General Relativity: Exercises 6

Till: June 27, 2011

Homework 1: De Sitter solution

Consider vacuum Einstein equations with non-zero cosmological constant

$$R_{\mu\nu} - \frac{1}{2} g_{\mu\nu} R + \Lambda g_{\mu\nu} = 0. \tag{1}$$

1) Find scalar curvature R in this case.
2) Assume spherical symmetry, i.e. metric in form

$$ds^2 = -e^{-\nu(r)} dt^2 + e^{\lambda(r)} dr^2 + r^2 (d\theta^2 + \sin^2\theta d\phi^2), \tag{2}$$

and solve vacuum Einstein equations with non-zero cosmological constant. You should be able to obtain solution

$$ds^2 = -(1 - \frac{\Lambda}{3} r^2) dt^2 + \frac{1}{(1 - \frac{\Lambda}{3} r^2)} dr^2 + r^2 (d\theta^2 + \sin^2\theta d\phi^2). \tag{3}$$

Hint: Components of Ricci tensor for metric (2) You can find for example in chapter 8.3 (review from 15th June)

Homework 2: Neutron star

In order to solve Tolman-Oppenheimer-Volkoff (TOV) Equation

$$\frac{dp(r)}{dr} = -\frac{G}{r} \frac{\left[\rho(r) + \frac{p(r)}{c^2}\right] \left[m(r) + 4\pi r^3 \frac{p(r)}{c^2}\right]}{\left[r - \frac{2Gm(r)}{c^2}\right]}, \tag{4}$$

and

$$m(r) = 4\pi \int_0^r \rho(r') r'^2 dr', \tag{5}$$

You need to know equation of state. Use most simple equation of state $\rho(p) = \rho_0 = 4.10^{17}$ kg/m^3, where ρ_0 is nuclear density, to determine maximal radius of neutron star.

Hint: Integrate TOV equation in order to find relation between pressure and radius. You require for physical neutron star that pressure at center of neutron star $p(0)$ should be finite. Find maximal radius of star at which "central" pressure will be finite.