9.3 solving the horizon problem

After inflation the Universe must enter the radiation dominated (RD) epoch. This process is called <u>reheating</u>. The temperature reached at the beginning of RD is called reheating temperature T_{reh}. Since BBN happened during RD, we must have $T_{reh.} > T_{BBN} \sim 1\,MeV$

Let t_e be the time at the end of inflation.

Present size of the region that was causally connected at time t_e:

$$d_{pe}(t_0) = a_0 \int_{t_{pe}}^{t_e} \frac{dt}{a(t)} = a_0 \int_{a_{pe}}^{a_e} \frac{da}{H a^2}$$

$$H = \frac{1}{a}\frac{da}{dt}$$

assuming that dominant contribution comes from lower end of integration region

$$d_{pe}(t_0) \simeq \frac{a_0}{a_{pe} H_{pe}}$$

$$\frac{d_{pe}(t_0)}{d_p(t_0)} \simeq \frac{a_0 H_0}{a_{pe} H_{pe}} = \frac{a_e H_e}{a_{pe} H_{pe}} \frac{a_0 H_0}{a_e H_e}$$

The horizon problem is solved if this is greater than 1 $\Longleftrightarrow$

$$(*) \quad \frac{a_e H_e}{a_{pe} H_{pe}} > \frac{a_e H_e}{a_0 H_0} \sim \frac{T_0}{T_{reh}} \frac{H_e}{H_0}$$

Write the increase of the scale factor during inflation as

$$\frac{a_e}{a_{pe}} = \exp(N)$$

N : number of e-foldings

Taking the log of $(*)$ gives

$$N > \log\left(\frac{T_0 H_{pe}}{T_{reh} H_0}\right) \simeq \log \frac{T_0}{H_0} + \log \frac{m_{pe}}{T_{reh}}$$

$$T_0 = 2.7\ K = 2.7 \times 8.6 \times 10^{-5}\ eV = 2.3 \times 10^{-4}\ eV$$

$$H_0 = h_{75}\ 0.75 \times [9.8 \times 10^9\ \pi\ 10^7 s]^{-1}$$

$$= h_{75} \times 2.4 \times 10^{-18} \underbrace{s^{-1}}_{= 6.6 \times 10^{-22} \times 10^6 eV}$$

$$= 1.6 \times 10^{-33}\ eV$$

$$\log \frac{T_0}{H_0} = 67$$

So for successful inflation one needs

$N > N_{min}$ with

$$N_{min} = 67 + \log \frac{m_{pe}}{T_{reh}}$$

$70 < N_{min} < 100$ for $1\ TeV < T_{reh} < m_{pe}$

9.4 Slow-roll inflation

If ρ is dominated by a cosmological constant, then ρ is constant and thus $H = \frac{8\pi}{3} G \rho$ is constant.

expansion: $\dot{a} = a H$ $\Rightarrow$ for constant $H > 0$ exponential expansion $a(t) = a_i \exp(Ht)$.

One obtains inflation if there is something like a cosmological constant that goes to zero at the end of inflation.

Assume that there is a scalar field $\varphi(t, \vec{x})$ with potential $V(\varphi)$. In sec 6.2 we found

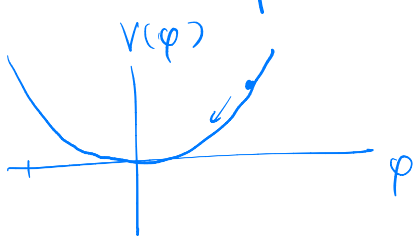
$$\rho = \frac{1}{2} \dot{\varphi}^2 + \frac{1}{2} (\nabla \varphi)^2 + V(\varphi)$$
$$P = \frac{1}{2} \dot{\varphi}^2 - \frac{1}{6} (\nabla \varphi)^2 - V(\varphi)$$

If $\varphi = \text{const}$, $\dot{\varphi} = 0$, $\nabla \varphi = 0$ then

$$\rho = V, \quad P = -V$$

like a cosmological constant. To end inflation, φ must be time dependent. Assume φ spatially constant

example



φ "rolls" down towards the minimum of V where $V = 0$.

If $|\dot{\varphi}|$ is small enough, we still have

$$P \simeq -\rho$$

This is called <u>slow roll inflation</u>

action for scalar field with gravity

$$S = \int d^4x \sqrt{-g} \left\{ \frac{1}{2} g^{\mu\nu} \partial_\mu \varphi \partial_\nu \varphi - V(\varphi) \right\}$$

$g = \det(g_{\mu\nu})$

Inserting the FRW metric and demanding $\delta S = 0$ gives (w/o calculation) for spatially constant φ

$$\ddot{\varphi} + 3H\dot{\varphi} + V'(\varphi) = 0$$

with $V'(\varphi) = \frac{dV}{d\varphi}$

This is the eq. of motion of a point particle in 1d moving in the potential V, subject to friction.

This <u>Hubble friction</u> gives rise to the decrease of energy density due to the Hubble expansion.

Slow-roll is satisfied when

(*) $$\boxed{|\ddot{\varphi}| \ll 3H|\dot{\varphi}|}$$

$$H^2 = \frac{8\pi}{3} G \left(\frac{1}{2}\dot{\varphi}^2 + V \right)$$

V will dominate of the kinetic energy when

(**) $$\boxed{\dot{\varphi}^2 \ll 2V}$$

If these cond's are satisfied, the eq. of motion simplifies to

$$\dot{\varphi} = -\frac{1}{3H} V'$$

$$H = \frac{1}{m_{pe}} \left(\frac{8\pi}{3} V \right)^{1/2} \qquad (G = m_{pe}^{-2})$$

What do these slow-roll cond's mean for the potential V?

(***) $$\dot{\varphi} = -\frac{1}{3} m_{pe} \left(\frac{8\pi}{3} V \right)^{-1/2} V' = -\frac{m_{pe}}{\sqrt{24\pi}} \frac{V'}{\sqrt{V}}$$

Then (**) becomes

$$m_{pe}^2 \frac{1}{24\pi V} (V')^2 \ll 2V \qquad \Leftrightarrow$$

(*) $$\boxed{\frac{m_{pe}^2}{48\pi} \left(\frac{V'}{V} \right)^2 \ll 1}$$

What does (*) require?

$\frac{d}{dt}$ (***) $\Rightarrow$

$$\ddot{\varphi} = -\frac{m_{pe}}{\sqrt{24\pi}} \dot{\varphi} \left(\frac{V''}{V^{1/2}} - \frac{1}{2} \frac{(V')^2}{V^{3/2}} \right)$$

$$\ddot{\varphi} = -\frac{m_{pl}}{\sqrt{24\pi}} \left[\frac{V''}{V} - \frac{1}{2} \left(\frac{V'}{V} \right)^2 \right] \sqrt{V}\, \dot{\varphi}$$

$$H = \frac{1}{m_{pl}} \left(\frac{8\pi}{3} V \right)^{1/2} \Rightarrow \sqrt{V} = m_{pl} \left(\frac{8\pi}{3} \right)^{-1/2} H$$

$$\ddot{\varphi} = -\frac{m_{pl}^2}{8\pi} \left[\frac{V''}{V} - \frac{1}{2} \left(\frac{V'}{V} \right)^2 \right] H \dot{\varphi}$$

This must be $\ll 3H|\dot{\varphi}|$.

Assuming ($\star$) is satisfied we need

$$\boxed{\left| \frac{V''}{V} \right| \ll \frac{24\pi}{m_{pl}^2}}$$

Example: $V(\varphi) = \frac{1}{2} m_\varphi^2 \varphi^2$, $V' = m_\varphi^2 \varphi$, $V'' = m_\varphi^2$

slow roll requires $\frac{\varphi^2}{4} \gg \frac{m_{pl}^2}{48\pi}$

Inflation takes place when $|\varphi| > m_{pl}$

<u>large field inflation</u>