

Tutorial sheet 1

Discussion topic: What does one call the “(relativistic) covariance of a theory”?

Aufgabe 1. Natural units

- i. Express the gravitational constant $G_N = 6.67 \times 10^{-11} \text{ m}^3 \text{ kg}^{-1} \text{ s}^{-2}$ in units of GeV. How big is the Planck mass $m_{\text{Pl}} = G_N^{-1/2}$?
- ii. Which length, duration, energy and mass (in SI units) correspond to 1 GeV (in natural units)?
- iii. Cross sections are often given in millibarns, with $1 \text{ mb} = 10^{-3} \text{ b} = 10^{-27} \text{ cm}^2$. How many millibarns correspond to a cross section of 1 GeV^{-2} ?
- iv. Along its trajectory from its creation point to a measurement apparatus, a neutrino changes its “flavor”: this phenomenon is referred to as *neutrino oscillation*.¹ The probability that a ν_e “oscillates” into a ν_μ reads

$$\mathcal{P}(\nu_e \rightarrow \nu_\mu) \propto \sin^2 \frac{\Delta m^2 L}{4E},$$

where L is the path length traveled by the neutrino, E its energy, and Δm^2 the difference between the squares of the masses of the mass-eigenstates. Show that the wavelength of the oscillation is given by

$$\lambda [\text{km}] = 2.48 \frac{E [\text{GeV}]}{\Delta m^2 [\text{eV}^2]}.$$

Aufgabe 2. Relative velocity

Consider two particles with the 4-momenta p_a^μ and p_b^μ . Find a covariant expression for their relative velocity, that is, the velocity of one of the particles in the rest frame of the other one.

Hint: Compute the scalar product $\mathbf{p}_a \cdot \mathbf{p}_b \equiv p_a^\mu p_{b,\mu}$ in the rest frame of one of the particles.

¹The topic will be presented in the lectures end of January.