Stochastic baryon charge transport in relativistic hydrodynamics

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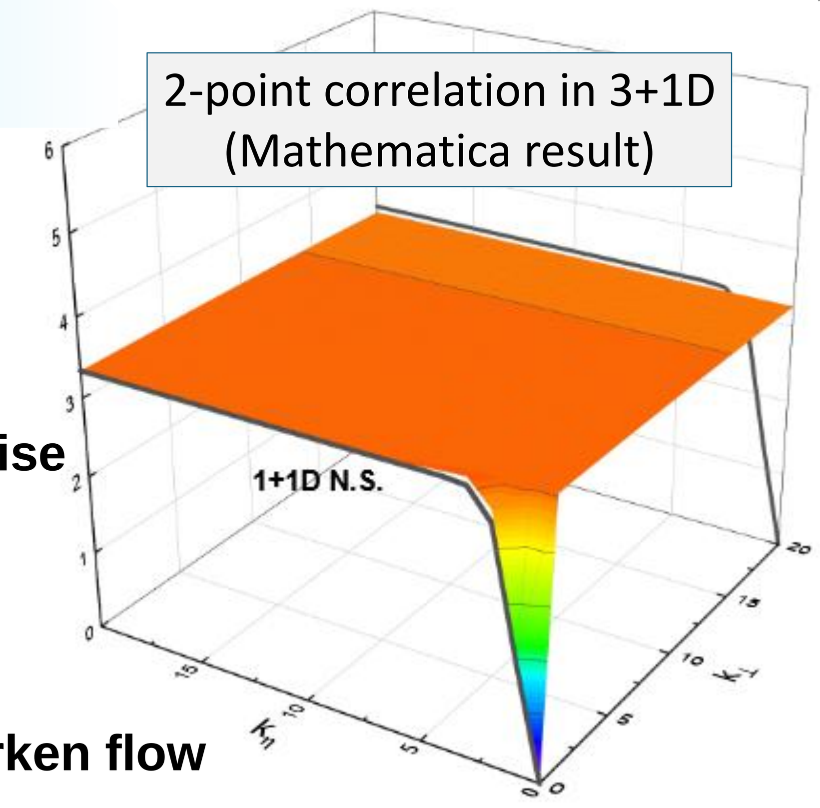
1. Introduction

- The conserved charge cumulant measurements in LHC show remarkable agreement with LQCD calculations (eq. & static). Can we understand it from dynamical models?
- For stochastic hydrodynamics, how can we improve the efficiency while keeping sufficient precision?

In this work

- **Linearized** stochastic equations for baryon transport.
- **Analytical solutions** in 1+1D (semi-analytical in 3+1D).
- **3+1D Numerical simulation**, reducing ~ 98 % computation time compared with traditional approach.

2. Analytical Solution



- **Conservation Equations**

$$\partial_\mu T^{\mu\nu} = 0 \quad \text{(energy-momentum)}$$
$$\partial_\mu N^{\mu} = 0 \quad \text{(baryon current)}$$

- **2nd order Baryon diffusion with noise**

$$\Delta^{\mu\nu} D q_\nu = -\frac{1}{\tau_q}\left(q^\mu - q^\mu_{NS} - \xi^\mu\right)$$

ξ^μ: stochastic noise

- **Linearized 1+1D solution with Bjorken flow**

$$\tilde{G}(k_\eta; s, s_0) = -2\sqrt{s s_0}\, e^{s_0 - s}\left[\tilde{I}_\nu(s_0)\tilde{K}_\nu(s) - \tilde{I}_\nu(s)\tilde{K}_\nu(s_0)\right]$$

$s = \tau/\tau_q$ (scaled proper time), $\tilde{I}_\nu(s), \tilde{K}_\nu(s)$: modified Bessel functions

- **3+1D solution available from Mathematica**

3. Model

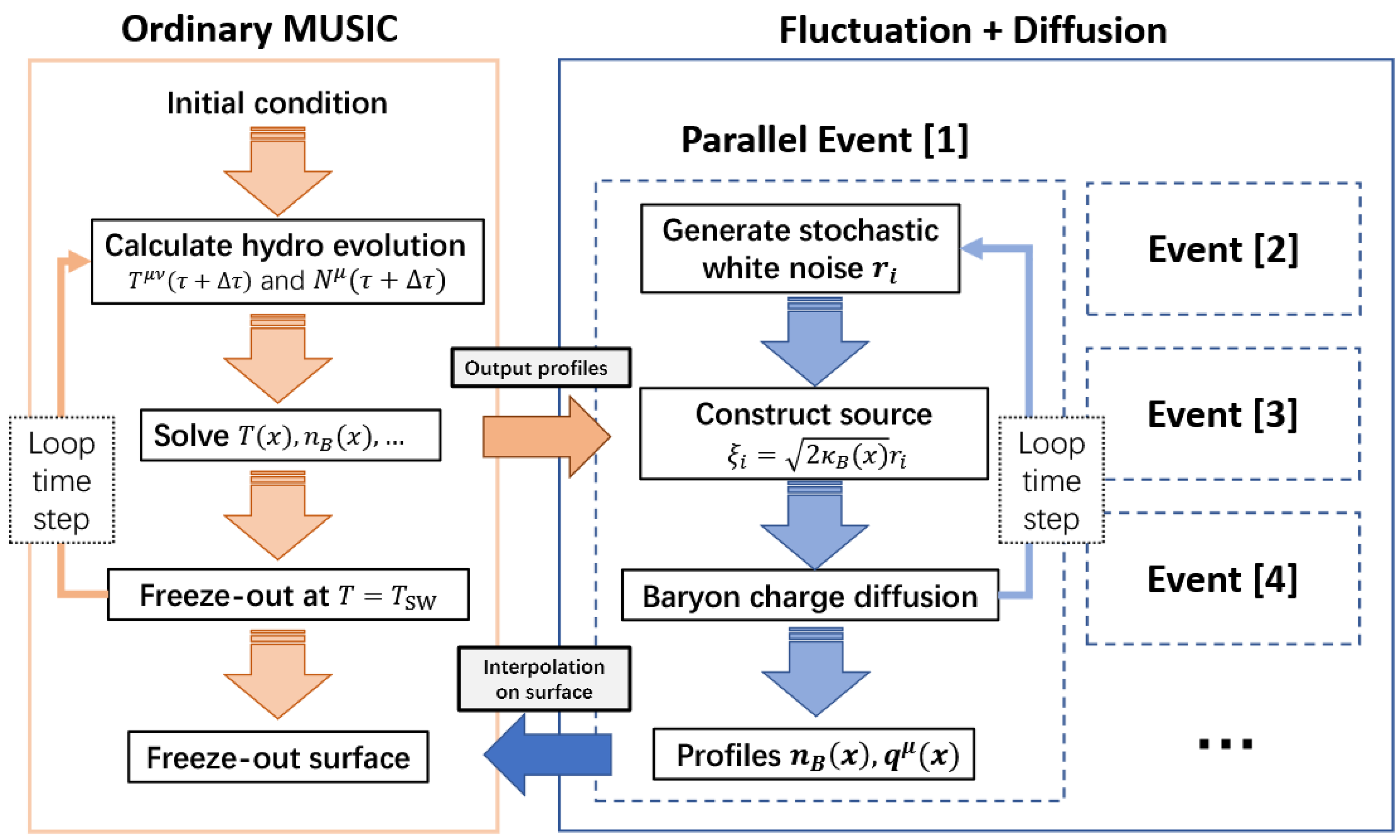
- Our model is built by two separated parts:

Ordinary MUSIC + **Stochastic Transport**

- **Ordinary MUSIC**:
  $T^{\mu\nu}$ evolution (w/o fluctuation)
- **Baryon Transport (in parallel)**:
  N^μ evolution with stochastic noise source terms
  - Stochastic white noise from FDR (κ_B from background)

$$\langle \xi^\mu(x) \rangle = 0$$
$$\langle \xi^\mu(x)\xi^\nu(x') \rangle = 2\kappa_B \Delta^{\mu\nu}\delta^{(4)}(x - x')$$

  - High frequency noise cut-off to avoid non-linear effects
  - Mapping the fluctuated $n_B(x)$ and $q_B^\mu(x)$ onto frz-surface
  - EoS: NEOS-B and ideal gas with linearized χ_B.

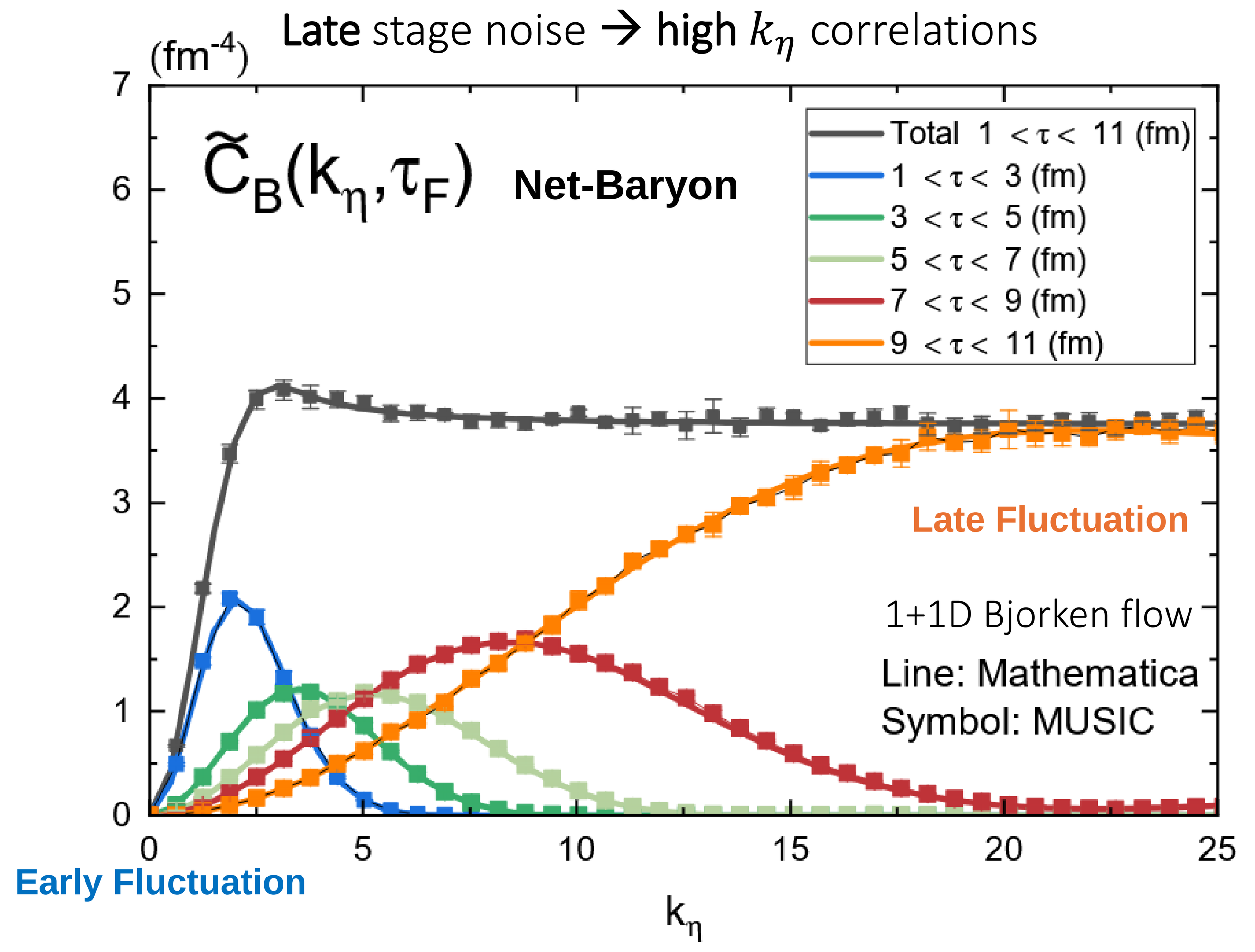


4. Results

1) 2-point correlation in Fourier space

- Numerical simulation matches the analytical solution.
- Reveals relation of noise creation time and correlation scale:

Early stage noise → low k_η correlations
Late stage noise → high k_η correlations

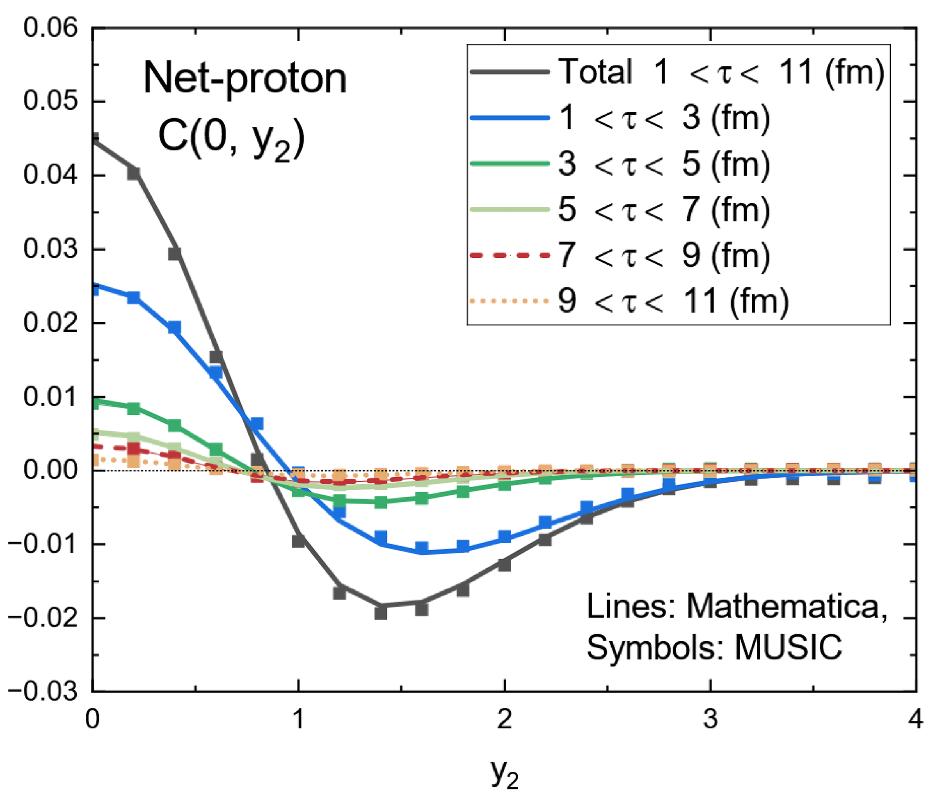


2) 2-particle correlation after freeze-out

- Cooper-Frye freeze-out for 1+1D numerical/analytical calculation

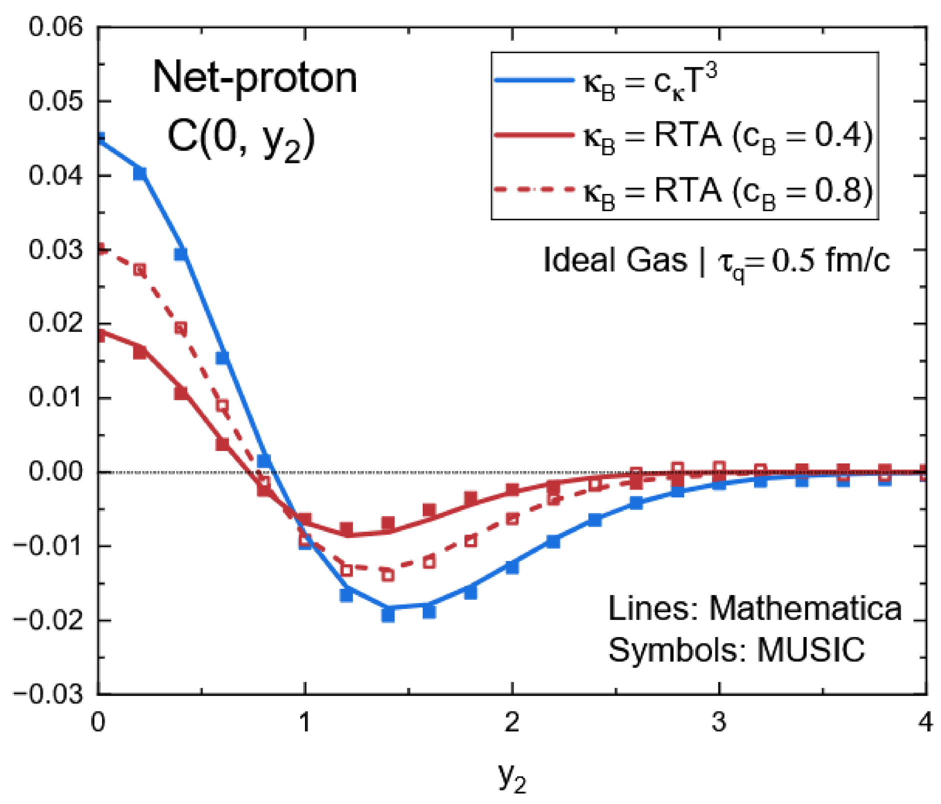
$$E\frac{dN_i}{d^3p} = \frac{g_i}{(2\pi)^3}\int_\Sigma p^\mu d\sigma_\mu f_i(x,p)$$

- The ordering (in left figure) is smeared due to the averaged freeze-out description



3) Sensitivities on EoS, τ_q and κ_B

- High k_η 2-point correlations are sensitive to EoS and relaxation time τ_q, which are smeared after Cooper-Frye freeze-out (not shown).
- Final particle correlation is sensitive to the diffusion constant $\boldsymbol{\kappa_B}$



5. Summary & Outlook

- We developed 3+1D linearized model "MUSIC + Stochastic Transport" to study the dynamical fluctuation and correlation of baryon charge.
- The charge correlation range depends on noise creation time.
- Two-particle correlation is sensitive to the diffusion constant κ_B.
- Future: 1) application in realistic 3+1D HIC, 2) diffusion with mixed charges

Reference:
[1] J. Kapusta, B. Muller, and M. Stephanov, Phys Rev C 85, 054906 (2012)
[2] K. Murase, Annals Phys, 411, 167969 (2019)