General Relativity: Exercises 3

Till: June 1, 2011

Homework 1: Ideal Fluid

Energy-Momentum tensor of ideal fluid in the frame which moves with velocity $\mathbf{v}(t, \mathbf{x})$ is

$$T^{\mu\nu} = p\eta^{\mu\nu} + (p + \rho)U^\mu U^\nu, \tag{1}$$

where p is pressure, ρ is density and $U^\mu = dx^\mu/d\tau$ is four-velocity. Show that from conservation law

$$\frac{\partial T^{\mu\nu}}{\partial x^\nu} = 0 \tag{2}$$

follows special relativistic equation for an ideal fluid. Find its form expressed in $\mathbf{v}(t, \mathbf{x})$, i.e.

$$\frac{\partial \mathbf{v}}{\partial t} + (\mathbf{v}\nabla)\,\mathbf{v} = ???. \tag{3}$$

Homework 2: Lie Derivative

a)Show that Lie derivative of vector field V obeys relation

$$\triangle_\epsilon V = [\epsilon, V], \tag{4}$$

where both V and ϵ are vector fields

$$V = V^\mu \partial_\mu, \qquad \epsilon = \epsilon^\mu \partial_\mu, \tag{5}$$

i.e., V^μ is component of vector field and ∂_μ it's basis.

b) Lie derivative tells us how some quantity change under change of coordinates. You know how Lie derivative acts on metric. Take metric on a 2-sphere and find such vector field

$$\epsilon = u\frac{\partial}{\partial\theta} + v\frac{\partial}{\partial\phi} \tag{6}$$

which leaves metric of a 2-sphere unchanged.

Note: There exists three indepenent solutions of equation which You will obtain.