Five lectures on

INTRODUCTION TO COSMOLOGY

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Lecture 1: The large picture

observations, cosmological principle, Friedmann model, Hubble diagram, thermal history

Lecture 2: From quantum to classical

cosmological inflation, isotropy & homogeneity, causality, flatness, metric & matter fluctuations

Lecture 3: Hot big bang

radiation domination, hot phase transitions, relics, nucleosythesis, cosmic microwave radiation

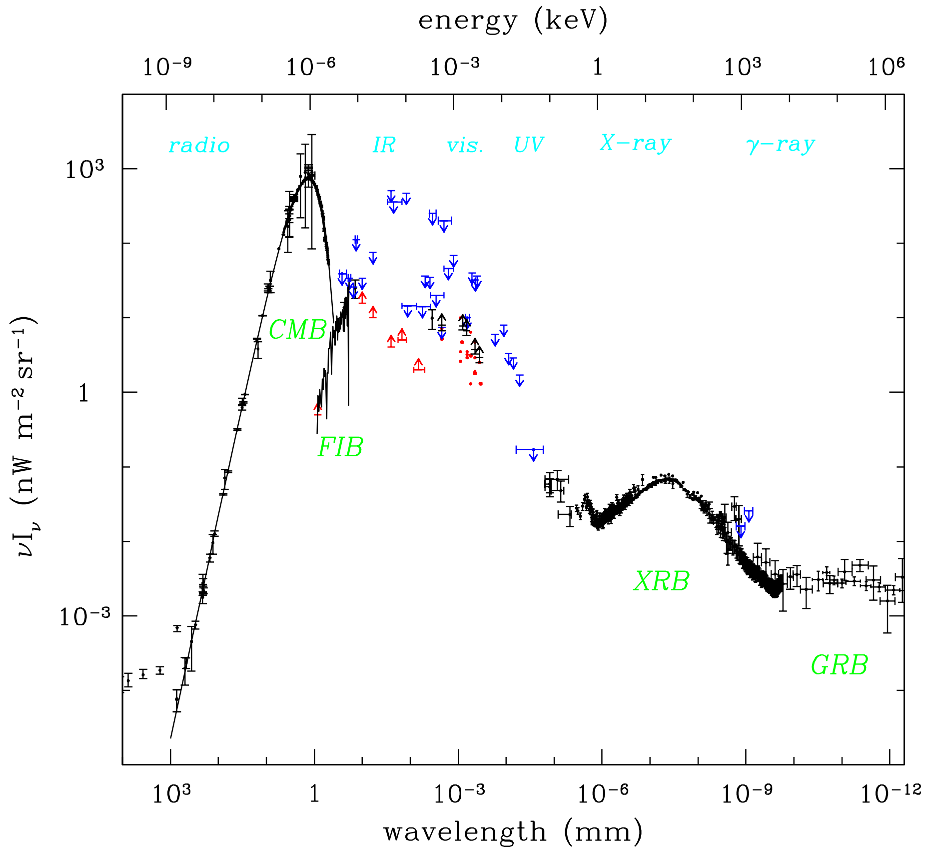
Lecture 4: Cosmic structure

primary and secondary cmb fluctuations, large scale structure, gravitational instability

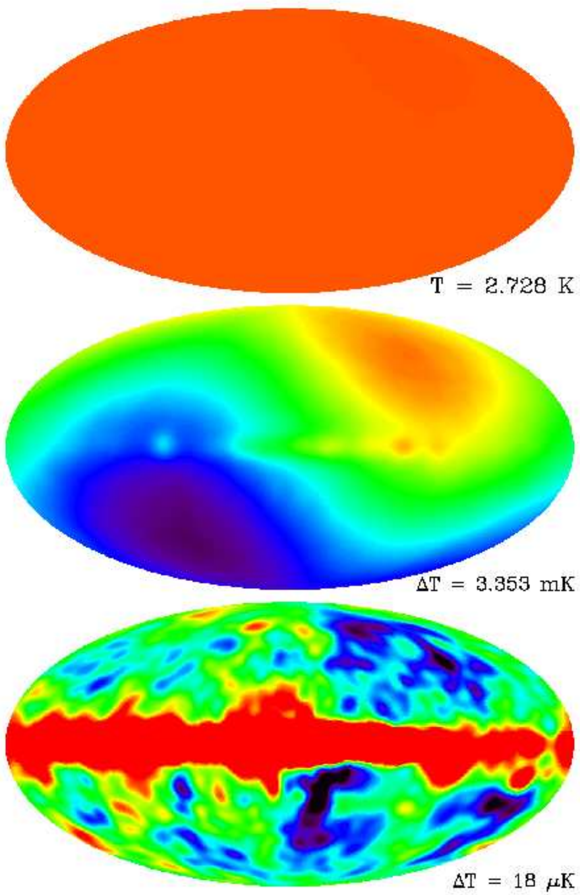
Lecture 5: Cosmic substratum

evidence and candidates for dark matter and dark energy, direct and indirect dm searches

Diffuse cosmic background radiation



Halpern & Scott 1999

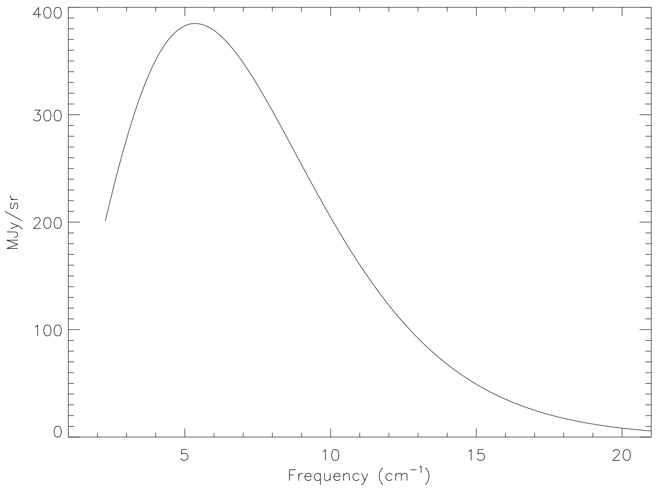


COBE/DMR: Bennett et al 1996

Cosmic microwave background

1. isotropic distribution

   Planck spectrum $T = 2.725 \pm 0.001$ K

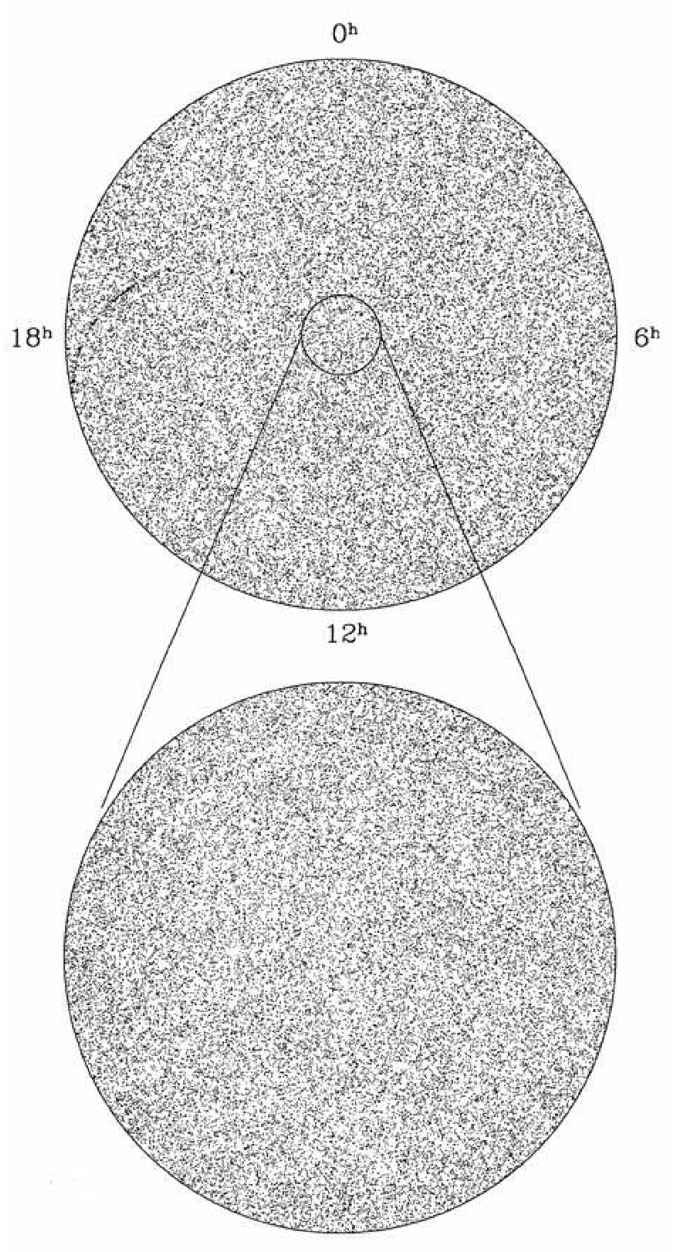


COBE/FIRAS: Fixsen et al 1996

2. dipole $\Delta T = 3.346 \pm 0.017$ mK
3. Milky Way and fluctuations

Nobel prize 1978: Penzias & Wilson

Nobel prize 2006: Mather & Smooth



Radio galaxies

isotropic distribution

$f = 1.4$ GHz

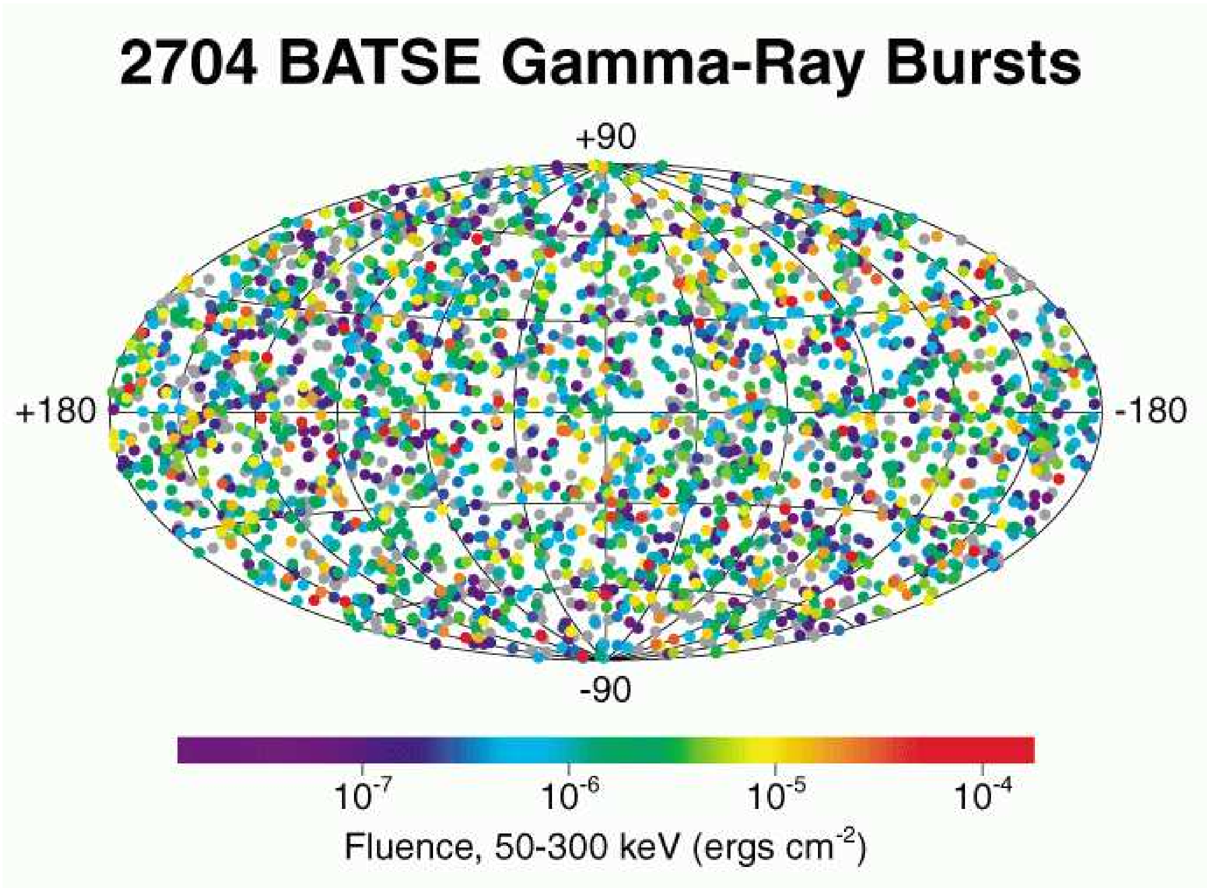
top: $S > 140$ mJy, $\delta > -40^\circ$

bottom: $S > 2.5$ mJy, $\delta > +75^\circ$

1 Jy $= 10^{-26}$ W m^{-2}Hz^{-1}

NRAO VLA Sky Survey

Condon 1999



isotropic distribution

Briggs 2000

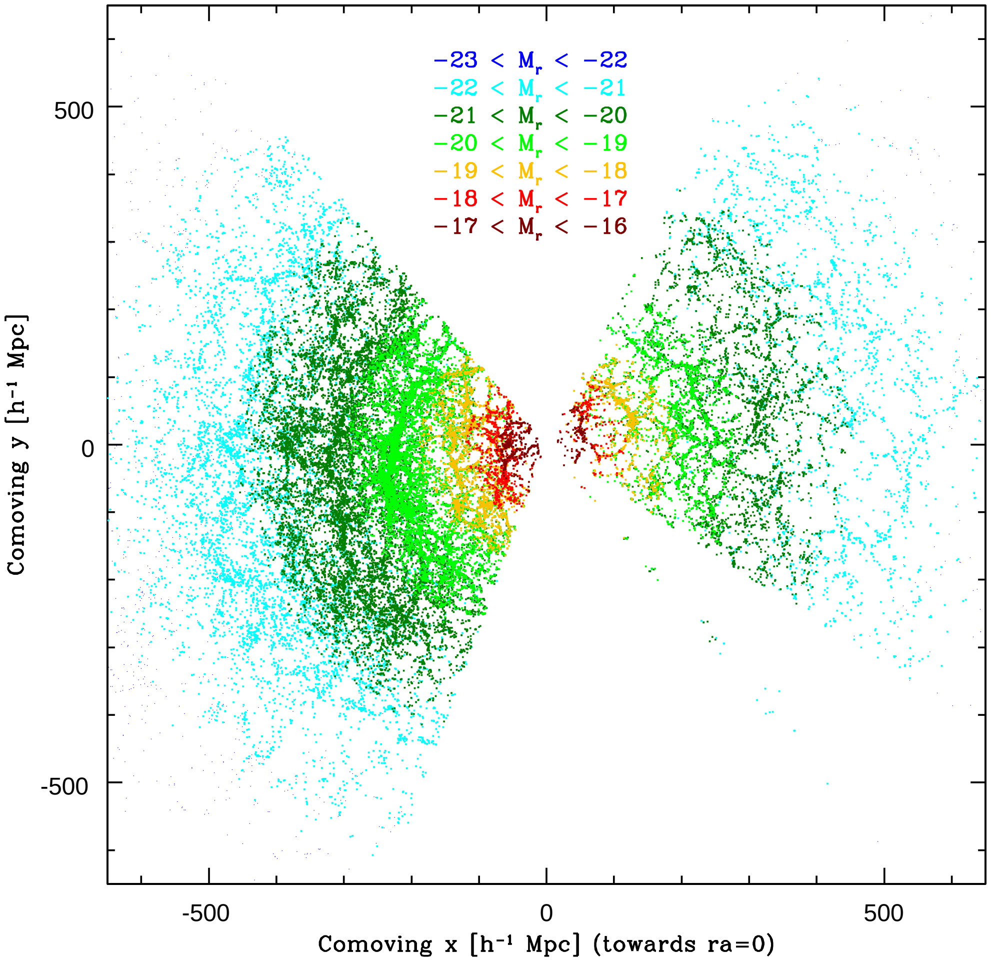
Isotropy

observational fact:
(statistically) isotropic distribution of light

possibilities:
1. isotropic around one point (we are at the centre)
2. isotropic around many points (we are preferred observers)
$\Rightarrow$ fractal space
3. isotropic around any point
(Copernican principle: we are typical observers)
$\Rightarrow$ continous homogenous space

Cosmological Principle (CP):
Universe is (statistically) isotropic and homogeneous

Large scale structure

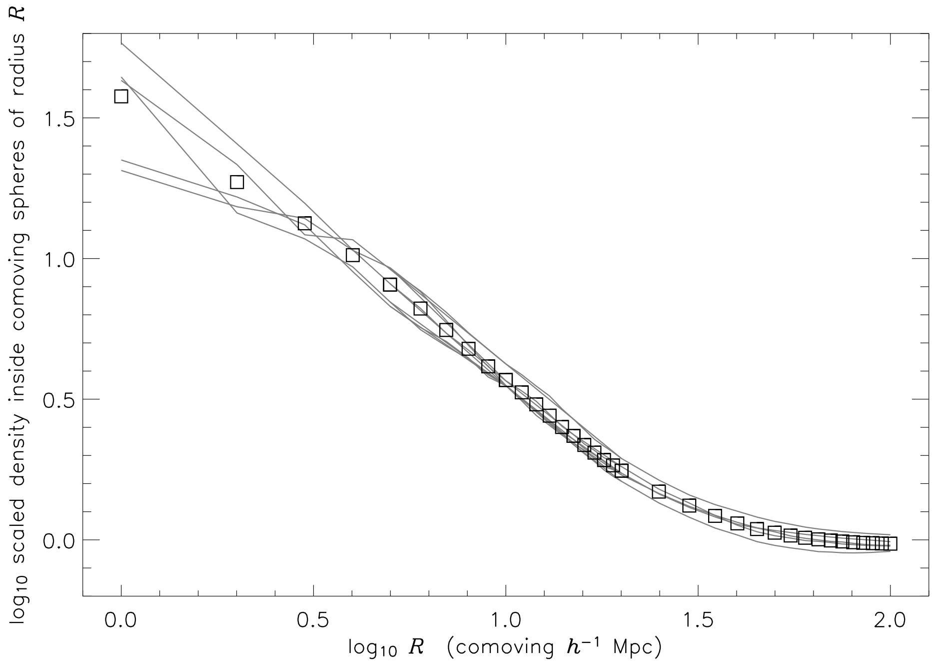


galaxies, visible light

Sloan Digital Sky Survey

Tegmark et al 2004

Homogeneity I



Hogg et al 2004

luminous red galaxies

Sloan Digital Sky Survey

$N(r)/r^3 \longrightarrow$ constant

for $r > 70h^{-1}$ Mpc

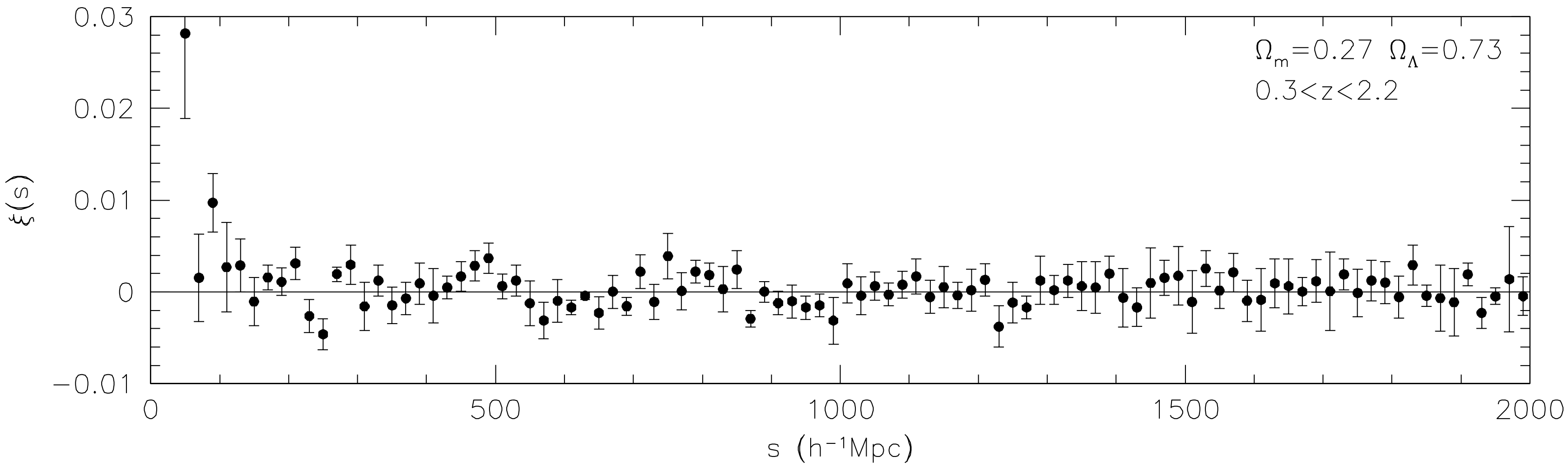
mean density exists!

fractal is excluded

probability distribution

$$\begin{aligned} \mathrm{d}P(\mathbf{x}) &= n(\mathbf{x})\mathrm{d}V \\ &= \bar{n}[1+\delta(\mathbf{x})]\mathrm{d}V \end{aligned}$$

Homogeneity II



2dF QSO redshift survey

Croom et al 2005

two-point correlation vanishes at $r > 100h^{-1}$Mpc $\Rightarrow$ homogeneity

$dP_{12} = \bar{n}^2[1 + \xi(r)]dV_1 dV_2$

Friedmann model I

assume cosmological principle holds for space-time itself

isotropic & homogeous line element ($c = 1$):

$$\mathrm{d}s^2 = -\mathrm{d}t^2 + a(t)^2 \left(\frac{\mathrm{d}r^2}{1 - Kr^2} + r^2 \mathrm{d}\Omega^2 \right)$$

a scale factor, K/a^2 spatial curvature ($K = -1, 0, +1$)
$H \equiv \dot{a}/a$ expansion rate

physical (radial) distance: $r_\mathrm{p} = a \sin(\sqrt{K}r)/\sqrt{K} = ar[1 - \frac{K}{6}r^2 + \mathcal{O}(K^2 r^4)]$

physical area (ball): $A_\mathrm{p} = 4\pi a^2 r^2$

physical volume (ball): $V_\mathrm{p} = \frac{4\pi}{3} a^3 r^3 [1 + \frac{3K}{10} r^2 + \mathcal{O}(K^2 r^4)]$

line element is unique up to coordinate transformations (Robertson & Walker)

Velocities and redshift

4-velocity of observer: $u^\mu \equiv \mathrm{d}x^\mu/\mathrm{d}\tau$, $u^\mu u_\mu = -1$ (time-like)
τ proper time of observer

free falling observers $u^\mu{}_{;\nu} u^\nu = 0$, are slowing down for growing $a(t)$:
$|u| \equiv \sqrt{g_{ij} u^i u^j} \propto 1/a$

free falling observers are asymptotically comoving ($u^i \equiv 0$)

photons are redshifted, i.e. $f \propto 1/a$

redshift $z \equiv \frac{f_\mathrm{e} - f_\mathrm{o}}{f_\mathrm{o}} = \frac{a_\mathrm{o}}{a_\mathrm{e}} - 1$

Luminosity distance and angular distance

comoving distance

$$d_{\mathrm{com}} \equiv r_{\mathrm{p}} = \frac{a_0}{\sqrt{K}} \sin\left(\frac{\sqrt{K}}{a_0}\int_0^z \frac{\mathrm{d}z'}{H(z')}\right)$$

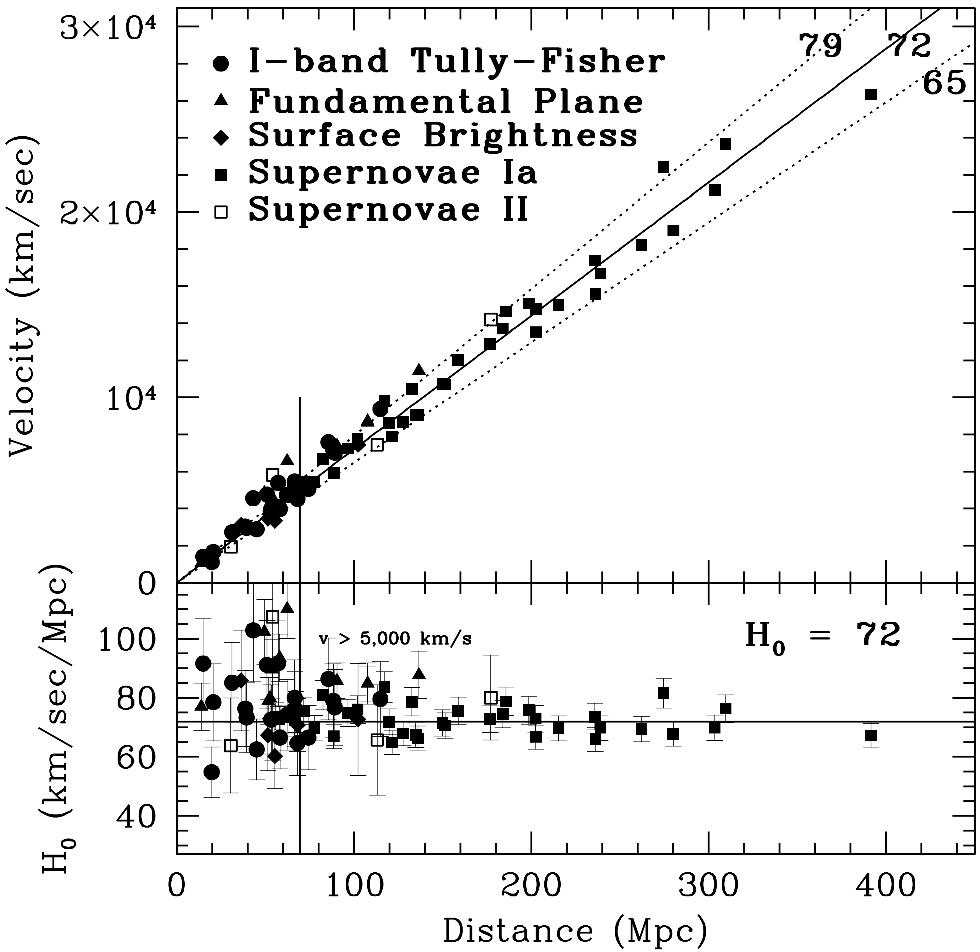
luminosity distance $d_{\mathrm{L}} \equiv \sqrt{\frac{L}{4\pi F}} = (1+z)d_{\mathrm{com}}$

Hubble diagram

angular distance $d_{\mathrm{a}} \equiv \frac{D}{\theta} = d_{\mathrm{com}}/(1+z)$

acoustic oscillations in cmb and lss

Expanding universe



red-shift

$$z \equiv \frac{f_e - f_o}{f_o} = \frac{a_o}{a_e} - 1$$

Hubble expansion

$$H_0 d_L = z + \mathcal{O}(z^2)$$

Red-shift and Hubble expansion are a direct consequence of the cosmological principle (without Einstein's equations)

$H_0 \equiv 100h$ km/s/Mpc

$h = 0.72 \pm 0.03 \pm 0.07$

Freedman et al 2001

Time and distance scales

astronomer's unit:

$1 \text{ pc} \equiv 1 \text{ AU}/1 \text{ arc sec}$
$1 \text{ Mpc} = 3.086 \times 10^{22} \text{ m} \approx 3.262 \times 10^{6}$ light-years
typical distance between two galaxies

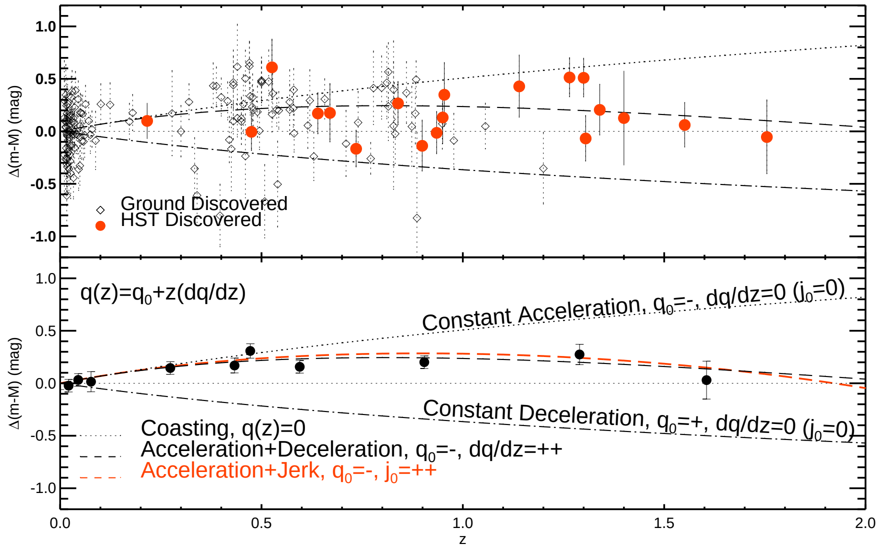
natural units of cosmology:
Hubble time, Hubble distance

$t_{\mathrm{H}} \equiv 1/H_0 = 9.78 h^{-1}\mathrm{Gyr}$, $d_{\mathrm{H}} \equiv t_{\mathrm{H}} = 3000 h^{-1}\mathrm{Mpc}$

H_0 sets the time and length scale of local causal processes

curvature radius: $r_{\mathrm{C}} \equiv a_0/\sqrt{|K|}$

Acceleration of the expansion



here $K = 0$; SN Ia Riess et al. 2004

luminosity distance at $z \ll 1$

$$d_{\mathrm{L}}(z) = \frac{1}{H_0}\left[z + (1-q_0)\frac{z^2}{2} + \left(-j_0 + 3q_0^2 + q_0 - 1 - \frac{K}{a_0^2}\frac{c^2}{H_0^2}\right)\frac{z^3}{6} + \mathcal{O}(z^4)\right]$$

deceleration $q \equiv -(\ddot{a}/a)/H^2$, jerk $j \equiv (\dddot{a}/a)/H^3$ SN Ia data suggest $q_0 < 0$

Phase space distribution of matter and light

Planck spectrum of CMB: equilibrium?

isotropy and homogeneity on large scales: equilibrium?

number of quanta in phase space at time t

$\mathrm{d}N_t = f_t(\mathbf{x}, \mathbf{p})\mathrm{d}\mathbf{x}\mathrm{d}\mathbf{p}$

homogeneity and isotropy: $f_t(\mathbf{x}, \mathbf{p}) = f_t(p)$

gravitational interaction only: $L(f) = 0 \Rightarrow$

equilibrium in an expanding Friedmann Universe possible for

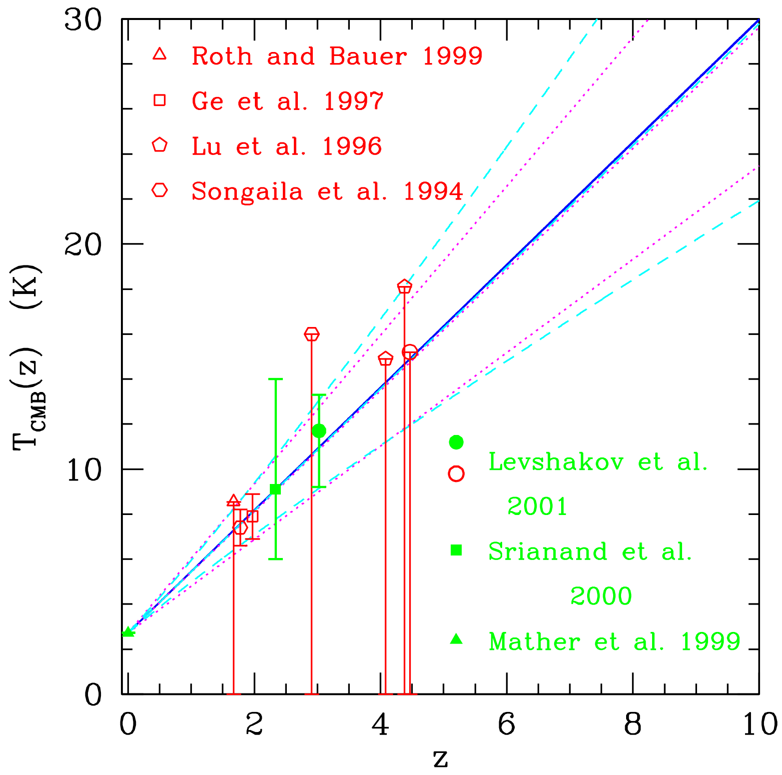
1. massless particles; Bose-Einstein or Fermi-Dirac $T \propto 1/a$, $\mu \propto 1/a$
2. massive particles, iff $m \gg T$; Maxwell-Boltzmann $T \propto 1/a^2$, $\mu \approx m$

"Hot Big Bang"

Universe expands:
cosmic Joule-Thomson effect

$$T(z) = T_0(1+z)$$

measure CMB temperature
in distant molecular clouds
via absorption spectra



LoSecco, Mathews & Wang 2001

Radiation domination in the early Universe

photons: $\epsilon = \frac{2\pi^2}{30} T^4$ (Stefan-Boltzmann law) $\propto a^{-4}$

dust (matter): $\epsilon = mn \propto 1/V_\mathrm{p} \propto a^{-3}$

curvature: $K a^{-2}$

cosmological constant: $\Lambda \propto a^0$

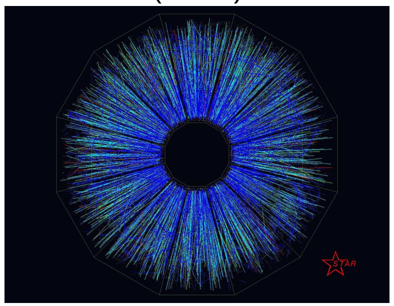
as $a \ll a_0$, at $T > T_\mathrm{eq}$: radiation (γ, νs, etc.) dominates early on

$T_\mathrm{eq} \equiv (1 + z_\mathrm{eq}) T_0, \quad (1 + z_\mathrm{eq}) \equiv \frac{\epsilon_\mathrm{m0}}{\epsilon_\mathrm{r0}} \sim 4000$ matter-radiation equality

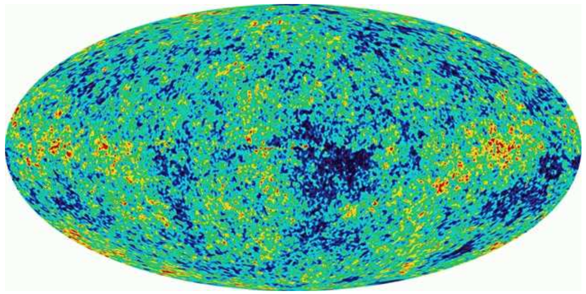
History of the Universe

LHC dipole

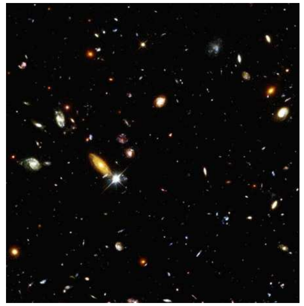
RHIC-event (STAR)

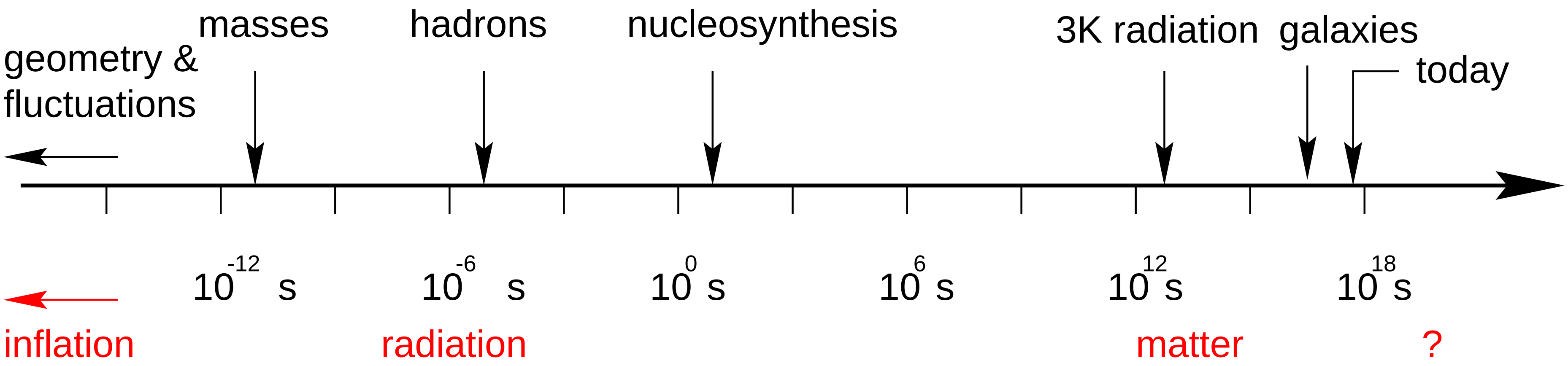


Sky from WMAP



Hubble Deep Field





Einstein equation

Lovelock's theorem:

1. covariant second order equation for the metric $g_{\mu\nu}$
2. covariant conservation of energy-momentum

$\Rightarrow$

$$G_{\mu\nu} - \Lambda g_{\mu\nu} = 8\pi G T_{\mu\nu}$$

G Newton constant

Λ cosmological constant

Einstein equations fix geometry, but not topology

assume trivial topology, no convincing observational evidence for more complex one

Friedmann model II

from d$U = -p$d$V + \delta Q$ (CP: $\vec{\nabla}\delta Q = 0^*$) and Einstein's equation

$$\dot{\epsilon} + 3H\,(\epsilon + p) = 0 \quad \text{and} \quad 3H^2 + \frac{3K}{a^2} - \Lambda = 8\pi G\epsilon$$

ϵ energy density, p pressure

to solve need equation of state $p = p(\epsilon)$

examples:
dust (matter) $p = 0$; radiation (light) $p = \epsilon/3$

*bulk dissipation is not excluded by CP, usually it is assumed to be irrelevant

Energy density and spatial curvature

$$\Omega \equiv \frac{\epsilon}{\epsilon_{\mathrm{C}}}, \qquad \text{with } \epsilon_{\mathrm{C}} \equiv \frac{3H^2}{8\pi G}$$

$$\Omega - 1 = \frac{K}{a^2 H^2}$$

to know Ω, H_0 must be known, thus measure $\omega \equiv h^2\Omega$

Einstein-de Sitter model

$\Lambda = 0$, flat dust solution

$$\epsilon(a) = \epsilon_0 \left(\frac{a_0}{a}\right)^3, \quad a(t) = a_0 \left(\frac{t}{t_0}\right)^{2/3}, \quad t_0 = \frac{2}{3} t_\mathrm{H}, \quad q_0 = \frac{1}{2}$$

in conflict with age of Universe $t_0 \geq 12\mathrm{Gyr}$ (oldest stars) and
in conflict with Hubble diagram $q_0 < 0$

Dark energy

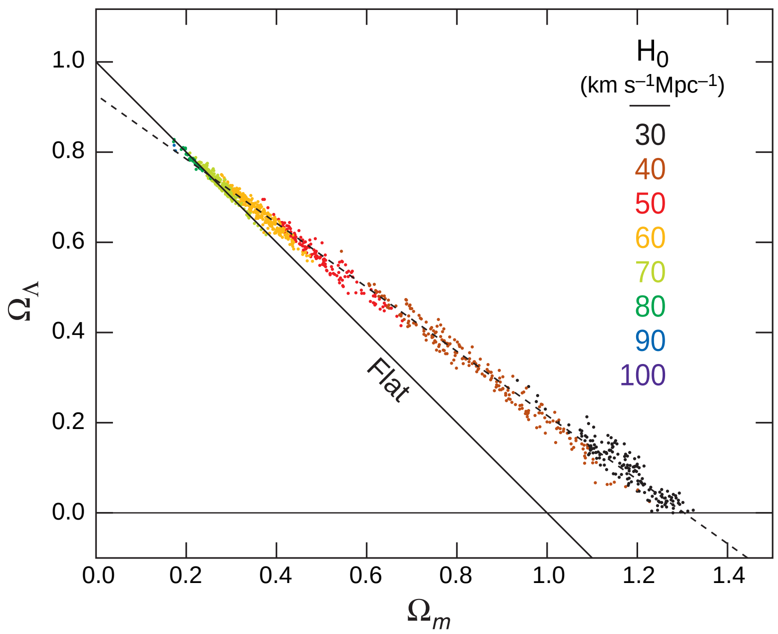
acceleration possible for

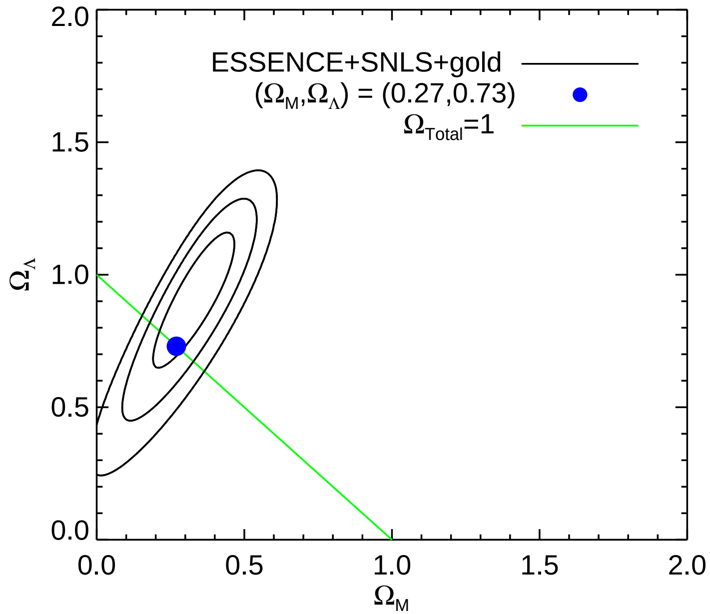
$$-3\frac{\ddot{a}}{a} = 4\pi G(\epsilon + 3p) - \Lambda < 0$$

cosmological constant or other form of “dark energy” required

simplest model: $\Lambda > 0, p = 0, K = 0$ flat ΛCDM

Cosmological parameters of ΛCDM: $h, \Omega_\Lambda, \Omega_{\rm m}$





Spergel et al. 2006

CMB (WMAP) and H_0 (HST key project)

$\Omega - 1 = -0.014 \pm 0.017 \Rightarrow r_{\rm c} > 21 \rm Gpc$

Wood-Vasey et al. 2007

supernovae Ia

$\Omega_\Lambda > 0$

Summary of 1st lecture

statistical isotropy is an observational fact

cosmological principle implies redshift and Hubble expansion

hot big bang: radiation domination followed by matter domination

Einstein equations and the CP lead to Friedmann cosmology

SN 1a Hubble diagram indicates accelerated expansion and
dark energy domination today

minimal model of cosmology: $K = 0, p_{\mathrm{m}} = 0, \Lambda > 0$ plus radiation
free parameters of the minimal model: $T_0, h, \omega_{\mathrm{m}}$