General Relativity: Exercises 4

Till: June 6, 2011

Homework 1: Gravitation radius

On lecture was derived Schwarzchild solution, where important quantity is "gravitational" or "Schwarzchild" radius, which tells us what would be a radius of black hole with given mass. You have already found gravitational radius for Sun and Earth.

a) In centre of Milky Way it seems that there is supermassive black hole with mass of 4.1 million solar masses. Find gravitational radius of this black hole.

b) Neglect electric charge and angular momentum of electron and picture it as scalar particle with mass m_e. What would be gravitational radius of electron? Express this in terms of Planck length.

c) What would be gravitational radius of black hole with mass of Moon?

Homework 2: Proper and "radar" distance in Schwarzchild solution

There are two ways how to measure distance in Schwarzchil solution:

1) Proper distance between points r_1 and r_2, i.e.

$$l_p(r_1, r_2) = \int_{r_1}^{r_2} \sqrt{g_{rr}} dr, \tag{1}$$

what corresponds to distance which we would measure by ruler between points r_1 and r_2.

2) "Radar" distance, i.e. distance measured by light. Imagine that observer at r_1 has flashlight and at r_2 is a mirror. Observer would point his flashlight in direction of miror and he would count how long does it take to light to make roundtrip. Then what he would call "radar" distance is simply distance which light would travel during this time (actually half of it because light makes roundtrip). You know that light travel along null-geodetic, i.e.

$$0 = g_{tt} dt^2 + g_{rr} dr^2. \tag{2}$$

Task of homeworks is:

a) Find expression for "radar" distance

$$l_R(r_1, r2) =? \tag{3}$$

b) Imagine that observer is sitting at $r_1 = a$, where a is gravitational radius of black hole and r_2 is some distance $r > a$. Find expressions for both proper distance and "radar" distance between points r_1 and r_2.

c) Discuss difference between these two kinds of distances in case b). Which of them is finite?