General Relativity: Exercises 2

Till: May 25, 2011

Homework 1: Foucault Pendulum

Take spherical coordinates on 2-sphere from last homeworks (ϕ, θ) and at some point $(\theta = \theta_0, \phi = 0)$ consider vector V with components $V = (V^\phi, V^\theta) = (0, 1)$. Write components of this vector parallel transported to point $(\theta = \theta_0, \phi)$. Show that this vector after paralel transport along the full circle $(\phi = 0$ to $\phi = 2\pi)$ will be rotated by angle

$$\alpha = 2\pi \cos \theta_0 \qquad (1)$$

This is the same angle by which Foucault pendulum will be rotated in one day. Explain this.

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Hint: Equation of paralel transport on 2-sphere are

$$\dot{V}^\mu + \Gamma^\mu_{\kappa\sigma} \dot{x}^\kappa V^\sigma = 0 \qquad (2)$$

where $\Gamma^\mu_{\kappa\sigma}$ are Christoffel symbols on 2-sphere (see last two homeworks). Write explicitly these two equations for coordinates $(\theta = \theta_0, \phi)$ and choose parametrization $t = \phi$. Work out obtained differential equations.

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Note: Here angle θ is measured on z-axis from North Pole (i.e. equator is on $\theta = 90°$ and North Pole is on $\theta = 0$) while geographical latitute is measured from equator (i.e. equator is on $\theta = 0$ and North Pole is on $\theta = 90°$). So, when measured in this "geographical quantity" we are indeed obtaining known result for Foucault pendulum $\alpha = 2\pi \sin \theta_0$.

Homework 2: Twin Paradoxes

a) Satellite of GPS system carry atomic clock and orbits Earth with orbital radius $26,600$ km with corresponding orbital velocity. In Special Relativity is well-known *twin paradox*, i.e. clock carried by satellite tick at different rate than those on surface of Earth. Find time difference between these two clocks accumulated in one day.

b) In General Relativity exists similar effect and clock carried by satellite will be ticking also at different rate, than one at surface of Earth, because of gravitational field. Use knowledge from chapter on Newtonian limit and find how much time difference acummulate between these two clocks in one day due to this GR effect.

c) What is net time difference (SR and GR effects combined) between these two clocks acummulated in one day?