Quantum Mechanics: Exercises 9

Due to: January 8, 2013.

Problem 1

Prove the identity

$$(\boldsymbol{\sigma} \cdot \boldsymbol{A})(\boldsymbol{\sigma} \cdot \boldsymbol{B}) = \boldsymbol{A} \cdot \boldsymbol{B} + i\, \boldsymbol{\sigma} \cdot (\boldsymbol{A} \times \boldsymbol{B}), \tag{1}$$

where $\boldsymbol{\sigma} = (\sigma_x, \sigma_y, \sigma_z)$ and $\boldsymbol{A}$ and $\boldsymbol{B}$ are some vector operators which commute with σ but do not necessarily commute with each other.

Problem 2

Find expectation value of of J_x and standard deviation ΔJ_x in the state $|j, m\rangle$.

Problem 3

Using

$$\hat{J}_z |j, m\rangle = \hbar m |j, m\rangle \tag{2}$$

show that for $j = 1$ we have

$$\hat{J}_z = \hbar \begin{pmatrix} 1 & 0 & 0 \\ 0 & 0 & 0 \\ 0 & 0 & -1 \end{pmatrix} \tag{3}$$

and derive expressions for $\hat{J}_+$,$\hat{J}_-$,$\hat{J}_x$, $\hat{J}_y$ and $\hat{J}^2$ (correct answer is in the lecture note p. 77).
Show that $\hat{J}_x^2$, $\hat{J}_y^2$ and $\hat{J}_z^2$ are commutative.