COLLECTIVE FLOW
AND
MULTIPARTICLE AZIMUTHAL CORRELATIONS

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• Standard collective flow analysis
  → two-particle correlations
  – Limited sensitivity
  

• New method

  → multiparticle correlations

  – Integrated flow
  – Differential flow
  – Increased sensitivity
  – Acceptance corrections

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FLOW

Flow $\equiv$ azimuthal correlation with the reaction plane:

$$\frac{dN}{d\phi} = A (1 + v_1 \cos \phi + v_2 \cos 2\phi + \cdots)$$

where:

$$v_n = \langle e^{in\phi} \rangle.$$ 

$v_n(p_T, y)$ differential flow; $v_n(D)$ integrated flow.

$v_1$ “directed” flow, $v_2$ “elliptic” flow.

At CERN SPS, $v_1$ and $v_2 \simeq 3\%$ for pions and protons. PHENIX & STAR analyses: $v_2 \simeq 5 - 6\%.$
STANDARD FLOW ANALYSIS

Coefficient $v_n$ extracted from the measured two-particle azimuthal correlations:

$$\langle e^{in(\phi_1-\phi_2)} \rangle = \langle e^{in\phi_1} \rangle \langle e^{-in\phi_2} \rangle + \langle e^{in(\phi_1-\phi_2)} \rangle_c$$

$$\equiv v_n^2 + \langle e^{in(\phi_1-\phi_2)} \rangle_c.$$

[study of $\Delta \phi$ correlation (see Roy Lacey) or correlation between 2 subevents].

Expansion of two-particle correlations:

```
● ● ● = ○ ○ ○ + ● ● ●
```

measured  flow  nonflow

“STANDARD” ASSUMPTION: the measured two-particle azimuthal correlations are only due to flow:

$$v_n = \pm \sqrt{\langle e^{in(\phi_1-\phi_2)} \rangle}.$$ 

Other sources of two-particle azimuthal correlations are negligible:

$$v_n^2 \gg \langle e^{in(\phi_1-\phi_2)} \rangle_c.$$ 

Is that true?
TWO-PARTICLE NONFLOW ("DIRECT") CORRELATIONS

Many sources for $\langle e^{in(\phi_1 - \phi_2)} \rangle_c$:

- total momentum conservation;
- quantum "HBT" correlations;
- final state (strong / Coulomb) interactions;
- resonance decays;
- other sources? (minijets...)

$\Rightarrow$ the assumption $v_n^2 \gg \langle e^{in(\phi_1 - \phi_2)} \rangle_c$ underlying the standard analysis holds only if

$$v_n \gg \frac{1}{N^{1/2}}.$$ 

Possibility: compute and subtract nonflow correlations.

OK, but nonflow correlations may not be under control... 

Important: two-particle nonflow correlations scale as $\frac{1}{N}$

$\Rightarrow$ dominant for peripheral collisions.
STANDARD FLOW ANALYSIS AT SPS

“Standard” assumption: $v_n^2 \gg \left\langle e^{in(\phi_1-\phi_2)} \right\rangle_c \sim \frac{1}{N}$.

- $v_1$ and $v_2 \simeq 3\%$ for pions and protons;
- total multiplicity in the collision $N \simeq 2500$.

⇒ the assumption is not valid.

Pion directed flow at SPS

□: “data”
•: data − HBT
×: data − (HBT & $p_T$ conservation)
NEW METHOD

Idea: extract flow from multiparticle azimuthal correlations.

Method: compare flow with direct 3-particle correlations
⇒ eliminate (non-negligible) extra terms:

\[
\begin{align*}
\mathcal{O}(1) & = \sum \Delta v_n^3 + \sum \Delta v_{nN} + \sum \Delta v_{nN^2} + \mathcal{O}(1/N^2).
\end{align*}
\]

cumulant of the multiparticle correlations.
NEW METHOD: INTEGRATED FLOW $v(D)$

Cumulant of the four-particle azimuthal correlation:

$$
\left\langle e^{in(\phi_1-\phi_2+\phi_3-\phi_4)} \right\rangle \equiv \left\langle e^{in(\phi_1-\phi_2+\phi_3-\phi_4)} \right\rangle - 2\left\langle e^{in(\phi_1-\phi_2)} \right\rangle^2
\hspace{1cm} = -v_n^4 + O\left(\frac{1}{N^3}\right)
$$

Increased sensitivity: analysis valid if $v_n \gg \frac{1}{N^{3/4}}$. 

DIFFERENTIAL FLOW $v'(p_T, y)$

1. Measure the integrated flow $\langle e^{i n \phi} \rangle = v_n$ using many particles ("pions"): reaction plane determination.

2. Study the correlation between the azimuth $\psi$ of a given particle ("proton") and the reaction plane: $\langle e^{-i n \phi} e^{i n \psi} \rangle$.

\[
\left( \begin{array}{c}
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\bullet \\
\end{array} \right) = \left( \begin{array}{c}
\times \\
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\end{array} \right) + ... + 2 \left( \begin{array}{c}
\times \\
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\bullet \\
\end{array} \right) + ... + \left( \begin{array}{c}
\times \\
\bullet \\
\bullet \\
\bullet \\
\end{array} \right)
\]

\[
v_n^3 v'_n \langle e^{-i n \phi} e^{i n \psi} \rangle_c \langle e^{i n (\phi_1 - \phi_2)} \rangle_c = O\left( \frac{1}{N^3} \right)
\]

Idea: compare the flow term with the direct multiparticle azimuthal correlation.

$\Rightarrow$ Cumulant of the $(1+3)$-particle azimuthal correlation:

\[
\left\langle e^{i n (\phi_1 - \phi_2 - \phi_3)} e^{i n \psi} \right\rangle \equiv \langle e^{i n (\phi_1 - \phi_2 - \phi_3)} e^{i n \psi} \rangle - 2 \langle e^{-i n \phi} e^{i n \psi} \rangle \langle e^{i n (\phi_1 - \phi_2)} \rangle = -v_n^3 \left[ v'_n + O\left( \frac{1}{N v_n^3} \right) \right].
\]
COLLECTIVE FLOW
AND
MULTIPARTICLE AZIMUTHAL CORRELATIONS

• At SPS energies, two-particle azimuthal correlations due either to collective flow or nonflow effects are of the same magnitude. ⇒ the standard analysis is close to its validity limit $v_n \gg 1/N^{1/2}$.

• New method, using four-particle azimuthal correlations, allows measurements of smaller integrated flow values $v_n \gg 1/N^{3/4}$.

  Sensitivity can still be improved, with multiparticle (involving $2k$ particles, $k > 4$) correlations.

• Detector acceptance corrections.

• Differential flow.

Two different methods to extract flow are available...

⇒ HANDS WANTED!

Both methods may yield different results...

“NEW” (unthought of) two-particle correlations!
**EVENT FLOW VECTOR**

For a given event:

\[ Q_n = \frac{1}{\sqrt{M}} \sum_{k=1}^{M} e^{in\phi_k} \]

\( M \) as large as possible.

- **Flow** ⇔ \( \langle Q_n \rangle = \sqrt{M} v_n \neq 0 \): random walk with a preferred direction;
- Powers of \( |Q_n|^2 \) involve multiparticle azimuthal correlations:
  \[ |Q_n|^2 = \frac{1}{M} \sum_{j,k=1}^{M} e^{in(\phi_j - \phi_k)} ; \]
- **Cumulants** of the \( |Q_n| \) distribution yield the flow:
  \[ \langle|Q_n|^4\rangle \equiv \langle|Q_n|^4\rangle - 2 \langle|Q_n|^2\rangle^2 = -\langle Q_n \rangle^4 + O\left(\frac{1}{M}\right) \]

Method valid if \( \langle Q_n \rangle^4 \gg \frac{1}{M} \iff v_n \gg \frac{1}{M^{3/4}} \)

- Increasing sensitivity using higher order cumulants \( \langle|Q_n|^{2p}\rangle \).
CUMULANTS $\langle |Q_n|^{2p} \rangle$: PRACTICAL FLOW ANALYSIS

“old version”: Phys. Rev. C63 (may 2001)

1. Compute $\bar{Q}_n = \frac{1}{\sqrt{M}} \sum_{k=1}^{M} e^{i n \bar{\phi}_k}$ for a given event ($\bar{\phi}_k$ measured angle).

2. Calculate the generating function $G(z) = e^{z^* \bar{Q}_n + z \bar{Q}_n^*}$, then average over events. Why? because $\langle G(z) \rangle = 1 + \cdots + |z|^2 \langle |\bar{Q}_n|^2 \rangle + \cdots + \frac{|z|^4}{4} \langle |\bar{Q}_n|^4 \rangle + \cdots$

3. Deduce the cumulants, taking $\ln \langle G(z) \rangle$:

$$\ln \langle G(z) \rangle = 1 + \cdots + |z|^2 \langle |\bar{Q}_n|^2 \rangle + \cdots + \frac{|z|^4}{4} \langle |\bar{Q}_n|^4 \rangle + \cdots$$

4. Extract the flow, using $\ln \langle G(z) \rangle = \ln I_0(2|z| \langle |Q_n| \rangle)$.

→ for instance, $\langle |\bar{Q}_n|^6 \rangle \equiv \langle |\bar{Q}_n|^6 \rangle - 9 \langle |\bar{Q}_n|^4 \rangle \langle |\bar{Q}_n|^2 \rangle + 12 \langle |\bar{Q}_n|^2 \rangle^3 = 4 \langle Q_n \rangle^6$.

5. Put your paper on nucl-ex.
INTERFERENCE BETWEEN \( v_1 \) AND \( v_2 \)

\[
- 2 \left( \begin{array}{c} \bullet \\ \bullet \end{array} \right)^2 \approx - \left( \begin{array}{c} \circ \\ \circ \end{array} \right) + \left( \begin{array}{c} \circ \\ \circ \end{array} \right) + \left( \begin{array}{c} \bullet \\ \bullet \\ \bullet \\ \bullet \end{array} \right)
\]

\[
\langle e^{i n (\phi_1 - \phi_2 + \phi_3 - \phi_4)} \rangle
\]

\[ v_n^4 \quad O\left(\frac{v_{2n}^2}{N^2}\right) \quad O\left(\frac{1}{N^3}\right) \]

\( \Rightarrow \) Measurements of \( v_n \) require \( |v_{2n}| \ll N v_n^2 \).

Problem for directed flow at RHIC, not for elliptic flow.
BETTER CUMULANTS: ANY HARMONIC

“new version”: keep an eye on nucl-th

1. Calculate the generating function \( G(z) = \prod_{k=1}^{M} \left( 1 + z^* e^{i\phi_k} + z e^{-i\phi_k} \right) \),
then average over events (\( \phi_k \) measured angle).

\[
\langle G(z) \rangle = 1 + \cdots + |z|^2 \left< \sum_{j \neq k} e^{i(\phi_j - \phi_k)} \right> + \cdots + \frac{|z|^4}{4} \left< \sum_{j,k,l,m} e^{i(\phi_j + \phi_k - \phi_l + \phi_m)} \right> + \cdots
\]

2. Deduce the cumulants, taking \( \langle G(z) \rangle^{1/M} - 1 \):

\[
\langle G(z) \rangle^{1/M} = 1 + \cdots + |z|^2 (M - 1) \left< e^{i(\phi_j - \phi_k)} \right> + \cdots
\]

3. Extract the flow, using \( \langle G(z) \rangle^{1/M} - 1 = I_0(2M v_n |z|)^{1/M} - 1 \).

Work still in progress (improving acceptance corrections).
WHY FLOW?

• Flow determination ⇒ equation of state:

Before the collision

\begin{align*}
\text{Au} & \quad \text{Au} \\
\text{out-of-plane emission} & \quad \langle \cos 2\phi \rangle < 0
\end{align*}

After the collision

\begin{align*}
\text{Au} & \quad \text{Au} \\
\text{in-plane emission} & \quad \langle \cos 2\phi \rangle > 0
\end{align*}

• Influence of flow on two-particle correlations (HBT, Coulomb...).

• Observation of possible parity violation requires accurate flow determination.