Anisotropic flow far from equilibrium: onset of collectivity

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Anisotropic flow far from equilibrium: onset of collectivity

- Do you need many collisions to build up "collective behavior"?

  flow of massless particles diffusing on fixed scattering centers

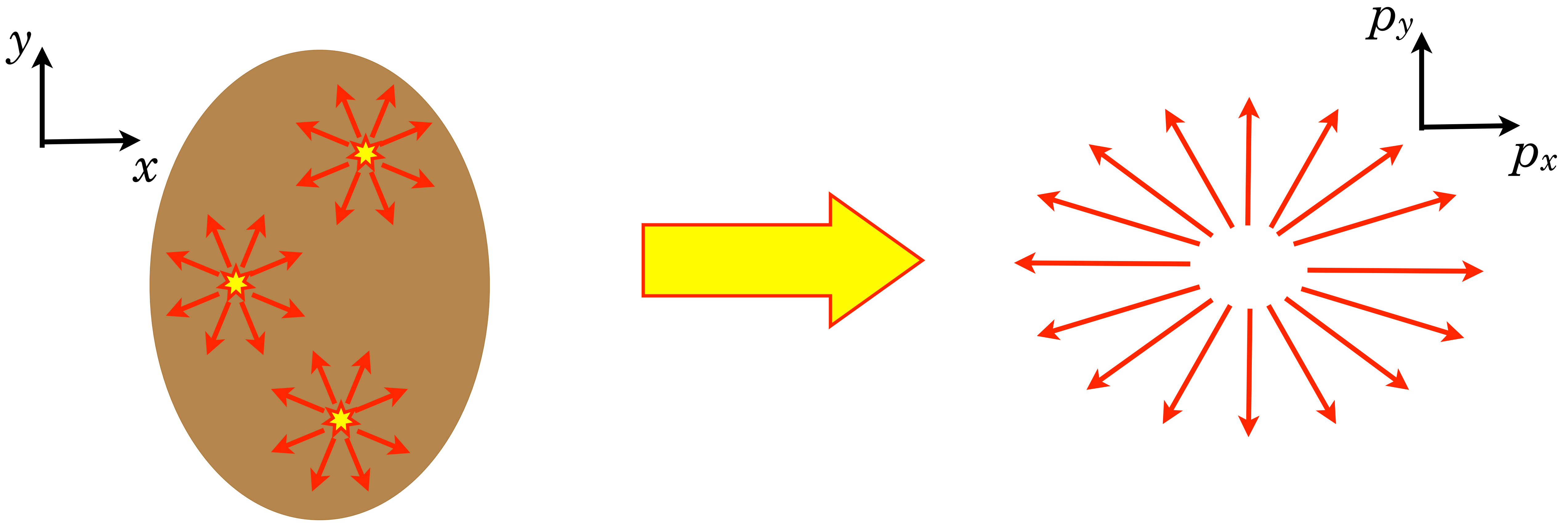
- Do you need the presence of a thermalized medium to obtain the "mass-ordering" of (elliptic) flow?

  flow of massive particles

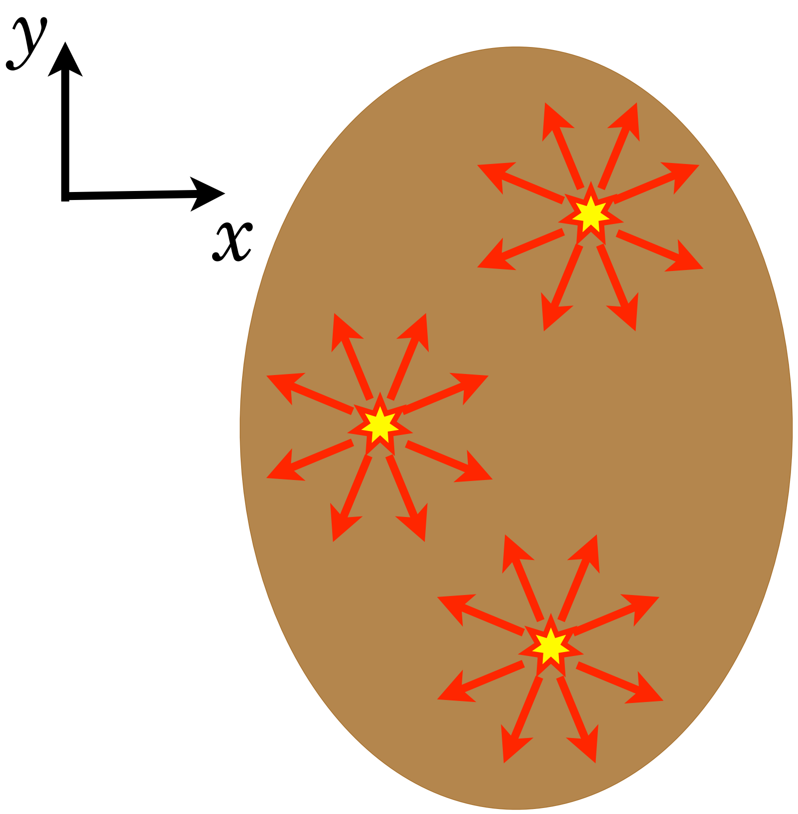
(a 25-minute summary of) N.B. & C.Gombeaud, arXiv:1012.0899

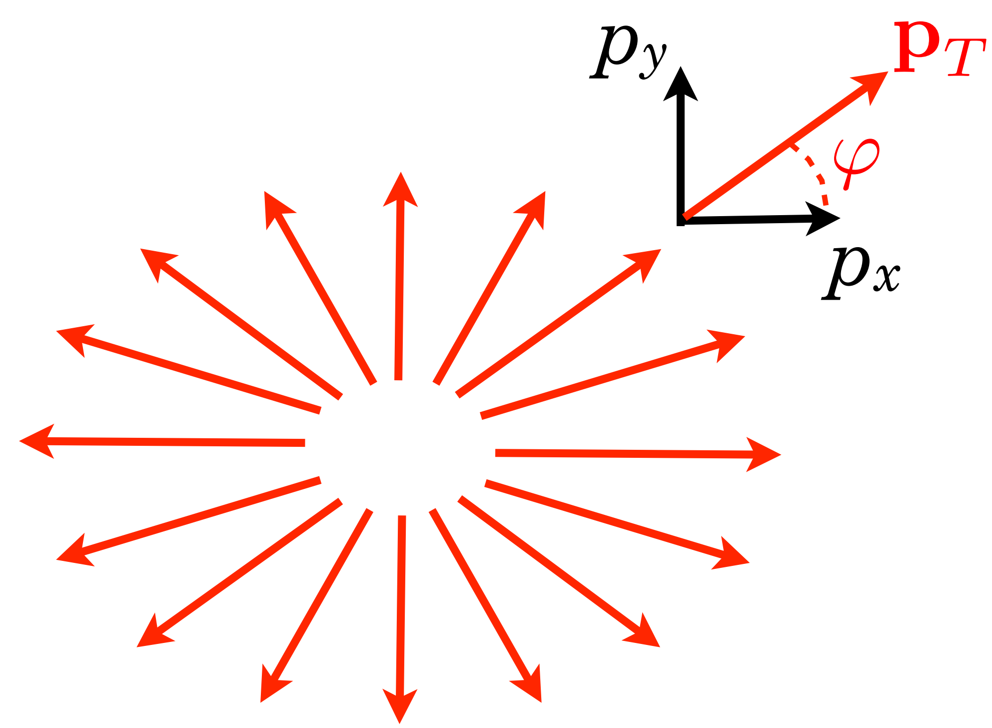
Anisotropic flow

In non-central nucleus-nucleus collisions, the initial spatial asymmetry of the overlap region in the transverse plane is converted by particle rescatterings into an anisotropic transverse-momentum distribution of the outgoing particles: anisotropic (transverse) flow.



Anisotropic flow





$$\epsilon_2 \equiv \frac{\langle y^2 - x^2 \rangle}{\langle x^2 + y^2 \rangle} \neq 0$$

$$\Rightarrow \quad v_n(p_T) \equiv \frac{\displaystyle\int d\varphi \, \frac{d^2N}{d^2\mathbf{p}_T} \cos n\varphi}{\displaystyle\int d\varphi \, \frac{d^2N}{d^2\mathbf{p}_T}} \neq 0$$

$$\frac{d^2N}{d^2\mathbf{p}_T} = \frac{1}{2\pi} \frac{dN}{p_T \, dp_T} \left[1 + \sum_{n=1}^{\infty} 2 v_n(p_T) \cos n\varphi \right]$$

Anisotropic flow far from equilibrium: onset of collectivity

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  flow of massless particles diffusing on fixed scattering centers

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  flow of massive particles

The model

- System: 2-dimensional dilute Lorentz "gas" of N_i massless particles, which scatter elastically on N_k fixed centers with an isotropic and constant differential cross-section $\sigma_{\rm d}$.
  - 2-dimensional: I'm only interested in the transverse expansion.
  - $\sigma_{\rm d}$ isotropic, constant, p_T-independent: a single parameter!
  - dilute system: kinetic description à la Boltzmann is meaningful.
  - distribution functions $f_i(t, \mathbf{x}, \mathbf{p}_i)$, $f_k(t, \mathbf{x}, \mathbf{p}_k = \mathbf{0})$.

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- Initial condition ($t = 0$): isotropic distribution in momentum space, asymmetric distribution in position space (identical for i and k).
  - in position space: Gaussian profile with mean square radii $R_x^2 < R_y^2$.

$$f(0, \mathbf{x}, \mathbf{p}_T) = \frac{N}{4\pi^2 R_x R_y} \tilde{f}(p_T) \exp\left(-\frac{x^2}{2R_x^2} - \frac{y^2}{2R_y^2} \right)$$

The model: initial condition

Remarks on the Gaussian profile

(independent of the choice of particle masses)

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Let $R_x^2 \equiv \frac{R^2}{1+\epsilon}, \quad R_y^2 \equiv \frac{R^2}{1-\epsilon}$; then $\epsilon_2(0) = \frac{\langle y^2 - x^2\rangle}{\langle x^2 + y^2\rangle} = \frac{R_y^2 - R_x^2}{R_x^2 + R_y^2} = \epsilon$!

(Note that $\epsilon_2 = -\frac{\langle r^2 \cos 2\varphi_r\rangle}{\langle r^2\rangle}$, where φ_r denotes the polar angle...)

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(Note that $\epsilon_2 = -\frac{\langle r^2\cos 2\varphi_r\rangle}{\langle r^2\rangle}$, where φ_r denotes the polar angle...)

Now, one finds $\epsilon_4 \equiv -\frac{\langle r^4\cos 4\varphi_r\rangle}{\langle r^4\rangle} = -\frac{\langle x^4-6x^2y^2+y^4\rangle}{\langle x^4+2x^2y^2+y^4\rangle} = -\frac{3\epsilon^2}{2+\epsilon^2}$,

that is ϵ_2 and ϵ_4 are of opposite signs.

The model

(independent of the choice of particle masses)

Once the distribution function $f(t, \mathbf{x}, \mathbf{p}_T)$ is known, the (transverse-) momentum distribution

$$\frac{\mathrm{d}^2 N}{\mathrm{d}^2\mathbf{p}_T}(t, \mathbf{p}_T) = \int \mathrm{d}^2\mathbf{x}\, f(t, \mathbf{x}, \mathbf{p}_T)$$

at time t follows at once.

One can thus obtain the time-dependence of the anisotropic flow coefficients $v_n(t, p_T)$.

The usual, experimentally accessible harmonic $v_n(p_T)$ is the large-time limit $v_n(t \to \infty, p_T)$.

The model: evolution equation

(independent of the choice of particle masses)

Boltzmann equation

$$\frac{\partial f_i}{\partial t} + \mathbf{v}_i \cdot \nabla_{\mathbf{x}} f_i = \int \mathrm{d}^2\mathbf{p}_k \, \mathrm{d}\Theta \, (f'_i f'_k - f_i f_k) \, v_{ik} \sigma_{\mathrm{d}}$$

- $f_i \equiv f_i(t, \mathbf{x}, \mathbf{p}_i)$, $\quad f_k \equiv f_k(t, \mathbf{x}, \mathbf{p}_k)$ distributions before the i+k → i+k collision;
- $f'_i \equiv f_i(t, \mathbf{x}, \mathbf{p}'_i)$, $\quad f'_k \equiv f_k(t, \mathbf{x}, \mathbf{p}'_k)$ distributions after the collision.
- $v_{ik} = \sqrt{(\mathbf{v}_i - \mathbf{v}_k)^2 - \frac{(\mathbf{v}_i \times \mathbf{v}_k)^2}{c^2}}$ relative velocity.
- Θ angle between $\mathbf{p}_k$ and $\mathbf{p}'_k$ (irrelevant hereafter: $\int \mathrm{d}\Theta = 2\pi$).

The model: evolution equation

(independent of the choice of particle masses)

Integrating the Boltzmann equation

$$\frac{\partial f_i}{\partial t} + \mathbf{v}_i \cdot \nabla_{\mathbf{x}} f_i = \int \mathrm{d}^2\mathbf{p}_k \, \mathrm{d}\Theta \, (f'_i f'_k - f_i f_k) \, v_{ik} \sigma_{\mathrm{d}}$$

over $\mathbf{x}$, the gradient part (odd function of $\mathbf{x}$) disappears:

$$\frac{\partial}{\partial t} \frac{\mathrm{d}^2 N_i}{\mathrm{d}^2\mathbf{p}_i} = \int \mathrm{d}^2\mathbf{x} \int \mathrm{d}^2\mathbf{p}_k \, \mathrm{d}\Theta \, (f'_i f'_k - f_i f_k) \, v_{ik} \sigma_{\mathrm{d}}$$

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$$v_n(p_T) \equiv \frac{\displaystyle\int \mathrm{d}\varphi \, \frac{\mathrm{d}^2 N}{\mathrm{d}^2\mathbf{p}_T} \cos n\varphi}{\displaystyle\int \mathrm{d}\varphi \, \frac{\mathrm{d}^2 N}{\mathrm{d}^2\mathbf{p}_T}}$$

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Multiplying with $\cos n\varphi_i$ and averaging over the azimuth φ_i yields the time derivative of the anisotropic flow coefficient $v_n(t, p_i)$.

Easy, no?

The model: first solution

(independent of the choice of particle masses)

If there are no rescattering between i and k particles: $\sigma_{\mathrm{d}} = 0$.

$$\frac{\partial f_i}{\partial t} + \mathbf{v}_i \cdot \nabla_{\mathbf{x}} f_i = 0$$

☞ free-streaming solutions:

$$f_i^{(0)}(t, \mathbf{x}, \mathbf{p}_i) = f_i^{(0)}(0, \mathbf{x} - \mathbf{v}_i t, \mathbf{p}_i)$$

If one starts with an isotropic distribution in momentum space, it remains so as the system evolves: no anisotropies develop...

$v_n(t, p_T) = 0$ at all times

Let's turn on the rescatterings...

(independent of the choice of particle masses)

... but only few of them!

New solution: $f_i(t, \mathbf{x}, \mathbf{p}_i) = f_i^{(0)}(t, \mathbf{x}, \mathbf{p}_i) + f_i^{(1)}(t, \mathbf{x}, \mathbf{p}_i) + \cdots$

with $f_i^{(1)} \ll f_i^{(0)}$, and so on.

☞ momentum anisotropies of f_i are those of $f_i^{(1)}$.

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$f_i^{(1)} \ll f_i^{(0)}$: need to ensure a small number of scatterings per particle.

Collision rate: $\dfrac{\mathrm{d}N_{\rm coll}}{\mathrm{d}t} = \int \mathrm{d}^2\mathbf{x} \int \mathrm{d}^2\mathbf{p}_i \, \mathrm{d}^2\mathbf{p}_k \, \mathrm{d}\Theta \, f_i f_k v_{ik} \sigma_{\rm d}$, which should be

integrated from $t = 0$ to ∞, with $f_i = f_i^{(0)}$, and be kept small.

Relation to anisotropic flow

(independent of the choice of particle masses)

Momentum anisotropies of f_i are those of $f_i^{(1)}$.

- the loss term of the Boltzmann equation does lead to anisotropies: the number of particles with azimuth φ_i lost in a rescattering is directly linked to the initial geometry.

Relation to anisotropic flow

(independent of the choice of particle masses)

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$$\frac{\partial v_n}{\partial t}(t,p_i) \propto -\int \mathrm{d}^2\mathbf{x}\,\mathrm{d}\varphi_i\,\mathrm{d}^2\mathbf{p}_k\,\mathrm{d}\Theta\, f_i^{(0)}(t,\mathbf{x},\mathbf{p}_i) f_k^{(0)}(t,\mathbf{x},\mathbf{p}_k)\, v_{ik}\sigma_{\mathrm{d}} \cos n\varphi_i$$

Thanks to the Gaussian spatial profile, the integral over $\mathbf{x}$ is trivial...

Lorentz gas

$$v_{ik} = \sqrt{(\mathbf{v}_i - \mathbf{v}_k)^2 - \frac{(\mathbf{v}_i \times \mathbf{v}_k)^2}{c^2}}$$

- massless diffusing particles: $|\mathbf{v}_i| = c$
- fixed scattering centers: $|\mathbf{v}_k| = 0$

☞ $v_{ik} = c$ ...much easier!

In particular, v_{ik} is independent of the particle azimuths.

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The integrals over $\mathbf{x}$, Θ, φ_k, $|\mathbf{p}_k|$ are easy or even trivial!

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The integrals over $\mathbf{x}$, Θ, φ_k, $|\mathbf{p}_k|$ are easy or even trivial!

Lorentz gas: the results

Let $\mathcal{C}(t,\mathbf{p}_i,\mathbf{p}_k) \equiv \int \mathrm{d}^2\mathbf{x}\,\mathrm{d}\Theta\, f_i^{(0)}(t,\mathbf{x},\mathbf{p}_k)\,f_k^{(0)}(t,\mathbf{x},\mathbf{p}_k)\, v_{ik}\sigma_{\mathrm{d}}$.

For the Lorentz gas,

$$\mathcal{C}(t,\mathbf{p}_i,\mathbf{p}_k) = \frac{N_i N_k \sigma_{\mathrm{d}} c\sqrt{1-\epsilon^2}}{8\pi^2 R^2}\,\tilde{f}_i(p_i)\,\tilde{f}_k(p_k)\,\exp\left[-\frac{c^2t^2}{4R^2}(1+\epsilon\cos 2\varphi_i)\right]$$

The integral over φ_k yields a factor 2π, those over $|\mathbf{p}_i|$ and $|\mathbf{p}_k|$ cancel the initial momentum distributions $\tilde{f}_i$, $\tilde{f}_k$ (which are normalized to 1).

The integral over φ_i gives a modified Bessel function I_0:

$$\frac{\mathrm{d}N_{\mathrm{coll}}}{\mathrm{d}t} = \frac{N_i N_k \sigma_{\mathrm{d}} c\sqrt{1-\epsilon^2}}{2R^2}\,\mathrm{e}^{-c^2t^2/4R^2}\, I_0\!\left(\frac{c^2t^2}{4R^2}\,\epsilon\right)$$

so that the total number of rescatterings is (K: elliptic integral)

$$N_{\mathrm{coll}} = \frac{N_i N_k \sigma_{\mathrm{d}}}{\sqrt{\pi}R}\sqrt{1-\epsilon}\,K\!\left(\sqrt{\frac{2\epsilon}{1+\epsilon}}\right)$$

Lorentz gas: number of rescatterings

The total number of rescatterings is (K: elliptic integral)

$$N_{\rm coll} = \frac{N_i N_k \sigma_{\rm d}}{\sqrt{\pi} R} \sqrt{1-\epsilon}\, K\!\left(\sqrt{\frac{2\epsilon}{1+\epsilon}}\right)$$

i.e. maximal for central collisions [$K(0) = \frac{\pi}{2}$] at a given cross-section:

the choice

$$\sigma_{\rm d}^{\rm max} = \frac{2}{N_k \sqrt{\pi}} R$$

ensures at most one rescattering per diffusing particle for all ϵ.

☞ consistency of the approach!

Lorentz gas: anisotropic flow

If we now multiply $\mathcal{C}(t, \mathbf{p}_i, \mathbf{p}_k)$ by $\cos n\varphi_i$ and then integrate over the azimuths and over $|\mathbf{p}_k|$ and divide by $N_i \tilde{f}_i$ — i.e. the denominator in the definition of the anisotropic flow coefficient — we get

(do not forget the − sign from our considering the loss term!)

$$\frac{\mathrm{d}v_n}{\mathrm{d}t} = (-1)^{\frac{n}{2}+1} \frac{N_k \sigma_{\mathrm{d}} c \sqrt{1-\epsilon^2}}{R^2} \, \mathrm{e}^{-c^2t^2/4R^2} I_{\frac{n}{2}}\!\left(\frac{c^2t^2}{4R^2}\epsilon\right)$$

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that is $$\sim (-1)^{\frac{n}{2}+1}\frac{N_k\sigma_{\mathrm{d}}c\sqrt{1-\epsilon^2}}{\left(\frac{n}{2}\right)!\,R^2}\left(\frac{ct\sqrt{\epsilon}}{4R}\right)^n \quad \text{for } t \ll \frac{2R}{c}$$

so that $v_n(t) \propto (-1)^{\frac{n}{2}+1} t^{n+1}$ at early times.

- behavior already seen in transport codes (Gombeaud & Ollitrault);
- differs from the slower rise $\propto t^n$ in fluid dynamics.

Lorentz gas: anisotropic flow

Integrating $\frac{\mathrm{d}v_n}{\mathrm{d}t}$ from $t = 0$ to ∞, one obtains v_n, e.g.

$$v_2(p_i) = \frac{N_k \sigma_\mathrm{d} \sqrt{\pi}}{8R} \sqrt{1-\epsilon^2}\, {}_2F_1\!\left(\frac{3}{4}, \frac{5}{4}; 2; \epsilon^2\right) \epsilon$$

Gauss hypergeometric function

Requiring at most one rescattering per diffusing particles, i.e. fixing σ_d to $\sigma_\mathrm{d}^\mathrm{max} = 2R/N_k\sqrt{\pi}$, gives the parameter-free results

$$v_2(p_i) = \frac{1}{4} \sqrt{1-\epsilon^2}\, {}_2F_1\!\left(\frac{3}{4}, \frac{5}{4}; 2; \epsilon^2\right) \epsilon$$

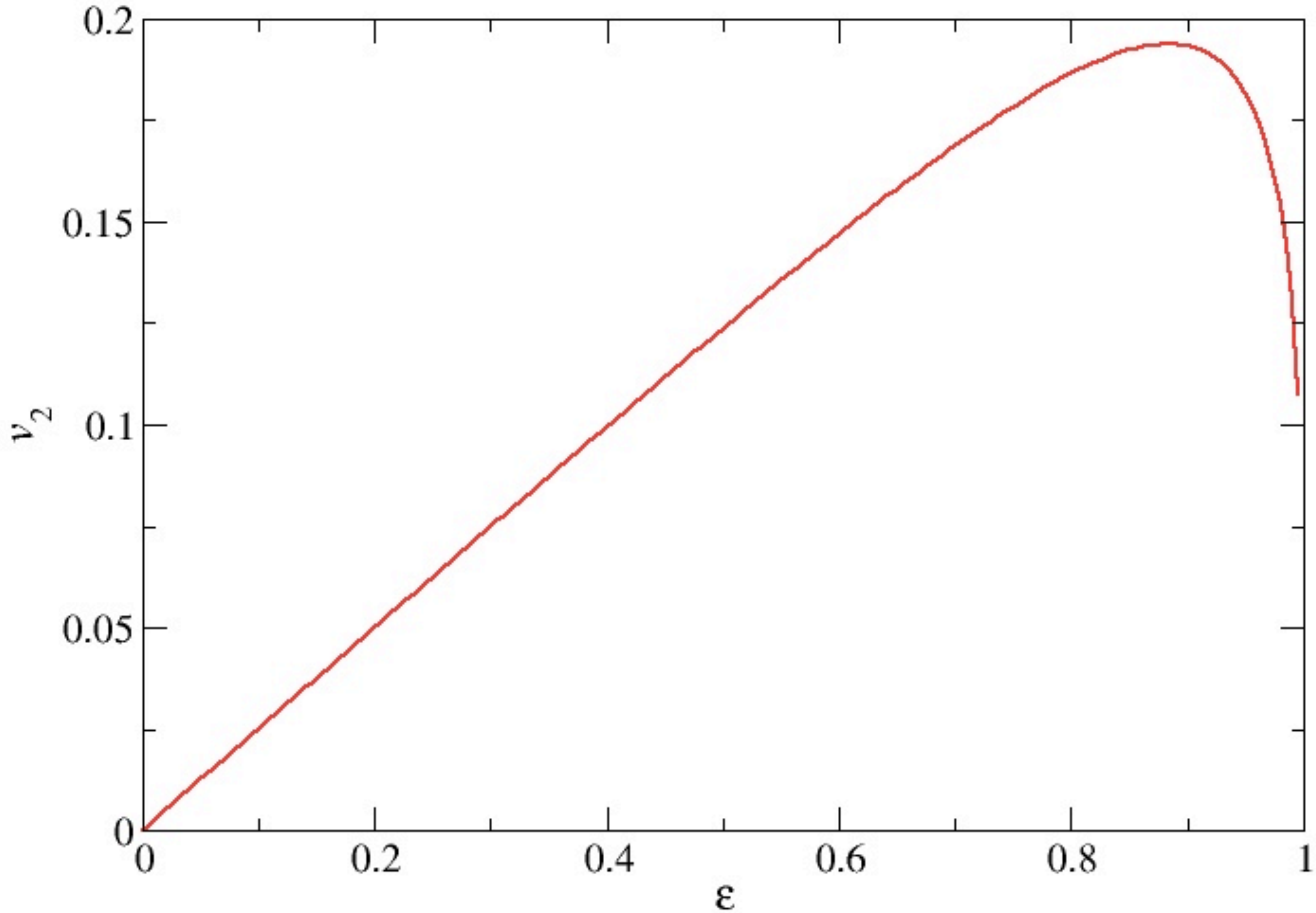
$$v_4(p_i) = -\frac{3}{32} \sqrt{1-\epsilon^2}\, {}_2F_1\!\left(\frac{5}{4}, \frac{7}{4}; 3; \epsilon^2\right) \epsilon^2$$

v_2 and v_4 are of opposite signs!

☞ reflects the opposite signs of ϵ_2 and ϵ_4: obvious (?)

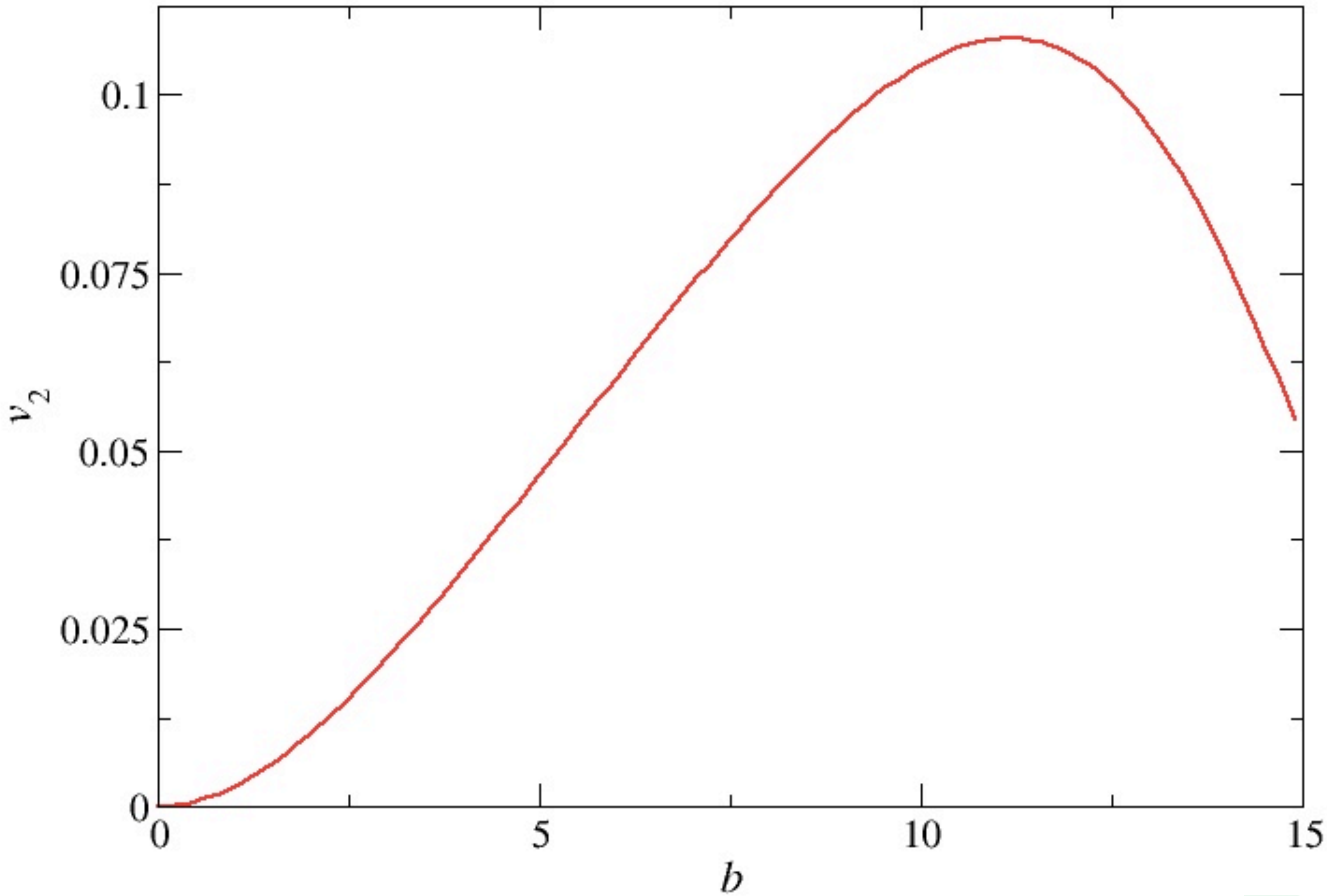
Lorentz gas: Centrality dependence of v_2

$$v_2(p_i) = \frac{1}{4}\sqrt{1-\epsilon^2}\,{}_2F_1\!\left(\frac{3}{4},\frac{5}{4};2;\epsilon^2\right)\epsilon$$

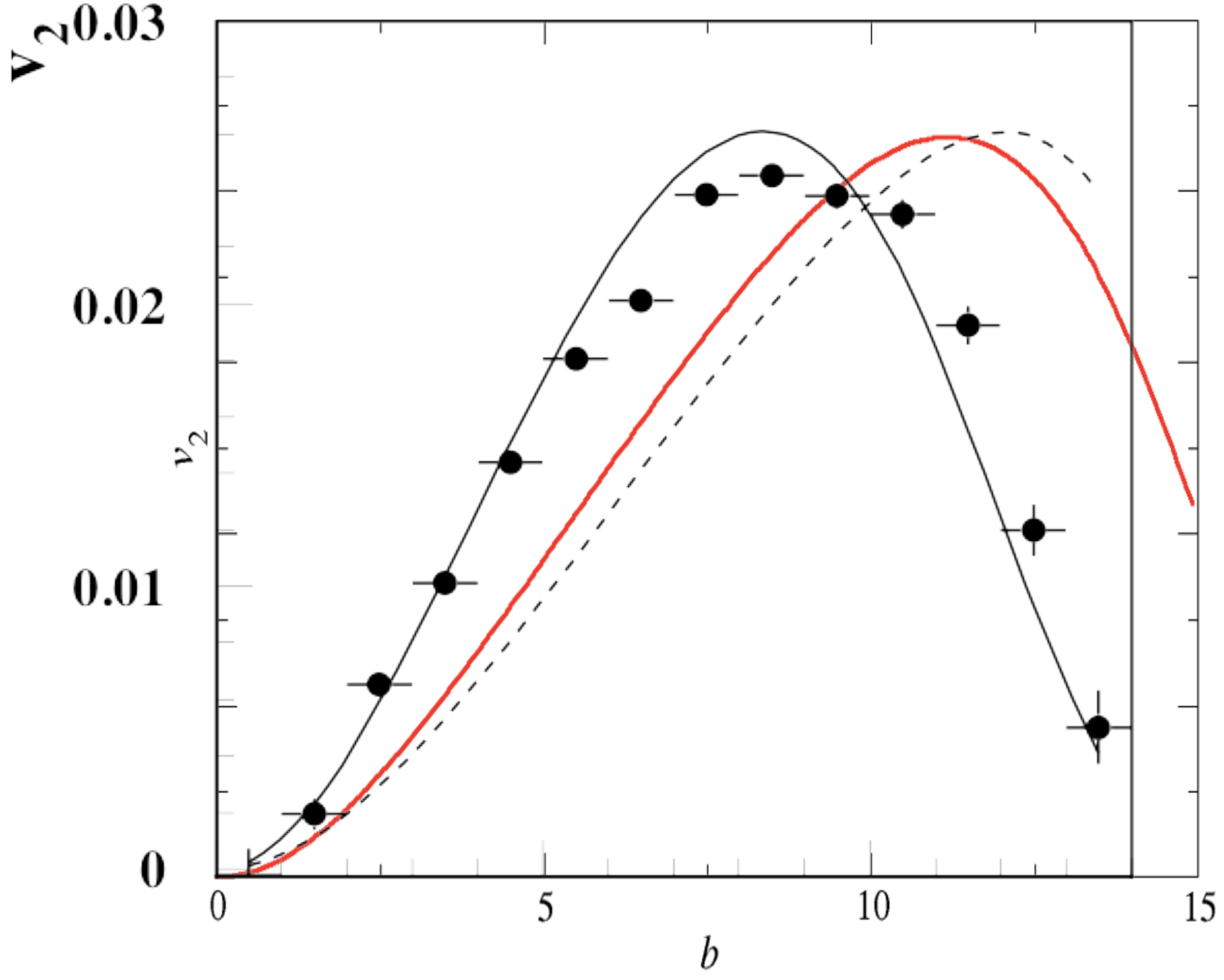


Lorentz gas: Centrality dependence of v_2

Glauber optical model to relate b and ϵ



Lorentz gas: Centrality dependence of v_2



Black curves (full: "LDL", dashed: hydro) and points (RQMD 2.3) from Voloshin & Poskanzer, Phys. Lett. B **474** (2000) 27

Anisotropic flow far from equilibrium: onset of collectivity

- Do you need many collisions to build up "collective behavior"?

flow of massless particles diffusing on fixed scattering centers

- Do you need the presence of a thermalized medium to obtain the "mass-ordering" of (elliptic) flow?

flow of massive particles

The model

- System: 2-dimensional dilute mixture of several components with different masses m_i, m_k... > 0, which scatter elastically on each other with an isotropic and constant differential cross-section $\sigma_{\rm d}$.

- Initial condition (t = 0): isotropic distribution in momentum space, asymmetric distribution in position space (identical for all species).
  - in position space: Gaussian profile with mean square radii $R_x^2 < R_y^2$.
  - in momentum space: no longer irrelevant!

- Evolution: Boltzmann equation

$$N_{\rm coll} = \int {\rm d}t\, {\rm d}^2\mathbf{x}\, {\rm d}^2\mathbf{p}_i\, {\rm d}^2\mathbf{p}_k\, {\rm d}\Theta\, f_i^{(0)}(t,\mathbf{x},\mathbf{p}_i) f_k^{(0)}(t,\mathbf{x},\mathbf{p}_k) v_{ik}\sigma_{\rm d}$$

$$v_n(p_i) \propto -\int {\rm d}t\, {\rm d}^2\mathbf{x}\, {\rm d}\varphi_i\, {\rm d}^2\mathbf{p}_k\, {\rm d}\Theta\, f_i^{(0)}(t,\mathbf{x},\mathbf{p}_i) f_k^{(0)}(t,\mathbf{x},\mathbf{p}_k)\, v_{ik}\sigma_{\rm d} \cos n\varphi_i$$

as before...

The model

Important complication: the relative velocity

$$v_{ik} = \sqrt{(\mathbf{v}_i - \mathbf{v}_k)^2 - \frac{(\mathbf{v}_i \times \mathbf{v}_k)^2}{c^2}}$$

$$= c\sqrt{\left[1-\beta_i\beta_k \cos(\varphi_i-\varphi_k)\right]^2 - (1-\beta_i^2)(1-\beta_k^2)}$$

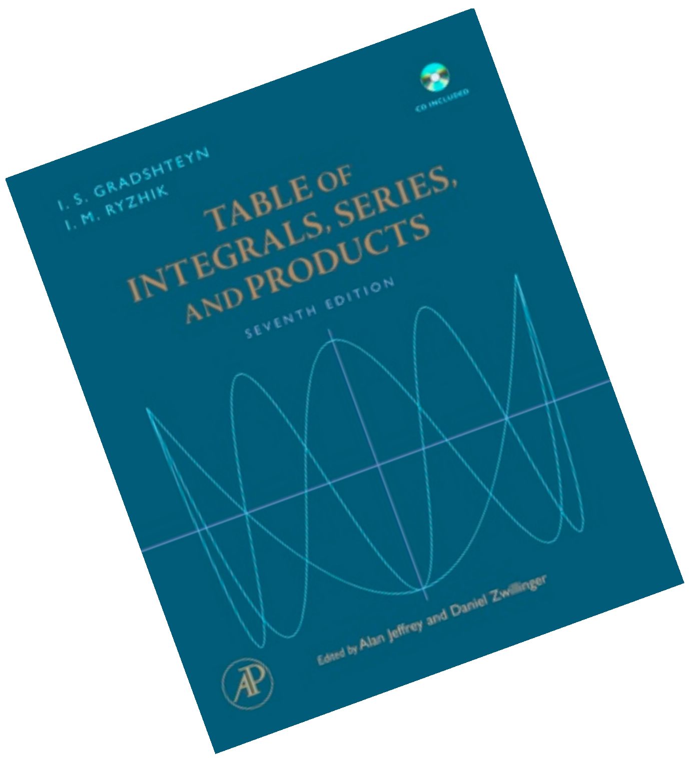
now depends on the particle azimuths...

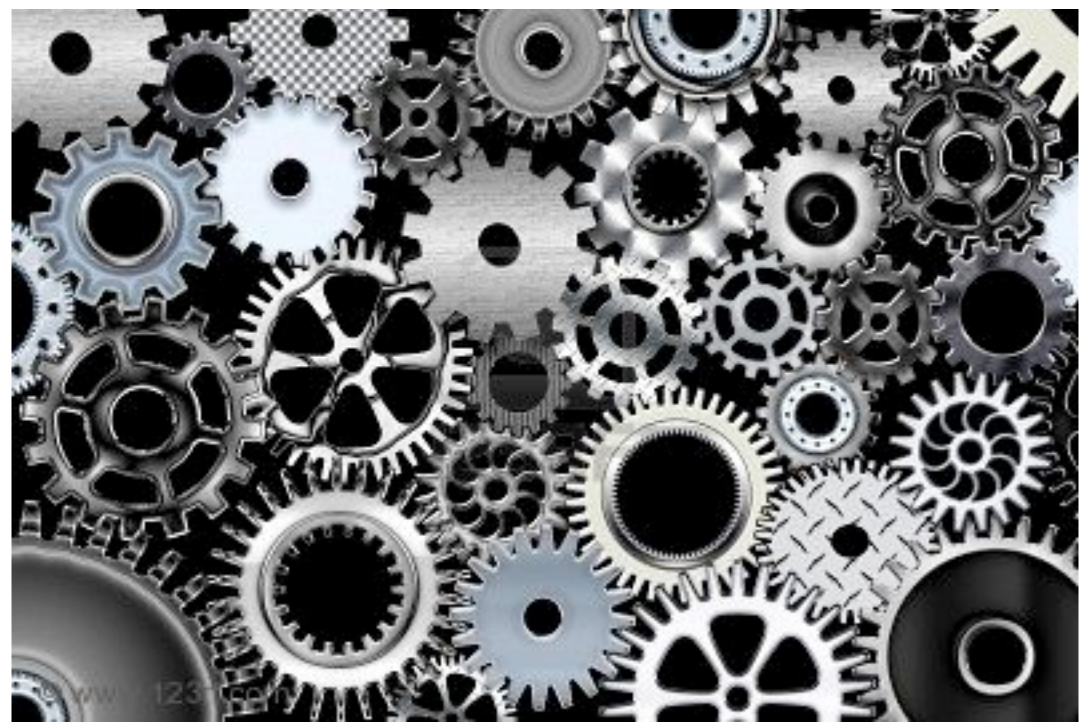
☞ Integrating over φ_i and φ_k is no longer straightforward.

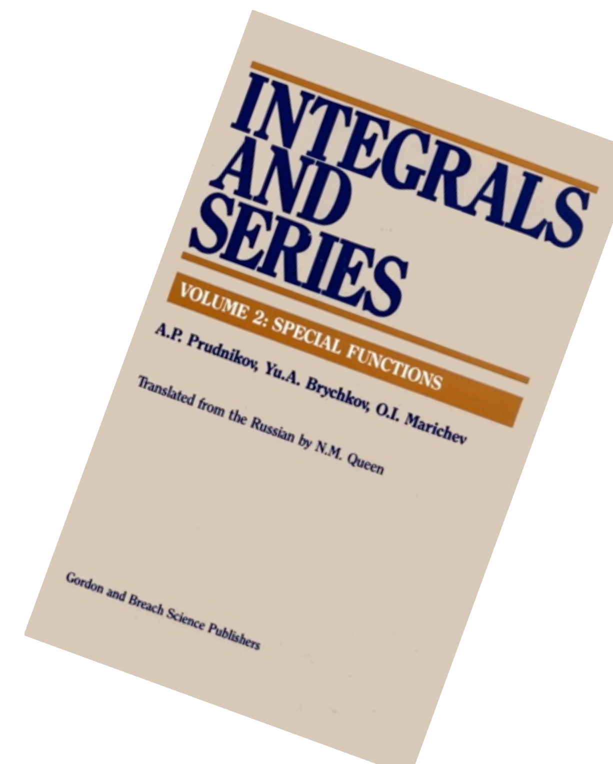
(in particular, the integral over time has to be performed "early" in the calculation: one loses the early-time dependence of $v_n(t)$.)

Mixture of massive components: anisotropic flow

$$v_n(p_i) \propto -\int \mathrm{d}t\,\mathrm{d}^2\mathbf{x}\,\mathrm{d}\varphi_i\,\mathrm{d}^2\mathbf{p}_k\,\mathrm{d}\Theta\, f_i^{(0)}(t,\mathbf{x},\mathbf{p}_i) f_k^{(0)}(t,\mathbf{x},\mathbf{p}_k)\, v_{ik}\sigma_{\mathrm{d}} \cos n\varphi_i$$







$$v_n(p_i) = \mathcal{N}_n \mathcal{K}_n(\epsilon) \int \mathrm{d}p_k\, p_k\, N_k\, \tilde{f}_k(p_k)\, \mathcal{F}_n(\beta_i, \beta_k)$$

Mixture of massive components: anisotropic flow

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- $\mathcal{K}_n(\epsilon)$: centrality dependence.
- $\tilde{f}_k(p_k)$: the momentum distribution of diffusing centers plays a role.
- $\mathcal{F}_n(\beta_i, \beta_k)$: universal function of the particle velocities.

  Boltzmann equation is kinetic: depends on velocities, not on momenta.

  ☞ $v_n(\beta_i)$ function of velocity, rather than momentum.

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  ☞ $v_n(\beta_i)$ function of velocity, rather than momentum.

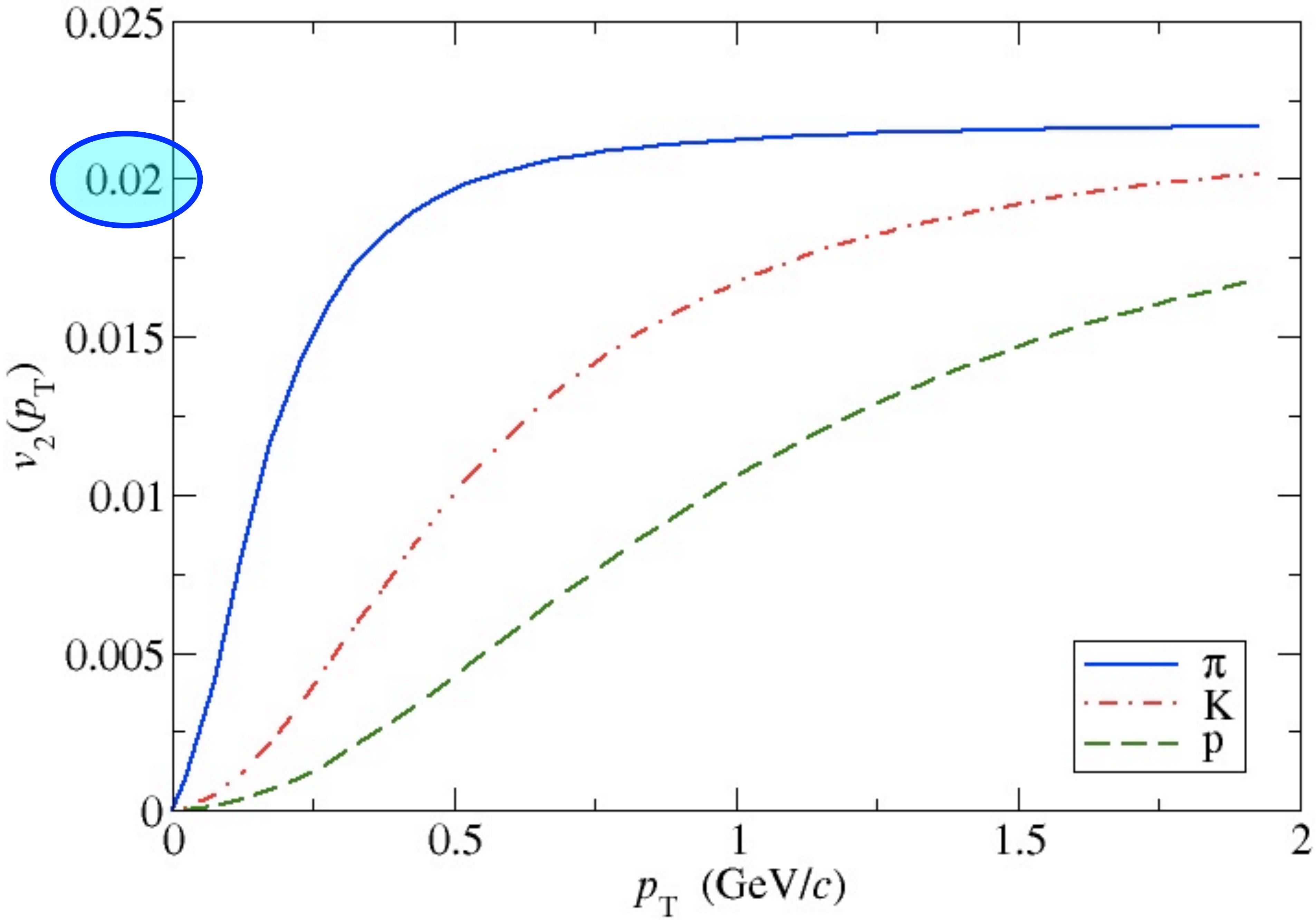
  - At a given momentum, heavier particles have smaller velocity
  - + v_2 increasing function of velocity

  ☞ $v_2(p_T)$ mass-ordered, irrespective of thermalization.

Mixture of massive components: v_2

thermal-like momentum spectrum assumed; one collision per particle

ϵ = 0.1



80% pions
12.5% kaons
7.5% protons

more plots in arXiv:1012.0899

Anisotropic flow far from equilibrium: onset of collectivity

- Do you need many collisions to build up "collective behavior"?

  flow of massless particles diffusing on fixed scattering centers

  $\frac{v_2}{\epsilon} \approx 0.2$ after a single collision per particle

- Do you need the presence of a thermalized medium to obtain the "mass-ordering" of (elliptic) flow?

  flow of massive particles

  v_n function of velocity, not momentum

Anisotropic flow far from equilibrium: phenomenological relevance?

- The model is very much simplified!
  - no longitudinal dilution;
  - universal, constant, isotropic cross-section for elastic collisions...

- Considering a single rescattering only may however be relevant for particles that are “destroyed” after a single collision:
  - high-momentum particles, which lose a sizable amount of their momentum, thus are gone from their initial $p_{\rm T}$ bin;
  - fragile states (quarkonia? ϕ-meson?).