Tutorial sheet 3

The exercises marked with a star are homework problems.

Discussion topic:

Can you give examples of linear Markovian nonequilibrium processes?

7. Transport of pseudoscalar quantities

Energy or (total) particle number density are scalar quantities, so that the corresponding flux densities are polar vectors—i.e. they change sign under spatial parity inversion, as does the position vector $\vec{r}$. One may however also consider the transport of *pseudoscalar* quantities—as e.g. the difference between the numbers of particles with spin up and spin down along a given axis.

i. Let $\chi_{\rm ps}$ denote the density of a pseudoscalar quantity. Convince yourself that the conjugate intensive quantity $\mathcal{Y}_{\rm ps}$ is also pseudoscalar, while the associated flux density $\vec{\mathcal{J}}_{\rm ps}$ is an axial vector (also called pseudovector).

ii. Consider a pseudoscalar density $\chi_{\rm ps}$ in a locally isotropic fluid, whose flow velocity $\vec{\mathsf{v}}$ is assumed to be a polar vector field. Can you write down a linear constitutive equation relating the flux density $\vec{\mathcal{J}}_{\rm ps}$ to the affinity $\vec{\nabla}\mathcal{Y}_{\rm ps}$? to the derivatives of the flow velocity? to the gradient of the (inverse of) temperature? Discuss the properties of the kinetic coefficients which you introduced.

8. Transport of time-reversal odd quantities

Assume that among the quantities characterizing a thermodynamic system, one ($\mathcal{A}$) is even under time reversal—for instance the internal energy—, while another one ($\mathcal{B}$) is time-reversal odd. Show that the linear kinetic coefficient $L_{\mathcal{AB}}$ does not contribute to entropy production.

*9. Conjugate variables in a moving fluid

Determine the intensive variable conjugate to particle number density in a moving fluid.

Hint: Remember the derivation of the intensive variable conjugate to momentum.

*10. Continuity equation for particle number

The single-particle phase-space density $f_1(t,\vec{r},\vec{p})$ of a collection of N particles is the ensemble average

$$f_1(t,\vec{r},\vec{p}) \equiv \left\langle \sum_{j=1}^{N} \delta^{(3)}\big(\vec{r}-\vec{r}_j(t)\big)\,\delta^{(3)}\big(\vec{p}-\vec{p}_j(t)\big) \right\rangle,$$

such that $f_1(t,\vec{r},\vec{p})\,\mathrm{d}^3\vec{r}\,\mathrm{d}^3\vec{p}$ is the number of particles in the phase space element $\mathrm{d}^3\vec{r}\,\mathrm{d}^3\vec{p}$ about the point $(\vec{r},\vec{p})$.

Show that the particle number density $n(t,\vec{r})$ and the particle current density $\vec{\mathcal{J}}_N(t,\vec{r})$ are given by

$$n(t,\vec{r}) = \int f_1(t,\vec{r},\vec{p})\,\mathrm{d}^3\vec{p}, \qquad \vec{\mathcal{J}}_N(t,\vec{r}) = \int f_1(t,\vec{r},\vec{p})\,\vec{v}\,\mathrm{d}^3\vec{p},$$

with $\vec{v}$ the velocity corresponding to momentum $\vec{p}$. Check that the continuity equation for particle number follows at once from these identities.