

General Relativity: Exercises 7

Till: July 6, 2011

Homework 1: Free fall to Black Hole

Far away (at infinity) from black hole is observer who observe test-particle free falling towards black hole with zero initial velocity. In Newtonian gravity velocity of test-particle observed by observer would be monotonically increasing quantity.

a) Find expression for velocity of free fall from infinity in Newtonian theory. What is velocity of test particle when it is at $2M$ distance from centre of black hole (i.e. distance which would correspond to black hole horizon)?

In GTR situation is different because distant observer would see test-particle initially accelerating towards black hole and then he would see test-particle to decelerate and finally to stop and hang over horizon *).

b) Use information that for particle free falling to black hole from infinity with initial zero velocity energy

$$\frac{E}{m} = \left(1 - \frac{2M}{r}\right) \frac{dt}{d\tau} = 1, \quad (1)$$

is conserved. Find expression for velocity of test-particle observed by observer at infinity.

c) Find at what distance from black hole horizon test-particle would have maximum free fall velocity (as observed by observer at infinity).

* Note: This is "little-bit" idealized: observer at infinity would not see particle hanging over horizon, because particle would get infinite red-shift and would also become "black" for distant observer. However, neglect this in following.