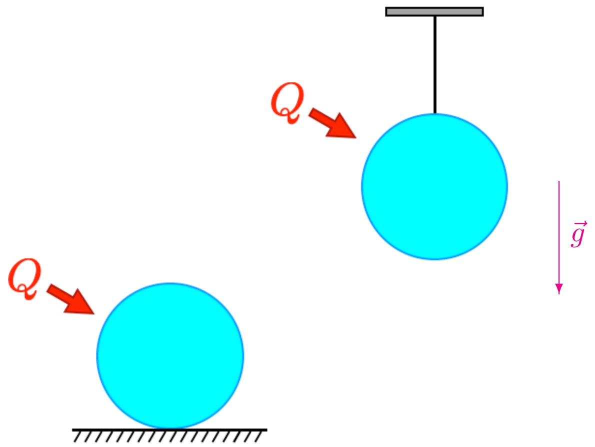
Tutorial sheet 1

The exercise marked with a star is homework for next week. The other problems are for in-class discussion (and homework for next week if necessary).

Discussion topics:
How does one describe a simple system at thermodynamic equilibrium? How is local thermodynamic equilibrium defined?

1. On the first law of thermodynamics[1]



Consider two perfectly identical spheres (same size, same material), initially at the same temperature. One is hanging from some support to which it is attached by a rigid thread, whereas the other lies on the ground. The same quantity of heat Q is given to each sphere. Assuming that there is no heat transfer to the environment (air, ground, thread...), which sphere will be hotter?

Hint: No calculation!

2. Flow of a gas between two containers at different temperatures and pressures

Consider a composite system made of two containers A and B with a classical ideal monoatomic gas of particles of mass m at temperatures $T^{(A)}$, $T^{(B)}$ and pressures $\mathcal{P}^{(A)}$, $\mathcal{P}^{(B)}$ respectively. Let $\Delta T \equiv T^{(B)} - T^{(A)}$ and $\Delta\mathcal{P} \equiv \mathcal{P}^{(B)} - \mathcal{P}^{(A)}$ denote the temperature and pressure differences.

i. Recall the expression of the Maxwell–Boltzmann distribution $p(\vec{v})$ for the velocities in an ideal gas at temperature T. Assuming that the particle density is uniform, write down the number density $f(\vec{r},\vec{v})\,\mathrm{d}^3\vec{v}$ of particles per unit volume with a velocity between $\vec{v}$ and $\vec{v}+\mathrm{d}^3\vec{v}$.

ii. Particle flow

A small hole of cross section $\mathcal{S}$ in the wall separating the containers allows gas to slowly flow from one container to the other. Where are at time t the gas particles that will traverse the hole with a given velocity $\vec{v}$ between t and $t+\mathrm{d}t$? Show that the number of particles flowing from container A to container B per unit time is

$$\mathcal{J}_N^{(A)} = \frac{\mathcal{P}^{(A)}\mathcal{S}}{\sqrt{2\pi m k_B T^{(A)}}}.$$

Hint: $\displaystyle\int_0^\infty x^3\,\mathrm{e}^{-ax^2}\,\mathrm{d}x = \frac{1}{2a^2}$.

[1] This exercise was taken from Kirone Mallick, who took it himself from Jean-Marc Victor.

Deduce the overall particle flux $\mathcal{J}_N \equiv \mathcal{J}_N^{(A)} - \mathcal{J}_N^{(B)}$ and express it as a function of ΔT and $\Delta\mathcal{P}$, assuming those differences are small.

iii. Energy flow

Show that the flow of (kinetic) energy through the hole from container A to container B per unit time is

$$\mathcal{J}_E^{(A)} = \sqrt{\frac{2k_BT^{(A)}}{\pi m}}\,\mathcal{P}^{(A)}\mathcal{S}.$$

Deduce the overall energy flux $\mathcal{J}_E \equiv \mathcal{J}_E^{(A)} - \mathcal{J}_E^{(B)}$ and express it as a function of ΔT and $\Delta\mathcal{P}$.

iv. The chemical potential of a classical ideal gas is given by

$$\mu(T, \mathcal{V}, N) = -k_BT \ln\left[\frac{\mathcal{V}}{N}\left(\frac{mk_BT}{2\pi\hbar^2}\right)^{3/2}\right].$$

Express $\mathcal{J}_N$ and $\mathcal{J}_E$ as a function of the differences (beware the signs!)

$$\Delta\left(\frac{1}{T}\right) \equiv \frac{1}{T^{(B)}} - \frac{1}{T^{(A)}} \quad \text{and} \quad \Delta\left(-\frac{\mu}{T}\right) \equiv \frac{\mu^{(A)}}{T^{(A)}} - \frac{\mu^{(B)}}{T^{(B)}}.$$

What do you recognize?

*3. Local thermodynamics in a simple fluid. Fundamental equation for the densities

Show that in a simple fluid the entropy density s is related to the local thermodynamic densities χ_a and to their conjugate intensive variables $\mathcal{Y}_a$ by

$$\sum_a{}' \mathcal{Y}_a \chi_a = s - \frac{\mathcal{P}}{T},$$

with $\mathcal{P}$ the pressure and T the temperature, while the sum runs over energy density, particle number density and the components of momentum density. Which known relation do you recognize?