Anisotropic flow from Lee–Yang zeroes

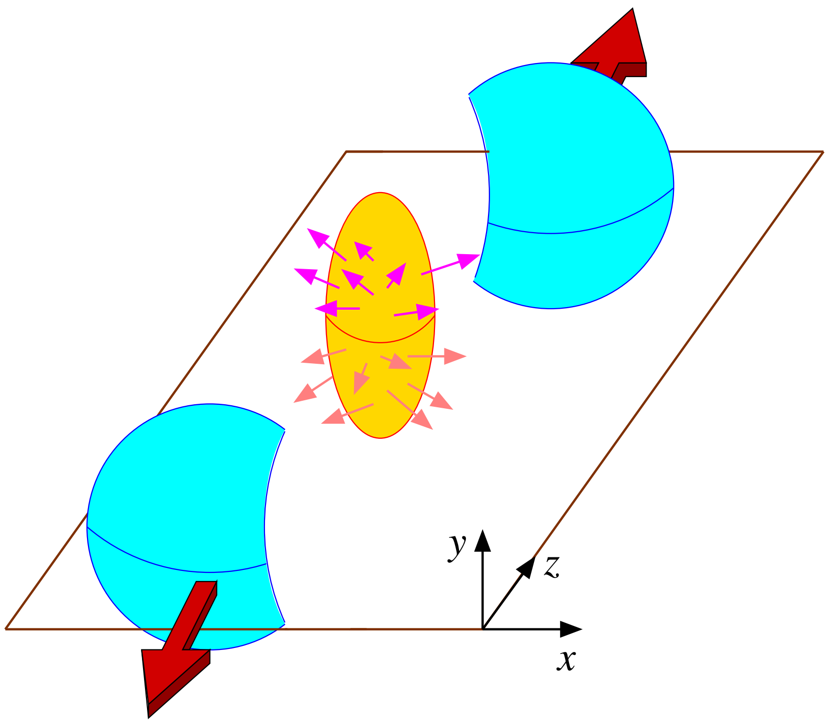
...or how to analyze v_2, v_4 $\ldots$ at RHIC & LHC

Nicolas Borghini[1],

Rajeev Bhalerao[2], Jean-Yves Ollitrault[1]

[1] CEA Saclay, [2] TIFR Mumbai

Anisotropic flow?



Source anisotropy

$\Rightarrow$ anisotropic emission of particles:
(transverse) FLOW

$$\frac{\mathrm{d}N}{\mathrm{d}\phi} \propto 1 + 2v_1 \cos(\phi - \Phi_R) + 2v_2 \cos 2(\phi - \Phi_R) + \ldots$$

v_1 "directed", v_2 "elliptic"

average over particles and events:

$$v_n = \langle \cos n(\phi - \Phi_R) \rangle$$

reaction plane, unknown

How is v_n measured?

- Reminder: analysis of v_n till now
- A new, simple method to analyze anisotropic flow
  - a simple recipe ☺
  - a quick justification: generating function, cumulants, zeroes
  - an enlightening analogy: Lee–Yang zeroes
- Summary

R.S. Bhalerao, N.B., J.-Y. Ollitrault, Nucl. Phys. A**727** (2003) 373

Methods used so far (RHIC)

Idea: measure v_n from interparticle correlations

- Two-particle methods:
  - Event-plane method ☞ PHENIX, PHOBOS, STAR
  - 2-particle correlation function & mixed events ☞ PHENIX
  - Cumulant of 2-particle correlations ☞ PHENIX, STAR

Measured 2-particle correlations plagued by nonflow effects 

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- Cumulants of 3-, 4-, 6-particle correlations ☞ STAR

Most nonflow effects subtracted 

An alternative to cumulants

NEW

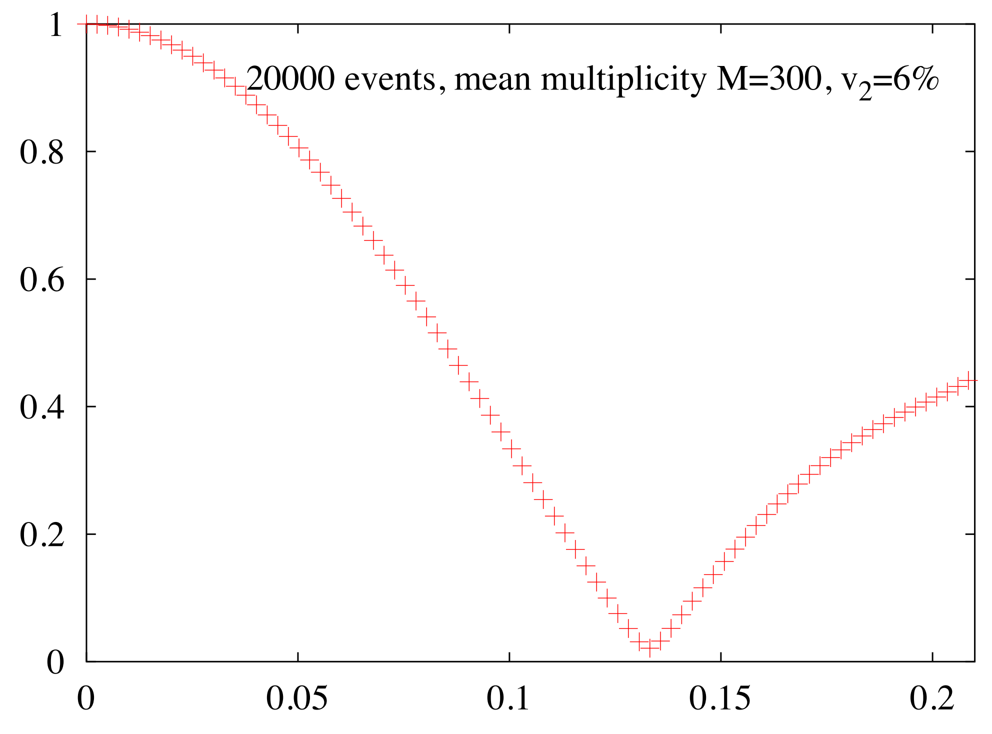
1. For each event, compute $g(\mathrm{i}r) \equiv \prod_j (1 + \mathrm{i}r \cos n\phi_j)$ for various r

lab. frame

2. Average over events:

$$G(\mathrm{i}r) \equiv \frac{1}{N_{\mathrm{evts}}} \sum_{\mathrm{evts}} g(\mathrm{i}r),$$

and plot $|G(\mathrm{i}r)|$



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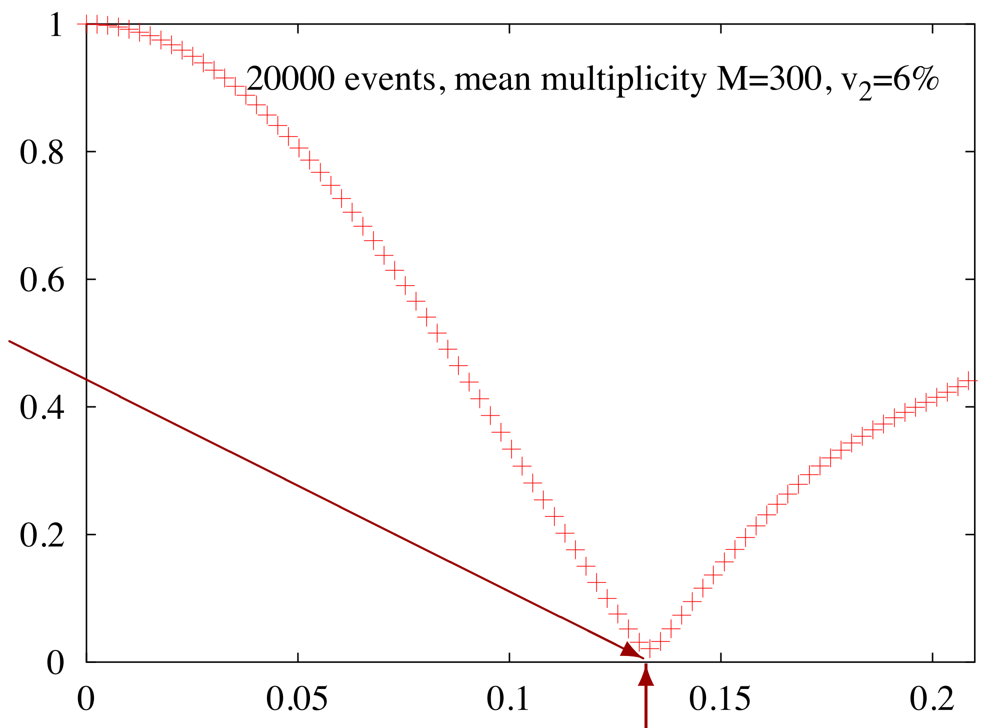
3. Read out the first minimum:

♡ r_0 is insensitive to nonflow correlations

♡ r_0 is related to $v_n = \dfrac{2.405}{M r_0}$



$r_0 = 0.133$

$\Rightarrow v_2 = 0.0603$

THAT'S ALL FOLKS!

Generating function, cumulants [part 1]

$$G(z) \equiv \left\langle \prod_{j=1}^{M} (1 + z \cos n\phi_j) \right\rangle$$ generating function (of multiparticle averages)

complex

Cumulants c_k are defined by $\ln G(z) \equiv \sum_{k=1}^{+\infty} c_k \frac{z^k}{k!}$

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ASSUMPTION: no flow = no collective behavior

system made of independent subsystems

$$\Rightarrow \quad G(z) = \prod_{\text{subsyst.}} G_{\text{sub.}}(z)$$

$$\ln G(z) = \sum_{\text{subsyst.}} \ln G_{\text{sub.}}(z) \quad \Rightarrow \quad c_k = \sum_{\text{subsyst.}} (c_k)_{\text{sub.}}$$

If there is no flow, all cumulants scale linearly with the system size M

Generating function, cumulants [part 2]

$$G(z) \equiv \left\langle \prod_{j=1}^{M} (1 + z \cos n\phi_j) \right\rangle , \qquad \ln G(z) \equiv \sum_{k=1}^{+\infty} c_k \frac{z^k}{k!}$$

cumulant

ASSUMPTION: system with flow

- Consider first collisions with the same reaction plane Φ_R:

  $\langle \cos n\phi \rangle = v_n \cos n\Phi_R \quad \ldots \quad G(z) = (\ldots)\, \mathrm{e}^{zMv_n \cos n\Phi_R}$

  a few omitted steps

- Then average over Φ_R: $\quad G(z) = (\ldots)\, I_0(\underbrace{zM}\, v_n)$

  $\Rightarrow z^k$ goes with a factor M^k

- Take the logarithm: the cumulant c_k scales like M^k

Generating function, zeroes

$$\left.\begin{array}{l}\text{nonflow effects } c_k \propto M \\ \text{collective flow } c_k \propto M^k\end{array}\right\}\text{to isolate flow, compute } c_k \text{ for large } k$$

Large k?

asymptotic behavior of the coefficients in the expansion of $\ln G(z)$

Entirely determined by the first zero of $G(z)$ in the complex plane!

See, for instance, $\ln\left(1-\frac{z}{z_0}\right)=\sum_{k=1}^{+\infty}\frac{z^k}{k\,{z_0}^k}$: large-order coefficients are controlled by z_0

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First zero? When there is flow, $G(z) = \underbrace{(\ldots)}_{\text{no zero}} I_0(zMv_n)$:

The first zero lies at $z_0 = \dfrac{2.405\mathrm{i}}{Mv_n}$: find z_0, and you get v_n

$$\left.\begin{array}{l}\text{finite number of events} \\ \text{uneven detector acceptance}\end{array}\right\} \text{☞ find first minimum of } |G(\mathrm{i}r)|\text{, rather than first zero of } G(\mathrm{i}r)$$

Analogy: Lee-Yang zeroes

Phys. Rev. 87 (1952) 404: a theory of phase transitions

- Grand partition function (T, V fixed): $\mathcal{Q}(\mu) = \sum_{N=0}^{+\infty} Z_N e^{\mu N/kT}$
- Take a reference value μ_c, define $z \equiv (\mu - \mu_c)/kT$
- Let $\mathcal{G}(z) \equiv \dfrac{\mathcal{Q}(\mu)}{\mathcal{Q}(\mu_c)} = \sum_{N=0}^{+\infty} P_N e^{zN}$: generating function

  P_N: probability to have N particles at $\mu = \mu_c$
- Let the volume V (= the system size) increase
  - if no phase transition, the zeroes are unchanged
  - if phase transition at $\mu = \mu_c$, the zeroes of $\mathcal{G}$ come closer to 0

    phase transition → long-range correlations, collective behavior

Anisotropic flow?

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Anisotropic flow → long-range correlations, collective behavior

Anisotropic flow analysis with Lee–Yang zeroes

- First application of Lee–Yang theory to experimental data
- Direct study of correlations involving many particles: conceptually, the best way to analyze collective flow
- Very simple implementation
- Various omitted features: (Nucl. Phys. A**727**, 373: 54 pages!)
  - Differential flow
  - Acceptance corrections
  - Stability with respect to nonflow effects
  - Statistical errors (for STAR, the same as with 4-particle cumulants!)
- THE method to analyze v_2 and v_4 at LHC

See also Phys. Lett. B**580** (2004) 157