Anisotropic flow at RHIC and prospects for LHC

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**Anisotropic flow at RHIC and prospects for LHC**

- **Anisotropic flow at RHIC**
  - An experimental success story
    - A wealth of data
    - As of end of November 2005, 13 PRL & 4 PRC
  - Theoretical challenges
    - Conflicting interpretations of the data: hydrodynamic expansion vs. out-of-equilibrium scenario

- **Anisotropic flow at LHC**
  - Very few theoretical predictions...
  - ... yet measuring flow with ALICE should be “easy”
Anisotropic flow in heavy-ion collisions

Non-central collision:

- Initial anisotropy of the source (in the transverse plane)
- \( \Rightarrow \) anisotropic pressure gradients, larger along the impact parameter \( \vec{b} \)
- \( \Rightarrow \) anisotropic emission of particles:
  - anisotropic (collective) flow

\[
E \frac{dN}{d^3p} \propto \frac{dN}{p_t \, dp_t \, dy} \left[ 1 + 2v_1 \cos(\phi - \Phi_R) + 2v_2 \cos 2(\phi - \Phi_R) + \ldots \right]
\]

“Flow”: misleading terminology; does NOT imply fluid dynamics!

N. Borghini – p.3/25
Anisotropic flow at RHIC: a short review [0/6]

RHIC experiments* have measured $v_1$, $v_2$, $v_4$, $v_6$

- for identified particles;
- as a function of the centrality of the collision;
- as a function of the particle transverse momentum;
- as a function of the particle (pseudo)rapidity;
- at 4 different center-of-mass energies;
- with different colliding nuclei.

and the first results came out quickly (Sep.2000).

*even those not designed to measure anisotropic flow
Anisotropic flow at RHIC: a short review [1/6]

RHIC experiments have measured the elliptic flow $v_2$ of photons, $e^{\pm}$, $\pi^{\pm}$, $\pi^0$, $K^{\pm}$, $K^0_S$, $p$, $\bar{p}$, $\phi$, $\Lambda + \bar{\Lambda}$, $\Xi^- + \bar{\Xi}^+$, $\Omega^- + \bar{\Omega}^+$ and deuterons.

“mass-ordering” of the $v_2(p_T)$
Flow has been measured over a wide range in (pseudo)rapidity

![Graph showing flow measurements at RHIC](chart.png)
Anisotropic flow at RHIC
a short review [3/6]

![Graph showing anisotropic flow at different energies](image)
Anisotropic flow at RHIC: a short review [4/6]

The measurements extend to high values of transverse momentum, and have been performed using different methods of analysis.
Anisotropic flow at RHIC: a short review [5/6]

\[ v_2(n) \]

\[ v_2(2) \quad v_2(4) \quad v_2(6) \]

% Most Central

\(0\) \(10\) \(20\) \(30\) \(40\) \(50\) \(60\) \(70\) \(80\)

\(0\) \(1\) \(2\) \(3\) \(4\) \(5\) \(6\) \(7\) \(8\)

N. Borghini – p.9/25
First measurement of $v_4$ + upper bounds on $v_6$ and $v_8$
Anisotropic flow at RHIC: phenomenology

What do we learn from the measurements of anisotropic flow at RHIC?

- "Low-$p_T$" region: more in the next slides!
- $2 \text{ GeV} \lesssim p_T \lesssim 5 \text{ GeV}$: coalescence picture

$$v_2(p_T) \approx n_q v_2 \left( \frac{p_T}{n_q} \right)$$

assuming hadrons are made of constituent quarks only.

- "High-$p_T$": anisotropic flow from jet-quenching

The amount of energy/momentum lost by a high-$p_T$ parton depends on the length of its in-medium path, hence on $\phi - \Phi_R$

$\Rightarrow$ at a given momentum, less depletion in-plane than out-of-plane

$$v_2(p_T) > 0$$

(See also the "jets in the wind" of Carlos & Urs, hep-ph/0411341)
Anisotropic flow at RHIC: the fashionable view

Ideal fluid dynamics reproduce both $p_t$ spectra and elliptic flow $v_2(p_t)$ of soft ($p_t \lesssim 2$ GeV/c) identified particles for minimum bias collisions, near central rapidity.
This agreement necessitates a soft equation of state, and very short thermalization times: $\tau_{\text{thermalization}} < 0.6$ fm/c.

$\Rightarrow$ strongly interacting Quark-Gluon Plasma
Fluid dynamics: various types of flow

- **Thermodynamic equilibrium?**
  - $Kn \gg 1$: Free-streaming limit
  - $Kn \ll 1$: Liquid (hydro) limit

- **Viscous or Ideal?**
  - $Re \gg 1$: Ideal (non-viscous) flow
  - $Re \leq 1$: Viscous flow

- **Compressible or Incompressible?**
  - $Ma \ll 1$: Incompressible flow
  - $Ma > 1$: Compressible (supersonic) flow

Knudsen number $Kn = \frac{\lambda}{L}$

Reynolds number $Re = \frac{\varepsilon Lv_{fluid}}{\eta}$

Mach number $Ma = \frac{v_{fluid}}{c_s}$
Fluid dynamics: various types of flow

Three numbers:

\[ Kn = \frac{\lambda}{L}, \quad Re = \frac{\varepsilon L v_{\text{fluid}}}{\eta}, \quad Ma = \frac{v_{\text{fluid}}}{c_s} \]

⇒ an important relation:

\[ Kn \times Re = \frac{\varepsilon \lambda v_{\text{fluid}}}{\eta} \sim \frac{v_{\text{fluid}}}{c_s} = Ma \]

Compressible fluid: “Liquids are Ideal”

Viscosity \equiv \text{departure from equilibrium}
Anisotropic flow: predictions of hydro

- Characteristic build-up time of $v_2$ is $\bar{R}/c_s$
  - typical system size
  - speed of sound

- $v_2/\epsilon$ constant across different centralities
  - system eccentricity

- $v_2$ roughly independent of the system size (Au–Au vs. Cu–Cu)

- $v_2$ increases with increasing speed of sound $c_s$

- Mass-ordering of the $v_2(p_T)$ of different particles
  - (the heavier the particle, the smaller its $v_2$ at a given momentum)

- Relationship between different harmonics: $\frac{v_4}{(v_2)^2} = \frac{1}{2}$
Dependence of $v_2$ on centrality

The natural time scale for $v_2$ is $\bar{R}/c_s$:

$$v_2 \text{ scaled by initial eccentricity} = \frac{\langle y^2 - x^2 \rangle}{\langle x^2 + y^2 \rangle}$$

Impact parameter dependence

$$c_s^2 = \frac{1}{3}$$
Dependence of $v_2$ on centrality

The natural time scale for $v_2$ is $\bar{R}/c_s$:

$$\frac{v_2}{\epsilon}$$

massless particles

$$c_s^2 = \frac{1}{3}$$

$v_2$ scaled by initial eccentricity

$$\frac{\langle y^2 - x^2 \rangle}{\langle x^2 + y^2 \rangle}$$

Impact parameter dependence

$\bar{b}=4$

$\bar{b}=2$
Dependence of $v_2$ on centrality

The natural time scale for $v_2$ is $\bar{R}/c_s$:

$$c_s^2 = \frac{1}{3}$$

Massless particles

Impact parameter dependence

$v_2$ scaled by initial eccentricity

$$\frac{v_2}{\epsilon}$$

$$\langle y^2 - x^2 \rangle$$

$$\langle x^2 + y^2 \rangle$$

$\epsilon$

$R$

$c_s t$

$0.5$

$1$

$1.5$

$2$

$2.5$

$3$

$0.1$

$0.2$

$0.3$

$0.4$

$0.5$

$0.6$

$0.7$

$0.8$

$b=6$

$b=4$

$b=2$
The natural time scale for $v_2$ is $\bar{R}/c_s$: massless particles

$$c_s^2 = \frac{1}{3}$$

$v_2$ scaled by initial eccentricity $\langle y^2 - x^2 \rangle / \langle x^2 + y^2 \rangle$.
Dependence of $v_2$ on centrality

The natural time scale for $v_2$ is $\bar{R}/c_s$:

$$v_2 \text{ scaled by initial eccentricity } \frac{\langle y^2 - x^2 \rangle}{\langle x^2 + y^2 \rangle}$$

Impact parameter dependence

$$c_s^2 = \frac{1}{3}$$

massless particles

$N. \text{ Borghini}$ – p.16/25
The natural time scale for $v_2$ is $\bar{R}/c_s$:

$$v_2 \text{ scaled by initial eccentricity} \quad \frac{\langle y^2 - x^2 \rangle}{\langle x^2 + y^2 \rangle}$$

$$\frac{v_2}{\epsilon}$$

Impact parameter dependence

- $b=12$
- $b=10$
- $b=8$
- $b=6$
- $b=4$
- $b=2$

massless particles

$$c_s^2 = \frac{1}{3}$$
Anisotropic flow: out-of-equilibrium scenario

An exact computation of the dependence of $v_2$, $v_4$ on the number of collisions per particle $Kn$ requires some cascade model...

...but we can guess the general tendency!

- in the absence of reinteractions ($Kn^{-1} = 0$), no flow develops
- the more collisions, the larger the anisotropic flow
- for a given number of collisions, the system thermalizes: further collisions no longer increase $v_2$
Anisotropic flow: out-of-equilibrium scenario

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...but we can guess the general tendency!

- In the absence of reinteractions ($Kn^{-1} = 0$), no flow develops
- The more collisions, the larger the anisotropic flow
- For a given number of collisions, the system thermalizes: further collisions no longer increase $v_2$ should be quantified!

![Graph showing $v_2$ vs. $Kn^{-1}$]

Fully thermalized (hydro)

Incomplete thermalization

$K n^{-1}$

$1$
Anisotropic flow: out-of-equilibrium scenario

\[ v_n \propto K n^{-1} \]

\[ \frac{v_4}{(v_2)^2} \propto \frac{1}{Kn^{-1}} \]

In the out-of-equilibrium scenario,

\[ \frac{v_4}{(v_2)^2} > \frac{1}{2} \]

STAR (PRC 72 (2005) 014904) & PHENIX (QM’05) find

\[ \frac{v_4}{(v_2)^2} \approx 1–1.5 \]
Out-of-equilibrium scenario: a control parameter

The natural time (resp. length) scale for $v_2$ is $\bar{R}/c_s$ (resp. $\bar{R}$)

$\Rightarrow$ number of collisions per particle to build up $v_2$:

$$Kn^{-1} \simeq \frac{\bar{R}}{\lambda} = \bar{R} \sigma n \left( \frac{\bar{R}}{c_s} \right) \simeq \frac{c_s}{c} \frac{\sigma}{S} \frac{dN}{dy}$$

$\sigma$ interaction cross section, $n(\tau)$ particle density, $S$ transverse surface

In the out-of-equilibrium scenario, $v_2$ depends on

- the system size $\bar{R}$
- breakdown of the scale-invariance of hydrodynamics
- the control parameter $\frac{1}{S} \frac{dN}{dy}$

Incomplete equilibration & RHIC data [1]

Centrality and beam-energy dependence:

Incomplete equilibration & RHIC data [1]

Centrality and beam-energy dependence:

![Graph showing $v_2/\epsilon$ versus $(1/S) dN_{ch}/dy$ for different RHIC and SPS data sets.]

Scaling law seems to work for RHIC data (+ matching with SPS)

$v_2(Kn^{-1})$ increases steadily (no hint at hydro saturation in the data)

Incomplete equilibration & RHIC data [2]

(Pseudo)rapidity dependence of $v_2$

Steve Manly (PHOBOS Coll.)
QM’05

$\rho_\eta$ and $dN/dy$ approximately proportional

$\Leftrightarrow v_2 \propto K n^{-1}$

Anisotropic flow at RHIC

Conflicting interpretations:

- **Perfect liquid**: \( \lambda \ll \bar{R} \)
  
  see e.g. T. Hirano, U. Heinz, D. Kharzeev, R. Lacey, Y. Nara, nucl-th/0511046

- **Out-of-equilibrium scenario**: \( \lambda \sim \bar{R} \)
  
  - rapidity dependence \( v_2(y) \)
  
  - dependence with the mean number of collisions \( v_2(Kn^{-1}) \)

\[
\frac{v_4}{(v_2)^2} > \frac{1}{2}
\]

Anisotropic flow at LHC: theoretical predictions

Very few predictions...

- D. Teaney, E.V. Shuryak, PRL 83 (1999) 4951: $v_2$ increases from SPS to LHC
- P.F. Kolb, J. Sollfrank, U. Heinz:
  - PLB 459 (1999) 667: $v_2, v_4$ constant from RHIC to LHC ($v_2$ smaller than at SPS);
  - PRC 62 (2000) 054909: $v_2$ increases from RHIC to LHC.
  - $v_2$ larger than at RHIC;
  - $\frac{v_4}{(v_2)^2}$ smaller than at RHIC, closer to $\frac{1}{2}$
- T. Hirano, preliminary results presented in Vienna, August 2005
Anisotropic flow at LHC: theoretical predictions

...stolen from T. Hirano’s extra slides in his Vienna talk:

CGC initial conditions + Hydro + Cascade

No jet component included;
systematic error from Cooper–Frye formula not estimated
Anisotropic flow at LHC: some high-odds bets

- $v_2$ larger than at RHIC
- hydro satisfactory?
- $v_2(p_T) \rightarrow 0$ at large $p_T$?
- $v_4$ sizeable $\frac{v_4(p_T)}{(v_2(p_T))^2}$?
- $v_1$ “smaller” than at RHIC
Anisotropic flow at LHC: some high-odds bets

- $v_2$ larger than at RHIC
- hydro satisfactory?
- $v_2(p_T) \to 0$ at large $p_T$?
- $v_4$ sizeable \[ \frac{v_4(p_T)}{(v_2(p_T))^2} \] ?
- $v_1$ “smaller” than at RHIC

- Multiplicity and $v_2$ larger than at RHIC
- ALICE will measure $v_2$ on day 1! (or not long after) using improved methods
- ALICE will be able to measure $v_4$, $v_1$ (and $v_6$, $v_8 \simeq 0$)
- Precise measurements of $v_2$, $v_4$ will be needed
- flow is a background for other studies (jets...)

ALICE Physics Week, Erice, December 9, 2005