9 Inflation [Mukhanov 5.1]

9.1 Homogeneity, isotropy problem

The early Universe was extremely homogeneous & isotropic:

fluctuations δT of the CMB temperature: $\frac{\delta T}{T} < 10^{-4}$

size of the visible Universe = size of causally connected region

$$d_p(t) = a_0 \int_0^t \frac{dt'}{a(t')} \qquad \text{(see Sec 1.6)}$$

particle horizon = boundary of this region

Present value:

$$d_p(t_0) \sim t_0 \sim H_0^{-1} \sim 10^{28}\,\text{cm} \qquad (c=1)$$

The Universe was homogeneous and isotropic at least in a comoving volume of at least this size

At some initial time t_i this volume had size

$$l_i \sim t_0 \cdot \frac{a_i}{a_0}$$

Assuming that the expansion did not smooth out inhomogeneities, the Universe must have been homogeneous and isotropic at this time.

Size of causally connected region at t_i: $d_{p\,i} \sim t_i$

$$(*) \quad \frac{\ell_i}{d_{pi}} \sim \frac{t_0}{t_i} \frac{a_i}{a_0}$$

The earliest time at which we can expect the Friedmann eqs. to be valid is the Planck time $t_i \sim 10^{-43}$ s

Assuming that the Universe was then filled with radiation with temperature $T_{PE} \sim M_{PE} \sim 10^{32}$ K $\Rightarrow$

$$\frac{a_i}{a_0} \sim \frac{T_0}{T_i} \sim \sim 10^{-32}$$

$$t_0 \sim 10^{17} \text{ s}$$

$$\frac{\ell_i}{d_{pi}} \sim \frac{10^{17}}{10^{-43}} 10^{-32} \sim 10^{28}$$

That is the observable Universe consisted of about

$$(10^{28})^3 \sim 10^{84}$$

causally disconnected regions. Why should the energy distribution be the same in all these regions that had no causal contact?

This is called the <u>horizon problem</u>.

Assuming that a goes with a power of t, one can estimate

$\frac{a}{t} \sim \dot{a}$. Then $(*)$ give

$$\frac{\ell_i}{d_{pi}} \sim \frac{\dot{a}_i}{\dot{a}_0}$$

If $\dot{a}$ decreases with time, then the homogeity scale was always larger than the causality scale

Flatness problem

$$H^2 + \frac{k}{a^2} = \frac{8\pi}{3} G \rho$$

$$\Omega = \frac{\rho}{\rho_c} \qquad \rho_c = \frac{3}{8\pi G} H^2$$

$$\rho - \rho_c = \frac{3}{8\pi G} \frac{k}{a^2}$$

$$\Omega - 1 = \frac{\rho - \rho_c}{\rho_c} = \frac{k}{a^2 H^2} = (\Omega_0 - 1)\left(\frac{a_0 H_0}{a \ H}\right)^2 = (\Omega_0 - 1)\left(\frac{\dot{a}_0}{\dot{a}}\right)^2$$

so for $t_i \sim t_{pl}$

$$\Omega_i - 1 = (\Omega_0 - 1)\, 10^{-56}$$

Ω_0 is very close to 1. So the initial cond. for Ω must be extremely fine-tuned.

Since $\Omega = 1$ corresponds to a flat Universe ($k = 0$)

this called flatness problem.

9.2 Inflation: basic idea [Gorbunov, Inflation 11.2]

We could trace the above problems back to the fact that $\dot{a}$ decreases. Thus they might be solved if there was an early period of accelerated expansion. Then $\frac{\dot{a}_i}{\dot{a}_0}$ may be less than 1.

We already know an example of accelerated expansion: A universe dominated by a cosmological constant.

Friedmann eqs $\quad H^2 + \frac{k}{a^2} = \frac{8\pi}{3} G\rho \quad , \quad \dot{\rho} = -3H(\rho + P)$

$$2H\dot{H} - 2\frac{k}{a^2}H = \frac{8\pi}{3} G\,(-3)\,H(\rho + P)$$

$$\dot{H} - \frac{k}{a^2} = -4\pi G(\rho + P)$$

$$\dot{H} = \frac{d}{dt}\frac{\dot{a}}{a} = \frac{\ddot{a}}{a} - \frac{\dot{a}^2}{a^2} = \frac{\ddot{a}}{a} - H^2 = \frac{\ddot{a}}{a} - \left(-\frac{k}{a^2} + \frac{8\pi}{3}G\rho\right)$$

$$\frac{\ddot{a}}{a} - \frac{8\pi}{3}G\rho = -\frac{4\pi}{3}G(3\rho + 3P)$$

$$\boxed{\frac{\ddot{a}}{a} = -\frac{4\pi}{3}G(\rho + 3P)}$$

Acceleration requires $\rho + 3P < 0 \iff P < -\frac{1}{3}\rho$

example: cosmological constant $P = -\rho = -\frac{\Lambda}{8\pi G}$

acceleration for $\Lambda > 0$, $H = \text{const}$