Jet broadening in a medium-modified parton shower approach

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Medium-induced modifications of jets

Jet physics in collisions of elementary particles

Modified Leading Logarithmic Approximation (MLLA)
(of QCD)

Towards a “medium-modified MLLA”: analytical approach

longitudinal distributions inside jets

N.B. & U.A. Wiedemann, hep-ph/0506218

transverse momentum distributions inside jets

N.B.... work in progress!

Analytical calculations only: no comparison with data!

intended as baseline for Monte Carlo implementations.
Jets “in vacuum”

Jet physics in collisions of elementary particles

longitudinal inclusive distributions inside jets

e^+e^- collisions, p\bar{p} collisions

particle with momentum \((k_0, \vec{k})\)

\[ \xi \quad \text{aka} \quad \ell \equiv \ln \frac{E_{\text{jet}}}{k_0} = \ln \frac{1}{x} \]
Jets in vacuum: successes of MLLA e^+e^- data

OPAL Collaboration, Phys. Lett. B 247 (1990) 617 (includes comparison with MLLA)
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Jets in vacuum: successes of MLLA

\( \bar{p}p \) data

data (here \( \theta_c = 0.47 \)) + comparison with MLLA: CDF Collaboration, Phys. Rev. D 68 (2003) 012003
Jets in vacuum

Jet physics in collisions of elementary particles

- longitudinal inclusive distributions inside jets
  - $e^+e^-$ collisions, $p\bar{p}$ collisions

- transverse momentum inclusive distributions inside jets
  - $p\bar{p}$ collisions

Particle with momentum $(k_0, \vec{k}_\parallel \approx k_0, \vec{k}_\perp)$

$$y \equiv \ln \frac{k_\perp}{Q_0} \approx \ln \frac{k_0\Theta}{Q_0}$$

$k_\parallel, \vec{k}_\perp, \Theta$: with respect to the jet axis (direction of energy flow)

$Q_0$: infrared cutoff
Jets in vacuum: successes of (N)MLLA p\bar{p} data

Jets “in vacuum”:
Ye big Booke of pQCD
(especially for jet calculus)

available @ http://www.lpthe.jussieu.fr/~yuri/BPQCD/cover.html
**MLLA in four/five slides**

Main ingredients:
- Resummation of double- and single-logarithms in $\ln \frac{1}{x}$ and $\ln \frac{E_{\text{jet}}}{Q_0}$;
- Takes into account the running of $\alpha_s$ along the parton shower evolution;
- Probabilistic interpretation (results from intra-jet colour coherence):
  - independent successive branchings $g \to gg$, $g \to q\bar{q}$, $q \to qg$;
  - with angular ordering of the sequential parton decays:
    - at each step in the evolution, the angle between father and offspring partons decreases.
- Includes in a systematic way next-to-leading-order corrections.

$O(\sqrt{\alpha_s})$!
Central object: generating functional \( Z_i[Q, \Theta; u(k)] \)

generates the various cross-sections (\( \rightarrow g g g \), \( \rightarrow g g q \bar{q} \) ...) for a jet initiated by a parton \( i (= g, q, \bar{q}) \) with energy \( Q \) in a cone of angle \( \Theta \).

\[
Z_i[Q, \Theta; u(k)] = e^{-\omega_i(Q, \Theta)} u(Q) \\
+ \sum_j \int_0^\Theta d\Theta' \int_0^1 d\epsilon e^{\omega_i(Q, \Theta') - \omega_i(Q, \Theta)} \frac{\alpha_s(k_\perp)}{2\pi} \\
\times P_{ji}(\epsilon) Z_j[zQ, \Theta'; u] Z_k[(1 - z)Q, \Theta'; u]
\]
Central object: generating functional $Z_i[Q, \Theta; u(k)]$

generates the various cross-sections ($\rightarrow ggg$, $\rightarrow gqq\bar{q}$ ...) for a jet initiated by a parton $i (= g, q, \bar{q})$ with energy $Q$ in a cone of angle $\Theta$.

$$Z_i[Q, \Theta; u(k)] = e^{-w_i(Q, \Theta)} u(Q) + \sum_j \int_0^{\Theta} d\Theta' \int_0^1 dz \left[ e^{w_i(Q, \Theta') - w_i(Q, \Theta)} \frac{\alpha_s(k_{\perp})}{2\pi} \right] \times P_{ji}(z) Z_j[zQ, \Theta'; u] Z_k[(1 - z)Q, \Theta'; u]$$
Central object: generating functional $Z_i[Q, \Theta; u(k)]$

generates the various cross-sections ($\rightarrow ggg, \rightarrow ggpq\bar{q} \ldots$) for a jet initiated by a parton $i$ ($= g, q, \bar{q}$) with energy $Q$ in a cone of angle $\Theta$.

$$Z_i[Q, \Theta; u(k)] = e^{-w_i(Q, \Theta)} u(Q)$$

$$+ \sum_j \int_0^\Theta d\Theta' \int_0^1 dz \ e^{w_i(Q, \Theta') - w_i(Q, \Theta)} \frac{\alpha_s(k_{\perp})}{2\pi}$$

$$\times P_{ji}(z) Z_j[zQ, \Theta'; u] Z_k[(1 - z)Q, \Theta'; u]$$

probability to have no branching with angle $< \Theta$
Central object: generating functional $Z_i[Q, \Theta; u(k)]$ generates the various cross-sections ($\rightarrow ggg$, $\rightarrow gqq\bar{q}$ ...) for a jet initiated by a parton $i (= g, q, \bar{q})$ with energy $Q$ in a cone of angle $\Theta$.

$$Z_i[Q, \Theta; u(k)] = e^{-w_i(Q, \Theta)} u(Q) + \sum_j \int^{\Theta} \frac{d\Theta'}{\Theta'} \int_0^1 dz e^{w_i(Q, \Theta') - w_i(Q, \Theta)} \frac{\alpha_s(k_{\perp})}{2\pi} \times P_{ji}(z) Z_j[zQ, \Theta'; u] Z_k[(1 - z)Q, \Theta'; u]$$

probability to have no branching with angle $< \Theta$ between $\Theta$ and $\Theta'$

angular ordering
Central object: generating functional \( Z_i[Q, \Theta; u(k)] \)

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\]

LO splitting function \( i \rightarrow jk \)
Central object: generating functional $Z_i[Q, \Theta; u(k)]$

Generates the various cross-sections ($\rightarrow ggg$, $\rightarrow ggq\bar{q}$ ...) for a jet initiated by a parton $i$ ($= g, q, \bar{q}$) with energy $Q$ in a cone of angle $\Theta$.

\[
Z_i[Q, \Theta; u(k)] = e^{-w_i(Q, \Theta)} u(Q) + \sum_j \int^{\Theta} d\Theta' \int^1_0 dz \frac{e^{w_i(Q, \Theta')-w_i(Q, \Theta)}}{\Theta'} \alpha_s(k_\perp) \frac{1}{2\pi} \times P_{ji}(z) Z_j[zQ, \Theta'; u] Z_k[(1-z)Q, \Theta'; u]
\]

Angular ordering

Probability to have no branching with angle $< \Theta$ between $\Theta$ and $\Theta'$

LO splitting function

$i \rightarrow jk$

$k_\perp \approx z(1-z)Q$
MLLA in four/five slides

The 1st derivative of the generating function gives (after some detours in Mellin space) the inclusive longitudinal distribution of partons inside a (gluon) jet:

"limiting spectrum"

\[
\tilde{D}_G^{\text{lim}}(x, Y_{\Theta}, Q_0) = \frac{4N_cY_{\Theta}}{bB(B+1)} \int_{i-\infty}^{\epsilon+i\infty} \frac{d\nu}{2\pi i} x^{-\nu} \Phi(-A + B + 1, B + 2; -\nu Y_{\Theta})
\]

with

\[
A \equiv \frac{4N_c}{b\nu}, \quad B \equiv \frac{a}{b}, \quad a \equiv \frac{11}{3}N_c + \frac{2N_f}{3N_c^2}, \quad b \equiv \frac{11}{3}N_c - \frac{2}{3}N_f
\]

( these coefficients follow from the prefactors of the leading-order splitting functions)

and

\[
Y_{\Theta} \equiv \ln \frac{E_{\text{jet}} \sin \Theta}{Q_0}
\]

Impressive expression... which can be dealt with!
For a 100 GeV parton:

“hump-backed plateau”
MLLA in four/five slides

For a 100 GeV parton:

\[ \frac{dN}{d \ln(1/x)} \]

- hard partons
- soft partons

"hump-backed plateau"

cutoff \( Q_0 \)
For a 100 GeV parton:

\[ \ln \frac{1}{x} = \frac{Y_\Theta}{2} \left( 1 + a \sqrt{\frac{\alpha_s}{8\pi N_c}} \right) \]

Note: the hump is dominated by the singular parts of the $P_{ji}(z)$.
MLLA in four/five slides

Modified Leading Logarithmic Approximation:

- Successive independent parton splittings, with a constraint on the emission angles

⇒ limiting spectrum \( \bar{D}^{\text{lim}}(x, Y_\Theta, Q_0) \)

- The spectrum is exact in the asymptotic \( Y_\Theta \to \infty \) limit and includes in a systematic way corrections to subleading order

\[ \mathcal{O}(\sqrt{\alpha_s}) \]
Modified Leading Logarithmic Approximation:

- Successive independent **parton splittings**, with a constraint on the emission angles

  ⇒ **limiting spectrum** \( \bar{D}^{\text{lim}}(x, Y_\Theta, Q_0) \)

- The spectrum is exact in the asymptotic \( Y_\Theta \to \infty \) limit and includes in a systematic way **corrections to subleading order**

  \( \mathcal{O}(\sqrt{\alpha_s}) \)

- What about hadronization? (\( \bar{D}^{\text{lim}}(x, Y_\Theta, Q_0) \) is a parton spectrum)

  Local parton-hadron duality (LPHD)

  \[
  \bar{D}^h(x, Y_\Theta, Q_0) = K^h\bar{D}^{\text{lim}}(x, Y_\Theta, Q_0)
  \]

  ⇒ two parameters \( Q_0 \) and \( K^h \).
Modeling the medium influence: a suggestion

The hump of the limiting spectrum is mostly due to the singular parts of the splitting functions.

In medium, the emission of a soft gluons by a fast parton increases.

One can model medium–induced effects by modifying the parton splitting functions $P_{ji}(z)$ and especially their singular parts:

$$P_{Gq}(z) = \frac{4}{3} \left[ \frac{2(1 + f_{\text{med}})}{z} - 2 + z \right]$$

and so on.

$f_{\text{med}} > 0 \Rightarrow$ Bremsstrahlung increases

I am messing up with the splitting functions...

usual sum rules no longer hold.
Medium-modified splitting functions vs. the “usual” approach to jet quenching

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QCD splitting/joining functions at finite temperature in the deep Landau-Pomeranchuk-Migdal regime

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There exist full leading-order-in-\( \alpha_s \) numerical calculations of the rates for massless quarks and gluons to split and join in the background of a quark-gluon plasma through hard, nearly collinear bremsstrahlung and inverse bremsstrahlung. In the limit of partons with very high energy \( E \), where the physics is dominated by the Landau-Pomeranchuk-Migdal effect, there are also analytic leading-log calculations of these rates, where the logarithm is \( \ln(E/T) \). We extend those analytic calculations to next-to-leading-log order. (…)

“My” constant \( f_{\text{med}} \) : unrealistic, but allows analytical computations, to check (future) Monte-Carlo cascades.
Medium-modified hump-backed plateau

inclusive longitudinal distribution

Partons are redistributed from large $x$ to small $x$. 

- medium-modified MLLA, $f_{\text{med}}=1$
- MLLA, $E_{\text{jet}} \sin \Theta = 100$ GeV
- OPAL, $\sqrt{s} = 192-209$ GeV
Probability $D^A_{A_0}(u, E\Theta_0, uE\Theta)$ that parton $A_0$ gives rise to $A$.

- $A$ splits into $B$ (energy fraction $z$) and $C$: splitting function $P_{BA}(z)$.
- Hadronization from $B$ into $h_1$ (energy $x_1E$) & $C$ into $h_2$ (energy $x_2E$).

- double differential $(x_1, x_2, \Theta)$ 2-particle cross-section
Integrating the double differential two-particle cross-section over one of the hadrons (weighted with its energy) yields the double differential single-particle inclusive distribution:

\[
\frac{d^2 N}{dx \, d \ln \Theta} = \frac{d}{d \ln \Theta} \left[ \sum_A \int du \, D_{A_0}^A (u, E\Theta_0, uE\Theta) \, D_{A}^h \left( \frac{x}{u}, uE\Theta, Q_0 \right) \right]
\]

\(\Theta\) angle with respect to the direction of energy flow (jet axis).

detailed computations in Pérez-Ramos & Machet, JHEP 04 (2006) 043
Integrating the double differential two-particle cross-section over one of the hadrons (weighted with its energy) yields the double differential single-particle inclusive distribution:

\[
\frac{d^2 N}{dx \, d \ln \Theta} = \frac{d}{d \ln \Theta} \left[ \sum_A \int du \, D_A^A(u, E\Theta_0, uE\Theta) \, \bar{D}^\lim \left( \frac{x}{u}, uE\Theta, Q_0 \right) \right]
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\(\Theta\) angle with respect to the direction of energy flow (jet axis).
Integrating the double differential two-particle cross-section over one of the hadrons (weighted with its energy) yields the double differential single-particle inclusive distribution:

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\frac{d^2 N}{dx \, d \ln \Theta} = \frac{d}{d \ln \Theta} \left[ \sum_A \int du \, D^A_{A_0}(u, E\Theta_0, uE\Theta) \tilde{D}^{\text{lim}} \left( \frac{x}{u}, uE\Theta, Q_0 \right) \right]
\]

\( \Theta \) angle with respect to the direction of energy flow (jet axis).

After some algebraic manipulations...

\[
\left( \frac{d^2 N}{dx \, d \ln \Theta} \right)_{G,q} = \frac{d}{d \ln \Theta} \left[ \frac{\langle C \rangle_{G,q}}{N_c} \tilde{D}^{\text{lim}} \left( \ln \frac{1}{x}, \ln \frac{E_{\text{jet}} \Theta}{Q_0}, Q_0 \right) \right]
\]

where \( \langle C \rangle_{A_0} \) is the average color current of partons.
Two further slides on MLLA...

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\[
\frac{d^2 N}{dx \, d \ln \Theta} = \frac{d}{d \ln \Theta} \left[ \sum_A \int du \, D_{A_0}^G (u, E_\Theta, uE_\Theta) \tilde{D}_{\lim} \left( \frac{x}{u}, uE_\Theta, Q_0 \right) \right]
\]

\(\Theta\) angle with respect to the direction of energy flow (jet axis).

After some algebraic manipulations...

\[
\left( \frac{d^2 N}{dx \, d \ln \Theta} \right)_{G,q} = \frac{d}{d \ln \Theta} \left[ \langle C \rangle_{G,q} \frac{\tilde{D}_{\lim} \left( \ln \frac{1}{x}, \ln \frac{E_{\text{jet}} \Theta}{Q_0}, Q_0 \right)}{N_c} \right]
\]

where \(\langle C \rangle_{A_0}\) is the average color current of partons.

Eventually

\[
\frac{dN}{d \ln \Theta} = \int dx \, \frac{d^2 N}{dx \, d \ln \Theta} \quad \text{... which I shall not do here!}
\]

detailed computations in Pérez-Ramos & Machet, JHEP 04 (2006) 043
\[ Y_{\Theta_0} \equiv \ln \frac{E_{\text{jet}} \sin \Theta_0}{Q_0 = \Lambda_{\text{QCD}}} = 7.5 \]
**Medium-modified double-differential single-particle distribution**

\[
Y_{\Theta_0} \equiv \ln \frac{E_{\text{jet}} \sin \Theta_0}{Q_0 = \Lambda_{QCD}} = 7.5
\]

\[f_{\text{med}} = 1\]
Medium-modified double-differential single-particle distribution

\[ Y_{\Theta_0} \equiv \ln \frac{E_{\text{jet}} \sin \Theta_0}{Q_0} = \Lambda_{\text{QCD}} = 7.5 \]

What's happening here?!!
Medium-modified double-differential single-particle distribution

\[ Y_{\theta_0} \equiv \ln \left( \frac{E_{\text{jet}} \sin \theta_0}{Q_0} \right) = \Lambda_{\text{QCD}} = 7.5 \]

Easy handwaving answer: should we trust results for \( k_\perp < 1 \text{ GeV/c} \)?

High-pT physics at LHC 09, Prague, February 4-7, 2009
Medium-modified double-differential single-particle distribution

\[ Y_{\Theta_0} \equiv \ln \frac{E_{\text{jet}} \sin \Theta_0}{Q_0 = \Lambda_{\text{QCD}}} = 7.5 \]

Redistribution from low to higher \( k_\perp \) ... or total nonsense.
Medium-modified double-differential single-particle distribution

Jet broadening? Guess why I was not eager to integrate \( \frac{d^2N}{dx \, d \ln \Theta} \) ...

Should I worry about negative probability distributions? (well, I do!)

- They also come up — albeit elsewhere — in the computations by Pérez-Ramos, Arleo & Machet that match the CDF data.

- They might become less severe... if I sit down, work harder and go to Next-to-MLLA.

- Better accounting of energy-momentum conservation at high \( z \);

- In the absence of a medium, this does not affect the longitudinal spectrum much, only the \( k_\perp \) distribution.

- But the problem might be much deeper...

High-pT physics at LHC 09, Prague, February 4-7, 2009
Does it all make sense?

9.2 QCD Portrait of an “Individual Jet”

Let us consider the general inclusive characteristics which may be called, in some sense, the characteristics of an isolated jet (neglecting the mutual influence of jets in their ensemble).

(...) The notion of the isolated jet makes sense, of course, if one does not deal with the azimuthal effects but considers only multiplicities, energy spectra and correlations, etc. In this case all the influence of the jet ensemble on a given jet may be encoded in a single parameter $\Theta_0$, the jet opening angle. This angle, in essence, is the angle between the considered jet and the nearest other one.

Multiplicity, energy spectra of particles and other characteristics of the QCD partonic cascade prove to depend not on the jet’s energy $E$ but on the hardness of the process producing this jet, i.e. on the largest possible transverse momentum of particles inside the jet, $Q = E\Theta_0$ at $\Theta_0 \ll 1$, which corresponds, of course, to the transverse momentum of the jet itself.