Onset of final-state-collectivity signals in nuclear collisions

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Strong-interaction matter under extreme conditions

Onset of final-state-collectivity signals in nuclear collisions

- Introduction and motivation
- Model
  - One approach, many possible setups
- A few generic results
  - Scaling behaviors

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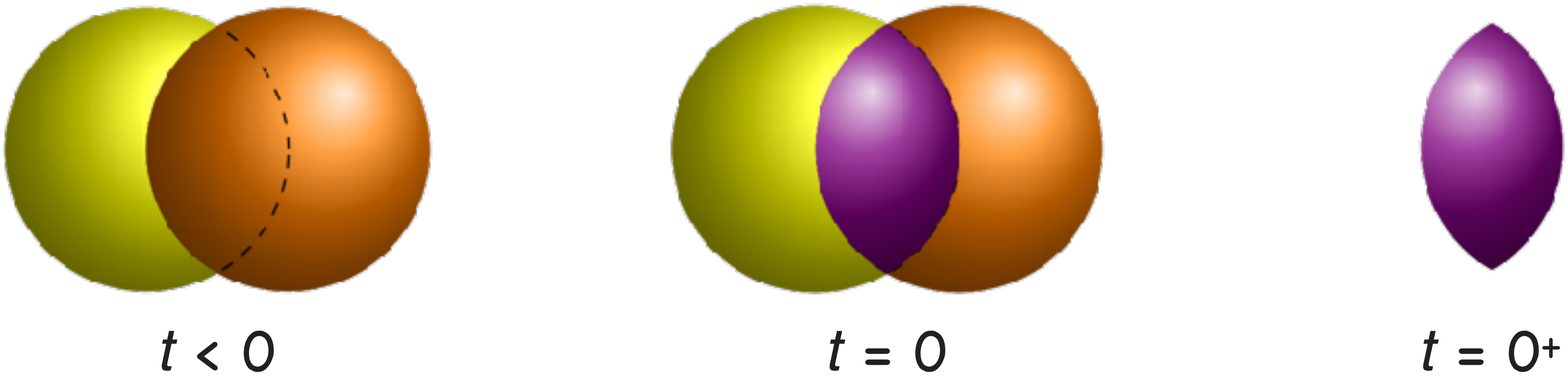
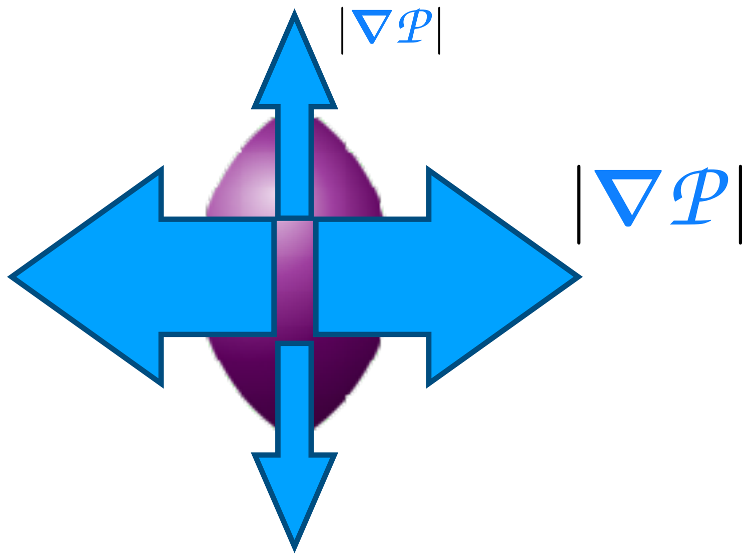
Preamble: Anisotropic collective flow

Classical idea (before ca.2010) in collisions of "large" systems:

Initially asymmetric collision zone (in the transverse plane)



$\Rightarrow$ larger pressure gradient along the impact parameter direction



$\Rightarrow$ larger fluid acceleration along the impact parameter direction

pictures shamelessly stolen from Matt Luzum

Preamble: Anisotropic collective flow

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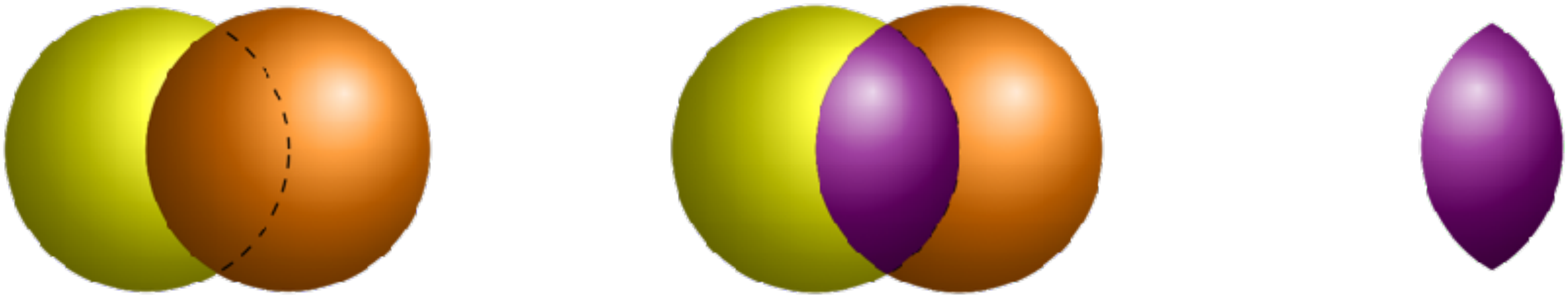
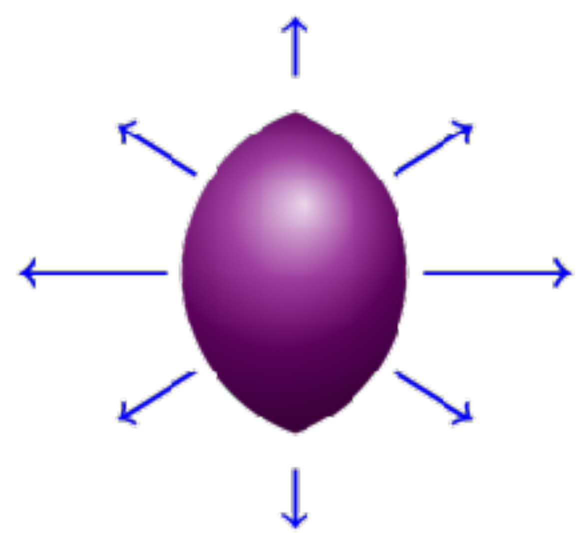
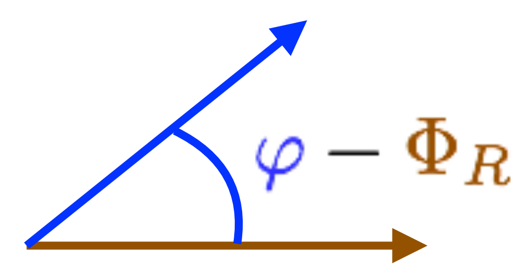


$t < 0$ $\qquad$ $t = 0$ $\qquad$ $t = 0^+$

$\Rightarrow$ anisotropic emission of particles





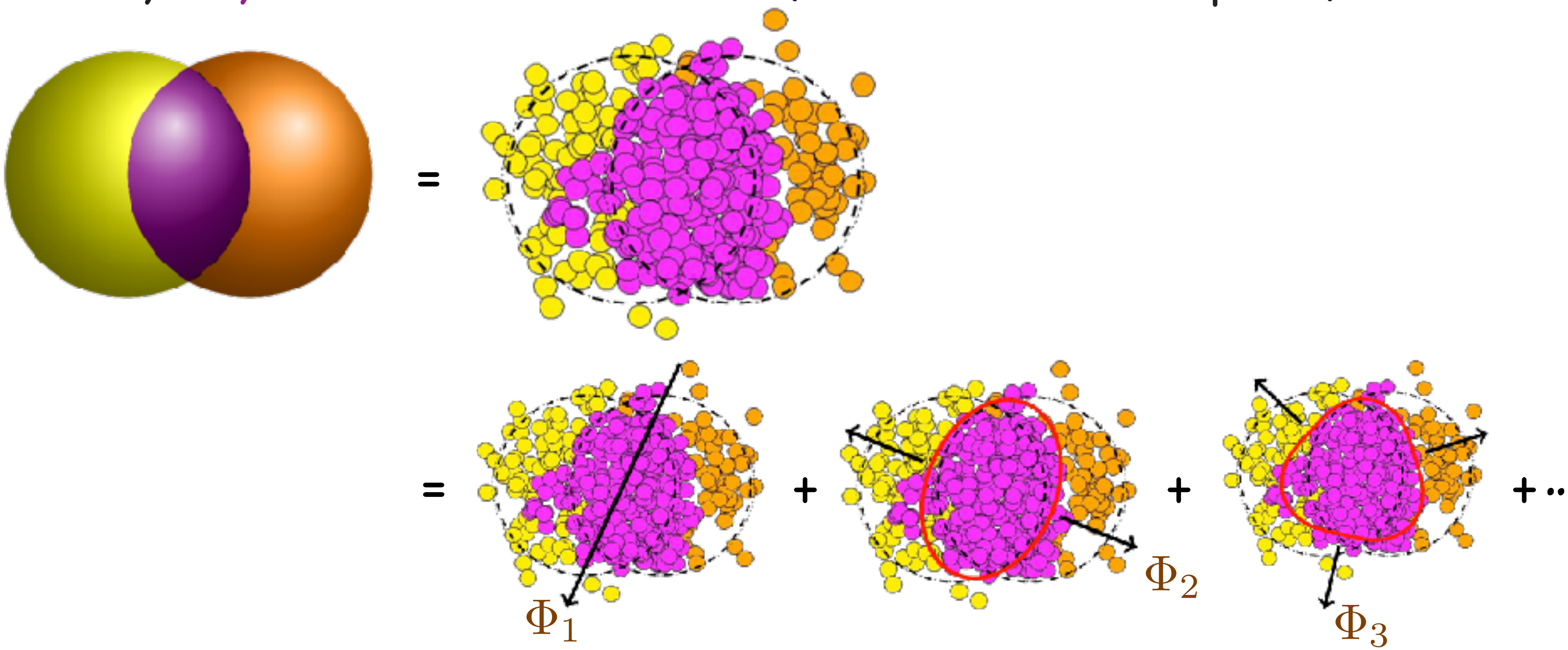
$$E\frac{\mathrm{d}N}{\mathrm{d}^3\mathbf{p}} \propto \frac{\mathrm{d}N}{p_T\mathrm{d}p_T\mathrm{d}y}\left[1 + 2v_1\cos(\varphi - \Phi_R) + 2v_2\cos 2(\varphi - \Phi_R) + \cdots\right]$$

pictures shamelessly stolen from Matt Luzum

Preamble: Anisotropic collective flow

Newer paradigm (since 2010) in collisions of "large" systems:

Initially asymmetric collision zone (in the transverse plane)



$\Rightarrow$ anisotropic emission of particles

$$E\frac{\mathrm{d}N}{\mathrm{d}^3\mathbf{p}} \propto \frac{\mathrm{d}N}{p_T\mathrm{d}p_T\mathrm{d}y}\left[1+2\sum_{n=1}^{\infty} v_n \cos n(\varphi-\Phi_n)\right]$$

pictures shamelessly stolen from Matt Luzum

Preamble: Anisotropic collective flow

The asymmetric initial-state (transverse) geometry, characterized by "eccentricities" ϵ_n:*

$$\epsilon_n\, \mathrm{e}^{\mathrm{i}n\Phi_n} \equiv -\frac{\langle r^n\, \mathrm{e}^{\mathrm{i}n\theta}\rangle_{\boldsymbol{r}}}{\langle r^n\rangle_{\boldsymbol{r}}}$$

is converted by the system evolution into final-state anisotropies in (transverse) momentum space, characterized by Fourier coefficients v_n.

$$v_n = f_n\big(\{\epsilon_m\}\big)$$

* and by other similar quantities

Anisotropic flow in fluid dynamics

$$v_n = f_n\big(\{\epsilon_m\}\big)$$

The "response functions" f_n still depend on the system size, but also on more intrinsic properties of the expanding fireball.

☞ in a (classical*) fluid-dynamical description of the fireball as a continuous medium, dependence on the equation of state and on the transport coefficients (η, ζ ...).

* i.e. relying on local thermodynamic equilibrium

Anisotropic flow in fluid dynamics

$$v_n = f_n\big(\{\epsilon_m\}\big)$$

Summary of findings within numerical fluid dynamical studies:*

- elliptic flow: $v_2 \simeq \mathcal{K}^{\rm hydro}_{2,2}\,\epsilon_2$
- triangular flow: $v_3 \simeq \mathcal{K}^{\rm hydro}_{3,3}\,\epsilon_3$
- quadrangular flow: $v_4 \simeq \mathcal{K}^{\rm hydro}_{4,22}\,\epsilon_2^2 + \mathcal{K}^{\rm hydro}_{4,4}\,\epsilon_4$
- pentagonal flow: $v_5 \simeq \mathcal{K}^{\rm hydro}_{5,23}\,\epsilon_2\epsilon_3 + \mathcal{K}^{\rm hydro}_{5,5}\,\epsilon_5$
- hexagonal flow: $v_6 \simeq \mathcal{K}^{\rm hydro}_{6,222}\,\epsilon_2^3 + \mathcal{K}^{\rm hydro}_{6,33}\,\epsilon_3^2 + \mathcal{K}^{\rm hydro}_{6,42}\,\epsilon_2\epsilon_4 + \mathcal{K}^{\rm hydro}_{6,6}\,\epsilon_6$

* discarding ϵ_1 and considering only the leading contributions

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- hexagonal flow: $v_6 \simeq \mathcal{K}^{\rm hydro}_{6,222}\,\epsilon_2^3 + \mathcal{K}^{\rm hydro}_{6,33}\,\epsilon_3^2$

* discarding $\epsilon_{1,4,5,6}$ and considering only the leading contributions in ϵ_2 and ϵ_3.

Anisotropic flow in fluid dynamics

$$v_n = f_n(\{\epsilon_m\})$$

Summary of findings within numerical fluid dynamical studies:*

- elliptic flow: $v_2 \simeq \mathcal{K}^{\rm hydro}_{2,2}\,\epsilon_2$
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nonlinear flow!

* discarding $\epsilon_{1,4,5,6}$ and considering only the leading contributions in ϵ_2 and ϵ_3.

Anisotropic flow in fluid dynamics

$$v_n = f_n\big(\{\epsilon_m\}\big)$$

In a fluid dynamical description of the heavy-ion fireball evolution, one finds both linear and nonlinear responses to the initial eccentricities.

Intuition / prejudice: the nonlinear response terms should emerge more slowly(?), and thus possibly probe a different stage of the evolution.

(N.B. & J.-Y.Ollitrault 2005: nonlinearity as a freeze-out effect?)

☞ How does it look like in other approaches to the fireball dynamics?

⇒ in a kinetic description à la Boltzmann?

Further motivation for such models: data from "small systems"

An apology...

Onset of final-state-collectivity signals in nuclear collisions

actually means

Anisotropic flow far from equilibrium

N.B., Steffen Feld & Nina Kersting, arXiv:1804.05729

N.Kersting & N.B., work (= more explicit calculations) in progress

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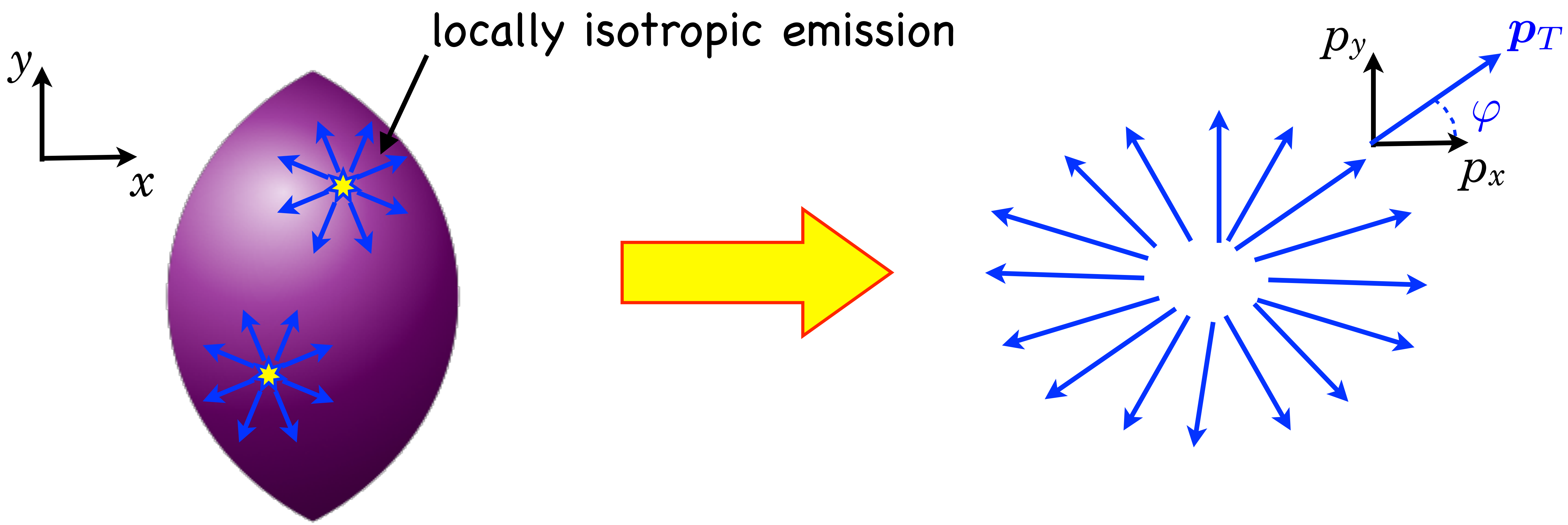
Anisotropic flow: the particle picture



In non-central nucleus-nucleus collisions, the initial spatial asymmetry of the overlap region in the transverse plane is converted by particle rescatterings into an anisotropic transverse-momentum distribution of the outgoing particles: anisotropic flow.

Kinetic theoretical approach

Ingredients for a classical* particle-based description:

- single-particle phase space distribution: $f(t, \mathbf{x}, \mathbf{p})$
  - total particle number $\int_{\mathbf{x},\mathbf{p}} f(t, \mathbf{x}, \mathbf{p}) = N(t)$
  - momentum distribution $\int_{\mathbf{x}} f(t, \mathbf{x}, \mathbf{p}) = E\frac{\mathrm{d}N}{\mathrm{d}^3\mathbf{p}}(t, \mathbf{p})$

$\Rightarrow$ anisotropic flow coefficients $v_n(t)$

final-state observables: let $t \to \infty$

* Quantum mechanical statistics (Pauli blocking, Bose–Einstein enhancement) allowed.

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$\Rightarrow$ anisotropic flow coefficients $v_n(t)$

final-state observables: let $t \to \infty$

Focus on transverse anisotropic flow: consider 2-dimensional system

* Quantum mechanical statistics (Pauli blocking, Bose–Einstein enhancement) allowed.

Kinetic theoretical approach

Ingredients for a classical* particle-based description:

- single-particle phase space distribution: $f(t, \mathbf{x}, \mathbf{p})$
- initial state: at some time t_0
  - eccentricities: "ellipticity" ϵ_2 , "triangularity" ϵ_3 ...
  - typical (transverse) size R

    $\Rightarrow$ typical particle number density $n \sim \dfrac{N}{R^2}$ (remember: 2D)

* Quantum mechanical statistics (Pauli blocking, Bose–Einstein enhancement) allowed.

Kinetic theoretical approach

Ingredients for a classical* particle-based description:

- single-particle phase space distribution: $f(t, \mathbf{x}, \mathbf{p})$
- initial state: at some time t_0
  - eccentricities: "ellipticity" ϵ_2, "triangularity" ϵ_3 ...
  - typical size R $\Rightarrow$ a possible setup (easy to work with analytically):

$$f(t_0, \mathbf{x}, \mathbf{p}) = F(\mathbf{p})\, \mathrm{e}^{-r^2/2R^2}\left[1 + \bar{\epsilon}_2\left(\frac{r}{R}\right)^2 \cos[2(\theta - \Phi_2)] + \bar{\epsilon}_3\left(\frac{r}{R}\right)^3 \cos[3(\theta - \Phi_3)]\right]$$

($\bar{\epsilon}_2$, $\bar{\epsilon}_3$) proportional to ϵ_2, ϵ_3

* Quantum mechanical statistics (Pauli blocking, Bose–Einstein enhancement) allowed.

Kinetic theoretical approach

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- single-particle phase space distribution: $f(t, \mathbf{x}, \mathbf{p})$
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require a cutoff

* Quantum mechanical statistics (Pauli blocking, Bose–Einstein enhancement) allowed.

Kinetic theoretical approach

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- initial state: at some time t_0
  - eccentricities: "ellipticity" ϵ_2, "triangularity" ϵ_3 ...
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$$f(t_0, \mathbf{x}, \mathbf{p}) = \underbrace{F(\mathbf{p})}\, \mathrm{e}^{-r^2/2R^2}\left[1 + \bar{\epsilon}_2\left(\frac{r}{R}\right)^2 \cos[2(\theta - \Phi_2)] + \bar{\epsilon}_3\left(\frac{r}{R}\right)^3 \cos[3(\theta - \Phi_3)]\right]$$

assumed to be isotropic: focus on generated anisotropies

* Quantum mechanical statistics (Pauli blocking, Bose–Einstein enhancement) allowed.

Kinetic theoretical approach

Ingredients for a classical* particle-based description:

- single-particle phase space distribution: $f(t,\mathbf{x},\mathbf{p})$
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  - eccentricities: "ellipticity" ϵ_2 , "triangularity" ϵ_3 ...
  - typical size R ⇒ a possible setup (easy to work with analytically):

$$f(t_0,\mathbf{x},\mathbf{p}) = \underbrace{F(\mathbf{p})}\,\mathrm{e}^{-r^2/2R^2}\left[1+\bar{\epsilon}_2\left(\frac{r}{R}\right)^2\cos[2(\theta-\Phi_2)]+\bar{\epsilon}_3\left(\frac{r}{R}\right)^3\cos[3(\theta-\Phi_3)]\right]$$

assumed to be isotropic: focus on generated anisotropies

+ independent of position

* Quantum mechanical statistics (Pauli blocking, Bose–Einstein enhancement) allowed.

Kinetic theoretical approach

Ingredients for a classical* particle-based description:

- single-particle phase space distribution: $f(t, \mathbf{x}, \mathbf{p})$
- initial state: at some time t_0
- evolution equation, of the kinetic type

$$p_\mu \partial^\mu f(t, \mathbf{x}, \mathbf{p}) = -\mathcal{C}_{\text{coll.}}[f]$$

$\Rightarrow$ various possible choices for the collision term ("setups")

- 2-to-2 elastic (Boltzmann), $2 \longleftrightarrow 3$ scatterings
- include / ignore Pauli blocking / Bose-Einstein enhancement
- rescattering cross section σ.

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Warm up: free streaming

Assume first that there are no rescatterings, $\mathcal{C}_{\text{coll.}}[f] = 0$.

$\Rightarrow$ the evolution equation becomes $\boxed{p_\mu \partial^\mu f(t, \mathbf{x}, \mathbf{p}) = 0}$,

with free-streaming solutions $f^{(0)}(t, \mathbf{x}, \mathbf{p}) = f^{(0)}(t_0, \mathbf{x} - \mathbf{v}(t - t_0), \mathbf{p})$.

integration over space

$$\frac{\mathrm{d}}{\mathrm{d}t}\left[\frac{\mathrm{d}N}{\mathrm{d}^2\mathbf{p}}(t, \mathbf{p})\right] = 0$$

Multiply by $\cos(n\varphi)$, average over φ, let $t \to \infty$...

$$v_n = v_n(t) = v_n(t_0)$$

Boring!

Few-scattering approach

Let us turn on particles rescatterings: $\mathcal{C}_{\rm coll.}[f] \neq 0$.

$\Rightarrow$ need an ansatz for the collision term (next-to-next slide)

If the number of rescatterings per particle is low enough, f should not differ much from the free-streaming solution $f^{(0)}$:

$$f = f^{(0)} + f^{(1)}$$

"low density limit": Heiselberg & Levy, PRC **59** (1999) 2716
"far from equilibrium": N.B. & Gombeaud, EPJC **71** (2011) 1612
"eremitic expansion", Romatschke, arXiv:1802.06804
"one hit dynamics", Kurkela, Wiedemann & Wu, arXiv:1803.02072

Note: (small) expansion parameter needed: wait till slide 20!

Few-scattering approach

Let us turn on particles rescatterings: $\mathcal{C}_{\text{coll.}}[f] \neq 0$.

$\Rightarrow$ need an ansatz for the collision term (next-to-next slide)

If the number of rescatterings per particle is low enough, f should not differ much from the free-streaming solution $f^{(0)}$:

$$f = f^{(0)} + f^{(1)}$$

In the absence of initial momentum-space anisotropy, the generated anisotropic flow is due to $f^{(1)}$, i.e. to rescatterings.

Few-scattering approach

If the number of rescatterings per particle is low enough, f should not differ much from the free-streaming solution $f^{(0)}$:

$$f = f^{(0)} + f^{(1)}$$

Insert into the kinetic evolution equation:

$$p_\mu \partial^\mu f^{(1)}(t, \mathbf{x}, \mathbf{p}) = -\mathcal{C}_{\rm coll.}[f^{(0)} + f^{(1)}]$$

$$\simeq -\mathcal{C}_{\rm coll.}[f^{(0)}]$$

Integrate over $\mathbf{x}$, multiply by $\cos(n\varphi)$, average over φ

$\Rightarrow$ anisotropic flow growth rate $\dfrac{\mathrm{d}}{\mathrm{d}t} v_n(t)$

Few-scattering approach

Let us turn on particles rescatterings: $\mathcal{C}_{\text{coll.}}[f] \neq 0$.

$\Rightarrow$ need an ansatz for the collision term

- relaxation time approximation $\mathcal{C}_{\text{coll.}}[f] = -\dfrac{p^\mu u_\mu}{\tau_{\text{rel.}}}\big(f - f_{\text{eq.}}[f]\big)$

Romatschke, arXiv:1802.06804

Kurkela, Wiedemann & Wu, arXiv:1803.02072

(nonlinearities hidden in u_μ and $f_{\text{eq.}}[f]$)

Few-scattering approach

Let us turn on particles rescatterings: $\mathcal{C}_{\text{coll.}}[f] \neq 0$.

$\Rightarrow$ need an ansatz for the collision term

- relaxation time approximation $\mathcal{C}_{\text{coll.}}[f] = -\dfrac{p^\mu u_\mu}{\tau_{\text{rel.}}}(f - f_{\text{eq.}}[f])$

Romatschke, arXiv:1802.06804
Kurkela, Wiedemann & Wu, arXiv:1803.02072

- collision integral, including a rescattering cross section, as e.g.

$$\mathcal{C}_{\text{coll.}}[f(\mathbf{1})] = \int_{\mathbf{p_2},\mathbf{p_3},\mathbf{p_4}} \left[f(\mathbf{3})f(\mathbf{4})w(\mathbf{3}{+}\mathbf{4} \to \mathbf{1}{+}\mathbf{2}) - f(\mathbf{1})f(\mathbf{2})w(\mathbf{1}{+}\mathbf{2} \to \mathbf{3}{+}\mathbf{4})\right]$$

Heiselberg & Levy, PRC **59** (1999) 2716
N.B. & Gombeaud, EPJC **71** (2011) 1612
N.B., Feld & Kersting, arXiv:1804.05729

Few-scattering approach

If the number of rescatterings per particle is low enough, f should not differ much from the free-streaming solution $f^{(0)}$:

$$f = f^{(0)} + f^{(1)}$$

What is the small parameter here?

Goal: small number of rescatterings per particle: $(\sigma R)\dfrac{N}{R^2} \ll 1$

$$N_{\rm resc.} \sim \frac{N\sigma}{R} \ll 1$$

For an explicit calculation, take an initial configuration, a given collision integral (with the free-streaming solution), integrate over $\mathbf{x}$, t, and the still unspecified momentum.

Scaling behaviors

$$f = f^{(0)} + f^{(1)}$$

- The free-streaming solution $f^{(0)}$ propagates the initial distribution, and thus only contains linear terms in the eccentricities ϵ_n.

$$p_\mu \partial^\mu f^{(1)}(t, \mathbf{x}, \mathbf{p}) = -\mathcal{C}_{\rm coll.}[f^{(0)} + f^{(1)}] \simeq -\mathcal{C}_{\rm coll.}[f^{(0)}]$$

- The first-order correction $f^{(1)}$
  - is linear in the cross section / the small parameter $N_{\rm resc.}$
  - contains nonlinear terms in the eccentricities ϵ_n.

$$\mathcal{C}_{\rm coll.}[f^{(0)}] \ni \int f^{(0)}(\mathbf{1}) f^{(0)}(\mathbf{2})\, \sigma \ni \int \underbrace{\epsilon_2 \cos(2\theta_1)\, \epsilon_2 \cos(2\theta_2)}\, \sigma$$

will contribute to v_4 $\longrightarrow$ $\ni \epsilon_2^2 \cos\left(4\frac{\theta_1+\theta_2}{2}\right)$

Scaling behaviors

$$f = f^{(0)} + f^{(1)}$$

- The free-streaming solution $f^{(0)}$ propagates the initial distribution, and thus only contains linear terms in the eccentricities ϵ_n.

$$p_\mu \partial^\mu f^{(1)}(t, \mathbf{x}, \mathbf{p}) = -\mathcal{C}_{\rm coll.}[f^{(0)} + f^{(1)}] \simeq -\mathcal{C}_{\rm coll.}[f^{(0)}]$$

- The first-order correction $f^{(1)}$
  - is linear in the cross section / the small parameter $N_{\rm resc.}$
  - contains $\underbrace{\text{linear and quadratic}}$ terms in the eccentricities ϵ_n.

  if the collision integral is quadratic in f

  (as in the Boltzmann ansatz)

Anisotropic flow far from equilibrium

Summary of findings with the Boltzmann collision kernel:

- elliptic flow: $v_2 = \mathcal{O}(N_{\rm resc.})\,\epsilon_2$
- triangular flow: $v_3 = \mathcal{O}(N_{\rm resc.})\,\epsilon_3$
- quadrangular flow: $v_4 = \mathcal{O}(N_{\rm resc.})\,\epsilon_2^2$
- pentagonal flow: $v_5 = \mathcal{O}(N_{\rm resc.})\,\epsilon_2\epsilon_3$
- hexagonal flow: $v_6 = \mathcal{O}(N_{\rm resc.})\,\epsilon_3^2$

Anisotropic flow far from equilibrium vs. in fluid dynamics

elliptic flow:	$v_2 = \mathcal{O}(N_{\rm resc.})\,\epsilon_2$	$v_2 \simeq \mathcal{K}^{\rm hydro}_{2,2}\,\epsilon_2$
triangular flow:	$v_3 = \mathcal{O}(N_{\rm resc.})\,\epsilon_3$	$v_3 \simeq \mathcal{K}^{\rm hydro}_{3,3}\,\epsilon_3$
quadrangular flow:	$v_4 = \mathcal{O}(N_{\rm resc.})\,\epsilon_2^2$	$v_4 \simeq \mathcal{K}^{\rm hydro}_{4,22}\,\epsilon_2^2$
pentagonal flow:	$v_5 = \mathcal{O}(N_{\rm resc.})\,\epsilon_2\epsilon_3$	$v_5 \simeq \mathcal{K}^{\rm hydro}_{5,23}\,\epsilon_2\epsilon_3$
hexagonal flow:	$v_6 = \mathcal{O}(N_{\rm resc.})\,\epsilon_3^2$	$v_6 \simeq \mathcal{K}^{\rm hydro}_{6,222}\,\epsilon_2^3 + \mathcal{K}^{\rm hydro}_{6,33}\,\epsilon_3^2$

reminder from slide 7

Anisotropic flow far from equilibrium vs. in fluid dynamics

elliptic flow:	$v_2 = \mathcal{O}(N_{\rm resc.})\,\epsilon_2$	$v_2 \simeq \mathcal{K}^{\rm hydro}_{2,2}\,\epsilon_2$
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not there at order $\mathcal{O}(N_{\rm resc.})$

☞ As anticipated(?), some of the nonlinear response terms seem to require more time (rescatterings) to be generated!

v_6 far from equilibrium

What about the contribution of order ϵ_2^3 to v_6?

Write $f = f^{(0)} + f^{(1)} + f^{(2)}$, insert into the evolution equation

$$p_\mu \partial^\mu [f^{(1)} + f^{(2)}] = -\mathcal{C}_{\rm coll.}[f^{(0)} + f^{(1)} + f^{(2)}]$$
$$\simeq -\mathcal{C}_{\rm coll.}[f^{(0)} + f^{(1)}]$$
$$= \underbrace{-\mathcal{C}_{\rm coll.}[f^{(0)}]}_{\text{yields } f^{(1)}} - \underbrace{\left(\mathcal{C}_{\rm coll.}[f^{(0)} + f^{(1)}] - \mathcal{C}_{\rm coll.}[f^{(0)}]\right)}_{\text{The terms } \mathcal{O}(N_{\rm resc.}^2) \text{ yield } f^{(2)}}$$

$f^{(2)}$ does contain a term in ϵ_2^3

☛ $v_6 = \mathcal{O}(N_{\rm resc.})\,\epsilon_3^2 + \mathcal{O}(N_{\rm resc.}^2)\,\epsilon_2^3$ nontrivial?

v_6 far from equilibrium

$$v_6 = \mathcal{O}(N_{\rm resc.})\,\epsilon_3^2 + \mathcal{O}(N_{\rm resc.}^2)\,\epsilon_2^3$$

How robust is this?

To get a contribution in ϵ_2^3 to v_6 already at order $\mathcal{O}(N_{\rm resc.})$, one must modify the collision integral:

- include $2 \longleftrightarrow 3$ scatterings (actually, only $3 \longrightarrow 2$ is useful)

  however $\sigma_{3\to2}$ is smaller than $\sigma_{2\to2}$: does not really work

- include Pauli blocking / Bose–Einstein enhancement factors

$$f(\mathbf{1})f(\mathbf{2})[1 \pm f(\mathbf{3})][1 \pm f(\mathbf{4})]$$

  relevant only for a dense system

☞ Does one remain in the "small number of rescatterings" regime?

Anisotropic flow far from equilibrium

One can find various scaling behaviors between initial eccentricities and final-state anisotropic flow harmonics.

- Are they confirmed by numerical studies?
  - Ultimately, how does the scaling with the number of rescatterings manifest themselves in the fluid-dynamical regime: dependence on viscosity of the proportionality coefficients $\mathcal{K}_{n,..}^{\rm hydro}$?
- Are they of any relevance for experimental data?
  - Small systems
  - Pre-hydrodynamization stage in heavy ion collision.

… And another thing…

An unpleasant curiosity?

A simple setup:

$$f(t_0,\mathbf{x},\mathbf{p}) = F(\mathbf{p})\,\mathrm{e}^{-r^2/2R^2}\left[1+\bar{\epsilon}_2\left(\frac{r}{R}\right)^2\cos[2(\theta-\Phi_2)]+\bar{\epsilon}_3\left(\frac{r}{R}\right)^3\cos[3(\theta-\Phi_3)]\right]$$

for massless particles with $2\longleftrightarrow 2$ elastic scatterings, with an isotropic cross section.

Leads to finite v_2, v_4, v_6 but vanishing v_3, v_5 at order $\mathcal{O}(N_{\rm resc.})$…

… And another thing…

$$f(t_0,\mathbf{x},\mathbf{p}) = F(\mathbf{p})\,\mathrm{e}^{-r^2/2R^2}\left[1+\bar\epsilon_2\left(\frac{r}{R}\right)^2\cos[2(\theta-\Phi_2)] + \bar\epsilon_3\left(\frac{r}{R}\right)^3\cos[3(\theta-\Phi_3)]\right]$$

The calculation involves that of the integral

$$\int f^{(0)}(t_0,\mathbf{x}-\mathbf{v}_1(t-t_0),\mathbf{p}_1)\,f^{(0)}(t_0,\mathbf{x}-\mathbf{v}_2(t-t_0),\mathbf{p}_2)\,\mathrm{d}^2\mathbf{x} = \int f^{(0)}(t_0,\mathbf{x},\mathbf{p}_1)\,f^{(0)}(t_0,\mathbf{x}-\mathbf{X},\mathbf{p}_2)\,\mathrm{d}^2\mathbf{x}$$

where $\mathbf{X} = (\mathbf{v}_2-\mathbf{v}_1)(t-t_0)$

A term linear in $\bar\epsilon_3$ comes with an odd contribution to the integrand, thus does not contribute to the integral… Leading to no v_3.

But do I understand the physics here?