Clumping of CDM from the cosmological QCD transition

Dominik J. Schwarz (Univ. Frankfurt)

C. Schmid and P. Widerin (ETH-Zürich)

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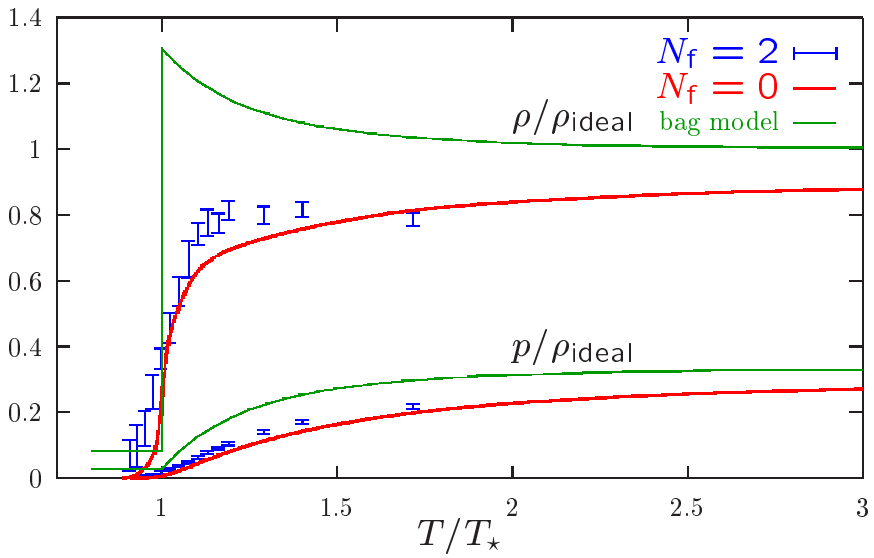
- 1st order QCD transition $\Rightarrow$ $c_s = 0$
  large amplification of subhorizon inhomogeneities $M_{\rm H}^{\rm CDM} \sim 10^{-8} M_\odot$

- peaks in the radiation fluid are erased at neutrino decoupling $M_\nu^{\rm CDM} \sim 10^{-5} M_\odot$

- neutralinos tightly coupled at $T \sim \Lambda_{\rm QCD}$

- peaks in kinetically decoupled CDM (axions, primordial black holes) survive
  $\Rightarrow$ CDM clumps with
  $10^{-20} M_\odot < M_{\rm clump} < 10^{-10} M_\odot$

QCD transition (quarks $\rightarrow$ hadrons)



Lattice QCD

$N_f = 0$: 1st order with $l \approx 1.4 T_\star^4 \approx 0.2 l^{\rm bag}$

$T_\star \approx 260$ MeV Boyd et al. 96, Beinlich et al. 96

$N_f = 2$: crossover $T_\star \approx 140$ MeV Bernard et al. 96

$N_f = 2 + 1$: 1st order or crossover ?

Iwasaki et al. 96

Bag model

$$p_{\rm QGP}(T) = p^{\rm ideal}_{\rm QGP}(T) - B, \quad \text{u, d, g}$$
$$p_{\rm HG}(T) = p^{\rm ideal}_{\rm HG}(T), \quad \pi$$

1st order with latent heat $l \equiv T_\star \Delta s = 4B$

Radiation fluid

High T: quark-gluon plasma

$$g_{\rm QGP} = 16 + (21/2)N_f$$

Low T: hadron gas $g_{\rm HG} = 3$

& photons and leptons (e, μ, ν) $g_{\gamma \rm l} = 14.25$

Scales: $T_\star \sim \mathcal{O}(\Lambda_{\rm QCD})$, we take $T_\star = 150$ MeV

$$t_{\rm H} \sim m_{\rm P}/T_\star^2 \sim 10^{-5}{\rm s}, \quad R_{\rm H} \sim 10{\rm km}$$

$$M_{\rm H}^{\rm RAD} \sim 1{\rm M}_\odot$$

Bubbles: $d_{\rm nuc} \sim 10^{-6} R_{\rm H}$ Ignatius et al. 94
based on small values for latent heat and surface tension from $N_f = 0$ lattice QCD
Beinlich et al. 96

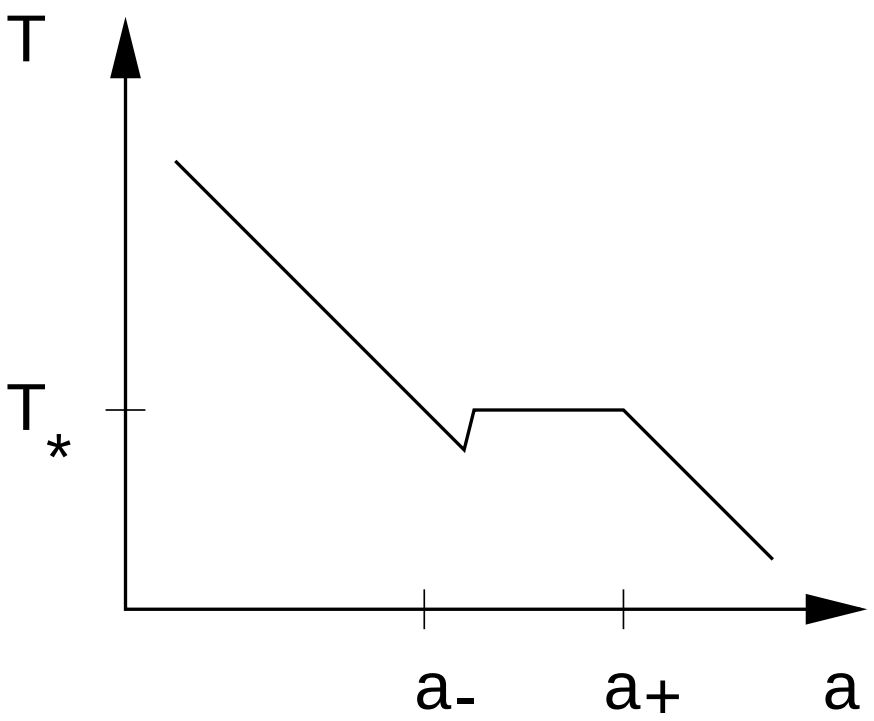
Interaction rates/expansion rate:

$$\begin{aligned} \Gamma_{\rm w}/H &\sim G_{\rm F}^2 m_{\rm P} T_\star^3 \sim 10^7 \\ \Gamma_{\rm e}/H &\sim \alpha^2 m_{\rm P}/T_\star \sim 10^{15} \\ \Gamma_{\rm s}/H &\sim \alpha_{\rm s}^2 m_{\rm P}/T_\star \sim 10^{20} \end{aligned}$$

Neutrino diffusion: $\lambda_\nu \sim \sqrt{\lambda_{\rm mfp} R_{\rm H}} \sim 10^{-4} R_{\rm H}$

for $\lambda > \lambda_\nu$: photons, leptons, and QGP-HG make up a single perfect fluid

First order QCD transition



quarks an hadrons coexist at

$$p_{\mathrm{QGP}}(T) \stackrel{!}{=} p_{\mathrm{HG}}(T)$$

$\Rightarrow$ expansion with $T = T_\star$ during coexistence

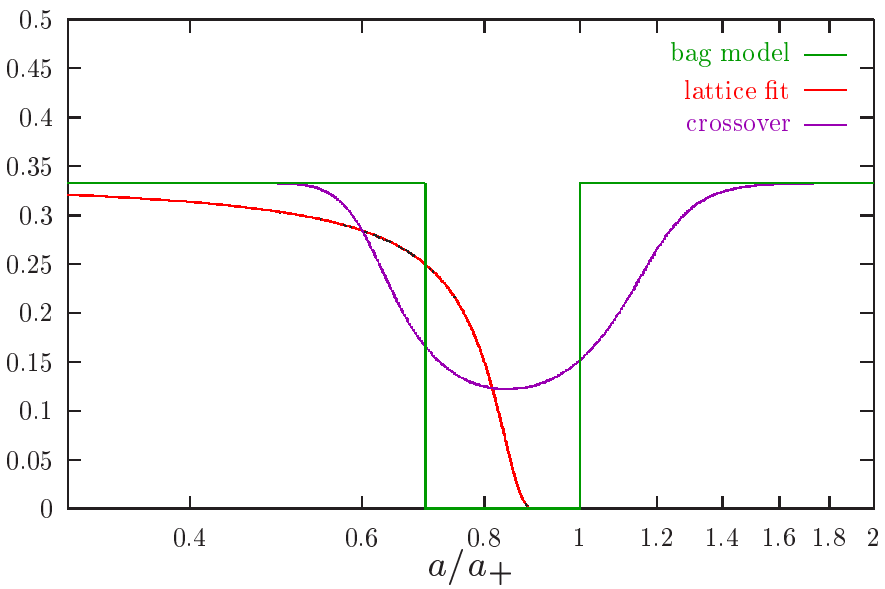
(negligible baryon chemical potential, $\mu_{\mathrm{B}}/T_\star \sim 10^{-8}$)

Sound speed

coexistence of QGP and HG during
$\Delta t \approx 0.3(0.1) t_{\mathrm{H}}$ for $l = 1(0.2) l^{\mathrm{bag}}$

for $\lambda > 10^{-6} R_{\mathrm{H}}$: $p(t) = p(T_\star)$ constant during $\Delta t \Rightarrow$ vanishing sound speed

$$c_s^2 \equiv \left(\frac{\partial p}{\partial \rho}\right)_s = \left.\frac{\mathrm{d}p/\mathrm{d}t}{\mathrm{d}\rho/\mathrm{d}t}\right|_{T_\star} = 0$$



no pressure gradients

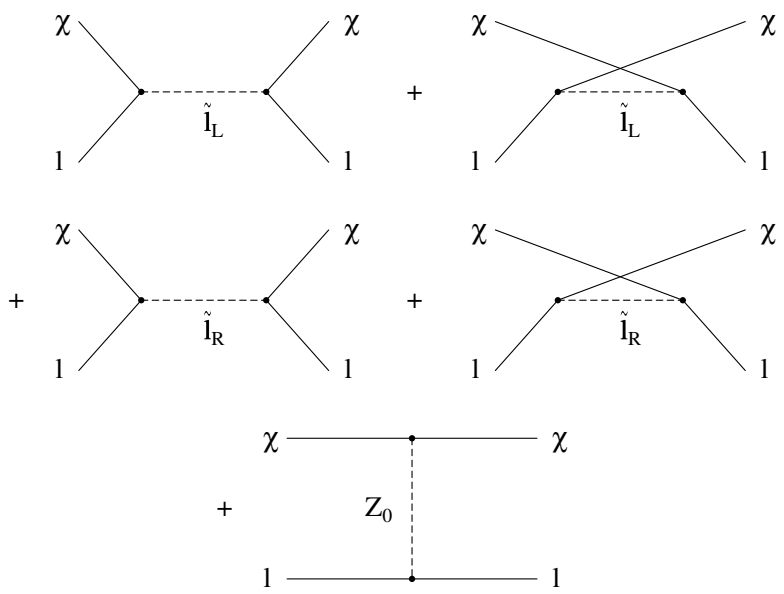
$$\delta p(\lambda) = c_s^2 \delta\rho(\lambda) + \underbrace{\tau \delta S(\lambda)}_{0 \text{ for } \lambda \gg d_{\mathrm{nuc}}}$$

Cold dark matter

Lightest SUSY particle in MSSM: neutralino
$m_\chi > 40$ GeV Ellis et al. 98

chemical freeze-out at $T \sim m_\chi/20 > 2$ GeV

but kinetically tightly coupled at $T_\star$
$\Gamma_{\chi l} = \langle \sigma v \rangle n_l \sim G_F^2 T^5$
$\Delta p_\chi / p_\chi \sim 1$ for $N \sim m_\chi / T$ collisions



e.g. bino $\Gamma_{\tilde{B}} \approx 4(\frac{M_W^2}{M_{\tilde{l}}^2 - M_{\tilde{B}}^2})^2 G_F^2 T^5$
$\Rightarrow$ neutralinos belong to the radiation fluid
(kinetic decoupling at ~ 10 MeV)

same for heavy neutrinos $m_\nu \sim 1$ TeV

Kinetically decoupled CDM

here CDM has to be decoupled kinetically well before $T_\star$

- WIMPzillas $m_{\rm WIMP} = 10^{12} - 10^{16}$ GeV
  Chung et al. 98

- axions $m_{\rm a} \sim 10^{-5}$ eV
  axion mass is switched on at $T_1 \sim 1$ GeV
  axion field oscillates coherently with $\tau_{\rm a} \sim 1/m_{\rm a} \sim 10^{-6} t_{\rm H}(T_\star) \Rightarrow \langle p_{\rm a} \rangle = 0$

- primordial black holes
  $M_{\rm BH} > 10^{-18} M_\odot$ if dark matter today
  pressureless gas at $T_\star$ for $M_{\rm BH} < 10^{-8} M_\odot$

$\Rightarrow$ two fluids: radiation and kinetically decoupled CDM

$$\rho^{\rm CDM}(T_\star) \approx 10^{-8} \rho^{\rm RAD}(T_\star)$$

Linear density perturbations

uniform expansion gauge $\delta[\mathrm{tr}(K)] \equiv 0$

Bardeen 88

Continuity equation:

$$\partial_t \delta\rho = -3H(\delta\rho + \delta p) - \Delta\psi - 3H(\rho + p)\alpha$$

Divergence of the Euler equation:

$$\partial_t \psi = -3H\psi - \delta p - (\rho + p)\alpha$$

Poisson equation:

$$(\Delta + 3\dot{H})\alpha = 4\pi G(\delta\rho + 3\delta p)$$

α ... perturbation of lapse

$\vec{v} = \nabla\psi/(\rho + p)$ ... fluid velocity

dimensionless variables:

$\delta \equiv \delta\rho/\rho$ ... density contrast

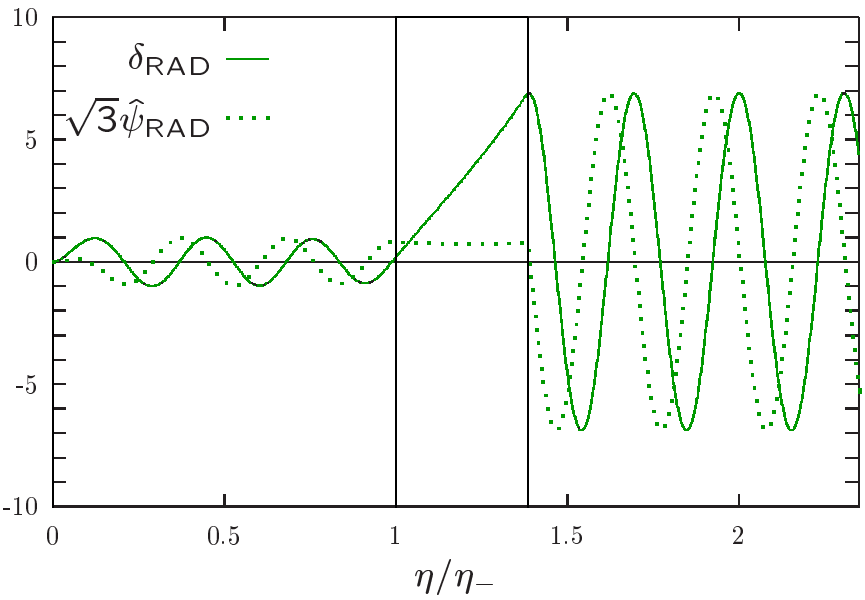
$\hat{\psi} \equiv k^{\mathrm{phys}}\psi/\rho \ldots \sim$ peculiar velocity

η ... conformal time, $k^{\mathrm{phys}} = k/a$ ... wavenumber

plane waves $\delta_k(\eta)\exp(i\vec{k}\vec{x})$

Time evolution in the bag model

for subhorizon scales $k^{\rm phys} \gg H$



$T \neq T_\star$: $c_s^2 = 1/3 \Rightarrow$ acoustic oscillations

$$\delta_k(\eta) \approx -A_{\rm in}\cos(c_s k\eta), \quad \hat{\psi}_k(\eta) \approx \delta_k'/k$$

$T = T_\star$: $c_s^2 = 0 \Rightarrow$ no restoring force $\Rightarrow$ density perturbations fall freely during $\Delta\eta$

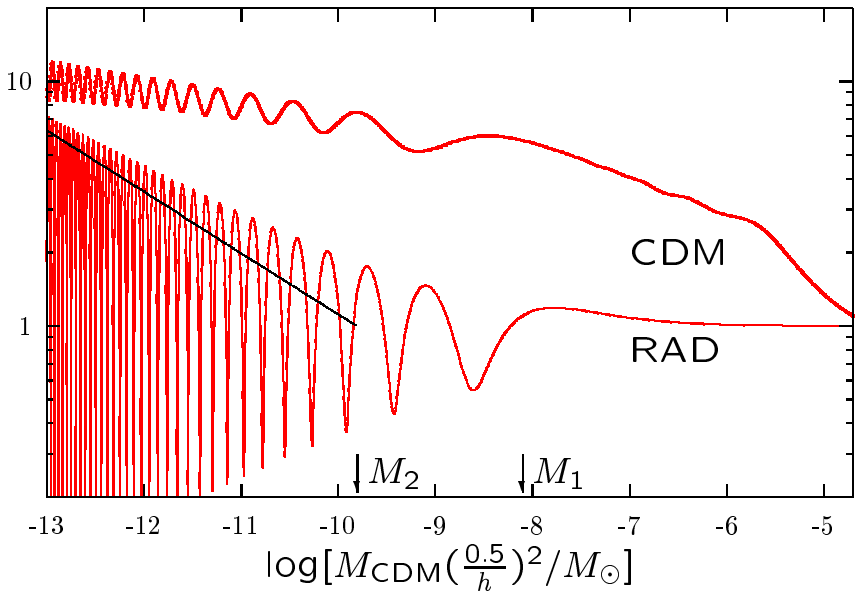
$$\hat{\psi}_k(\eta) = \hat{\psi}_k^- \text{ and } \delta_k \approx \delta_k^- + k\Delta\eta\hat{\psi}_k^-$$

large peaks and dips in $\delta \propto k$

$$\frac{A_{\rm out}}{A_{\rm in}} = \frac{\Delta\eta}{\sqrt{3}}k\sin(\varphi_-)$$

CDM falls into the gravitational potential wells of radiation during $\Delta\eta$

Transfer functions



1st order transition: large peaks and dips

Bag model: $\delta_k \propto k/k_1,\ k_1 \sim k_\star$

Lattice fit: $\delta_k \propto (k/k_2)^{3/4},\ k_2 < k_\star$

Crossover transition: peaks and dips at the horizon scale, no amplification at subhorizon scales ($c_s^2 > 0$)

Fate of radiation peaks

- $1M_\odot$ black hole production?

  need $\delta_{\rm rms} \approx 0.06$ to make black holes with probability $a_{\rm QCD}/a_{\rm eq} \approx 10^{-8}$
  this is a factor of ~ 600 above the COBE normalized HZ spectrum
  from the QCD transition the amplification at the Hubble scale is ~ 2
  unless primordial spectrum is doubly fine-tuned
  $\Rightarrow$ no BHs from the QCD transition

- Damping of peaks

  large peaks in δ are damped due to collisional damping by neutrinos

  $$\delta_k \propto \exp[-\int \Gamma {\rm d}t] = \exp[-(M_{\rm D}/M)^{1/4}]$$

  with $M_{\rm D}^{\rm CDM} \sim 10^{-5} M_\odot$
  all radiation peaks are gone by the time of nucleosynthesis

Clumps in CDM

when $(\delta\rho/\rho)_R \sim 1$, CDM overdensities collapse and virialize

without QCD transition

$$\left(\frac{\delta\rho}{\rho}\right)_R^{\rm CDM}(t_{\rm eq}) \approx 2\times 10^{-4}\left(\frac{k}{k_0}\right)^{(n-1)/2}\left[\ln(\frac{k}{k_{\rm eq}})\right]^{3/2}$$

first-order QCD transition enhances $(\delta\rho/\rho)_R$ for $R < R_{\rm H}$

$\Rightarrow$ CDM clumps $10^{-20}M_\odot < M_{\rm clump} < 10^{-10}{\rm M}_\odot$
take $M^{\rm CDM} = 10^{-15}(10^{-20})M_\odot$
QCD enhancement is a factor of 2(8)

$$\left(\frac{\delta\rho}{\rho}\right)_R^{3\sigma,n=1}(t_{\rm eq}) \approx 0.1(0.6)$$

$\Rightarrow$ nonlinear at $z \sim 600(3600)$ and collapse to $R_{\rm clump} \sim 14(0.04)R_\odot$
overdensity of $10^{10}(10^{12})$ today

Observations?

first-order QCD transition with kinetically decoupled CDM

- femtolensing $10^{-16}M_\odot - 10^{-13}M_\odot$
  Gould 92, Marani et al. 98
  need $R_{\rm clump} < R_{\rm E} \sim \sqrt{\frac{M}{10^{-15}M_\odot}}0.02R_\odot$
  could be satisfied for moderate tilt ($n = 1.2$) and large latent heat

- kinetically decoupled CDM is not distributed homogeneously in the galaxy $\Rightarrow$ affects probabilities to find CDM particles

  magnet-induced axion decay would tell whether the Earth is sitting in an axion cloud, but would not give a new limit on the axion mass
  Sikivie 83, Hagmann et al 98
  other clumping mechanism
  Hogan & Rees 88, Kolb & Tkachev 93

- BUT evolution of clumps at late times remains to be investigated (mergers, disruption)