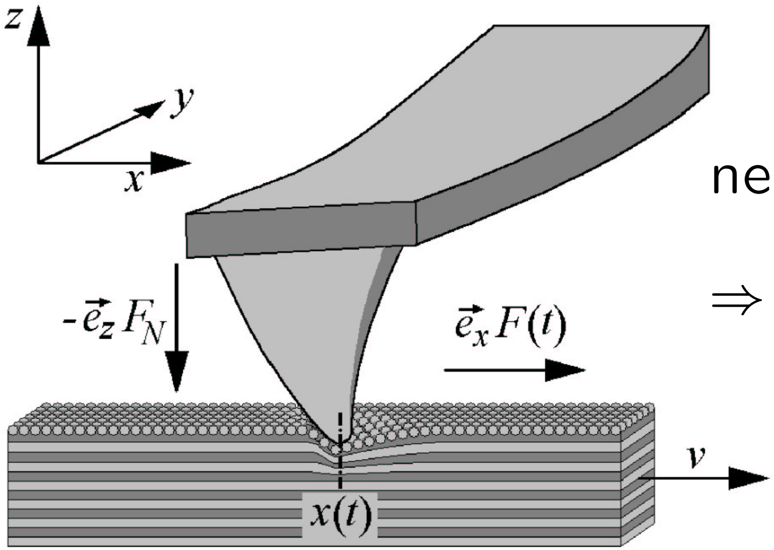
Riedo et al., PRL 91, 084502 (2003)

mica surface

$T = 293$ K

- "plateaux" at large v

Model



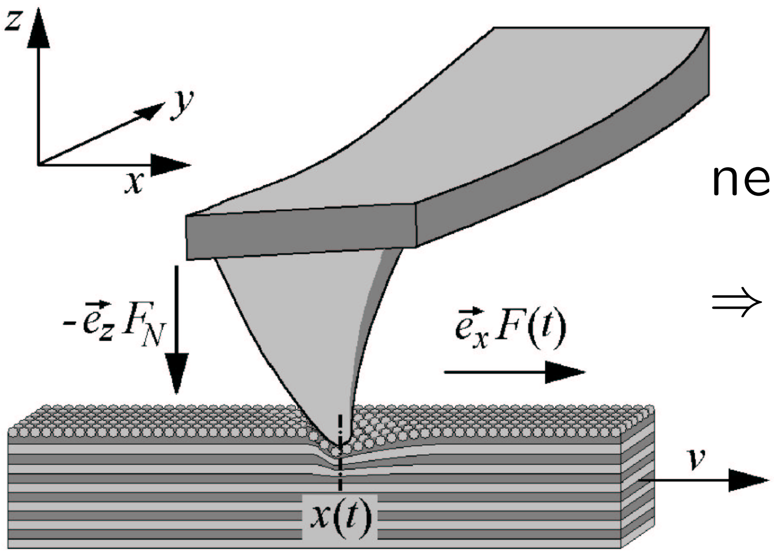
neglect "fast" thermal fluctuations of molecules

⇒ 2 "slow" state variables/collective coordinates:

x-coordinate of tip apex (rest position $x = 0$)

s : position of substrate along x-axis

Model



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- given x and s, the entire "global" configuration is uniqely fixed
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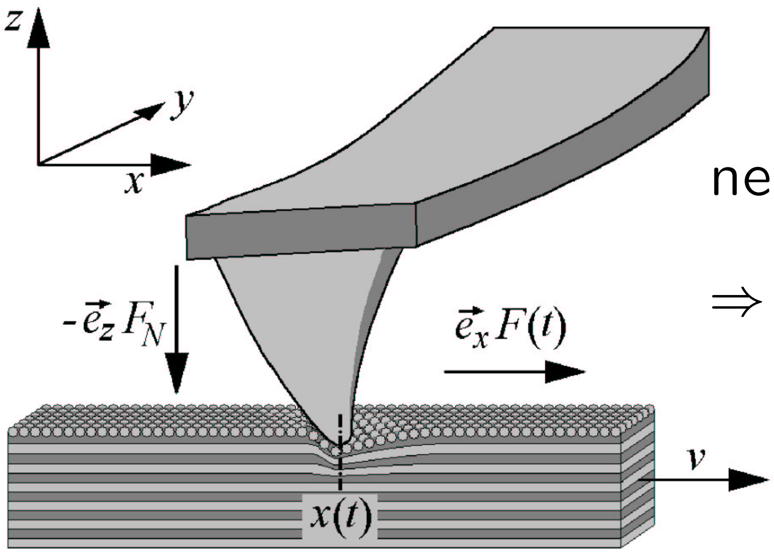

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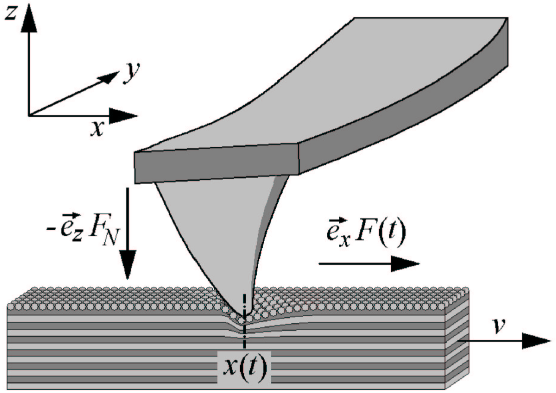
$\Rightarrow$ goal: equation of motion for $x(t)$



$x = x(t)$: position of tip apex (rest position $x = 0$)

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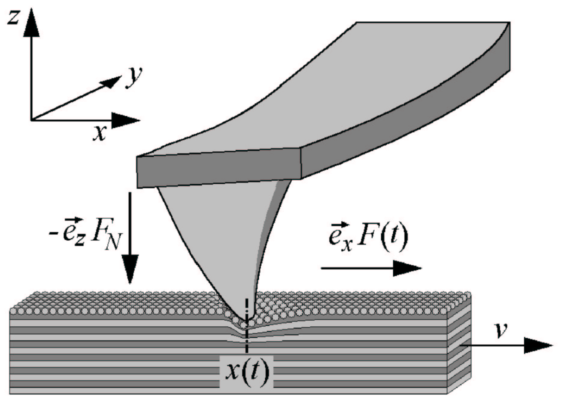
substrate potential $U(x-s)$ with $\boxed{U(x+L) = U(x)}$

elastic force $\boxed{-\kappa\,x(t) = -F(t)}$ [$\kappa \approx 1$ nN/nm]



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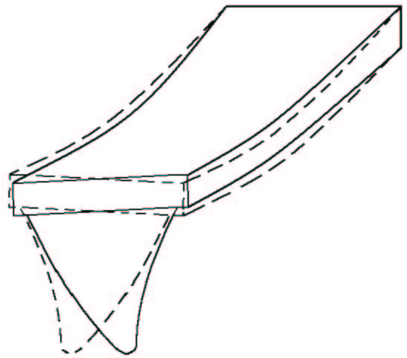
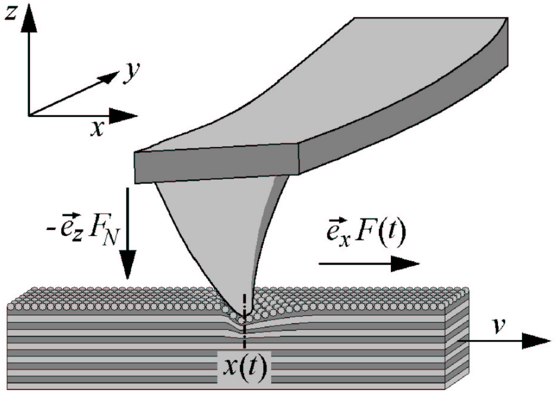
"fast" fluctuations of molecules $\Rightarrow$ thermal bath effects (close to eq.)

- dissipation of cantilever & tip $\boxed{-\eta_c\, \dot{x}(t)}$
- concomitant thermal noise $\boxed{\sqrt{2\eta_c k T}\, \xi_c(t)}$





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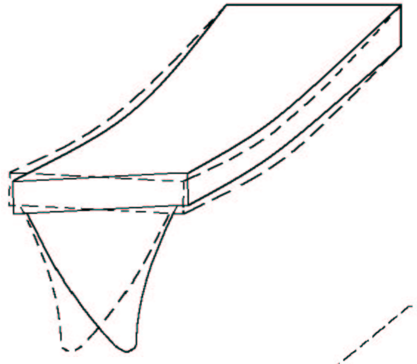

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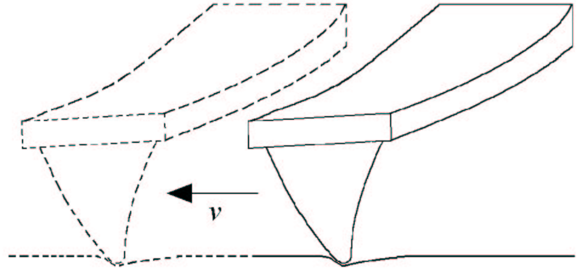
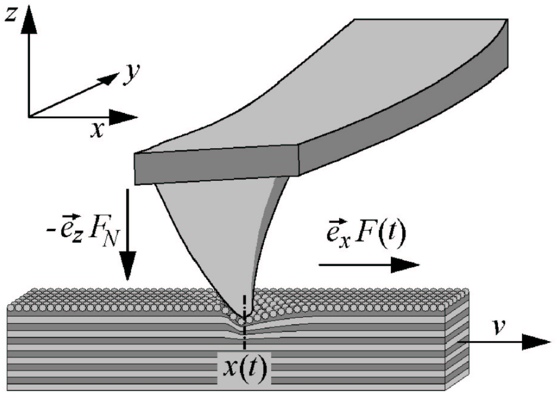


- dissipation of substrate $\boxed{-\eta_s\, (\dot{x}(t) - v)}$
- concomitant thermal noise $\boxed{\sqrt{2\eta_s kT}\, \xi_s(t)}$



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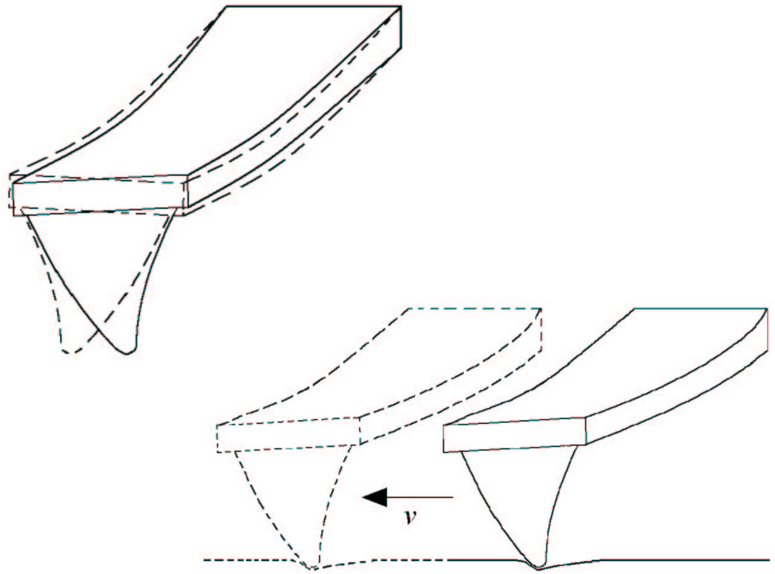
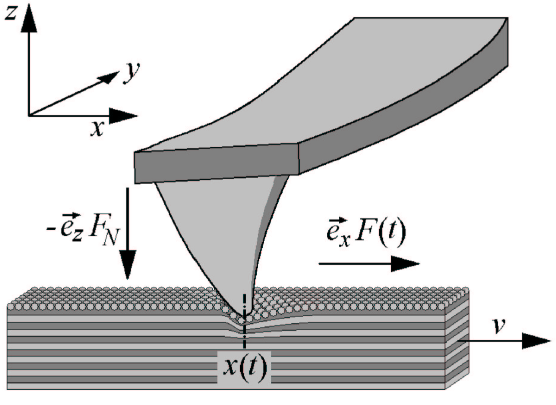


$$\boxed{m\ddot{x}(t) = -U'(x(t)-vt) - \kappa x(t) - \eta_c \dot{x}(t) + \sqrt{2\eta_c kT}\xi_c(t) - \eta_s(\dot{x}(t)-v) + \sqrt{2\eta_s kT}\xi_s(t)}$$



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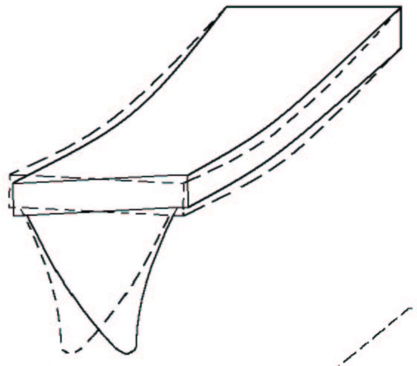

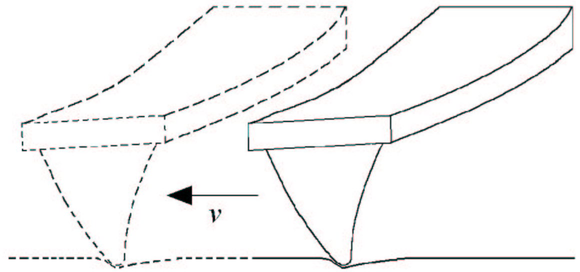

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$$\eta := \eta_c + \eta_s\ ,\quad \vartheta := \eta_c/\eta\ ,\quad X(t) := x(t) - v\,t\ ,\quad F(t) = \kappa\, x(t) = \kappa\,(X(t) + vt)$$

$$\boxed{\boldsymbol{m\,\ddot{X}(t) = -U'(X(t)) - F(t) - \vartheta\eta v - \eta\dot{X}(t) + \sqrt{2\eta kT}\,\xi(t)}}$$