8 Dark matter [Mukanov 4.6.1, Gorbunov, Rubakov 9.1]

Figs: Hubble diagrams from lec. 7

Fig: DESI 2024 VI fig. 4

reminder: $\Omega_i = \rho_i / \rho_{crit}$, $H^2 = \frac{8\pi}{3} G \rho_{crit}$

Hubble diagram + CMB fluctuations + baryon acoustic oscillations

$\rightarrow \quad \Omega_M \sim 0.25 - 0.3$

the contribution from baryonic matter:

$\Omega_B \approx \rho_B / \rho_{crit} \approx 5\%$

matter is mostly non-baryonic

a lot more evidence for dark matter from astronomy

However, we have no idea what dark matter (DM) is.

What we know is that it is electrically neutral (not visible) and stable on cosmological time scale.

Let us assume that DM was interacting with ordinary matter at some time in the early Universe.

At some point this interaction must have stopped, and since then the DM particles propagate freely.

Then one can imagine different types of scenarios.

If the DM particles decouple at $T \ll m$, they are non-relativistic at decoupling, called <u>Cold DM</u>.

If they decouple at $T \gtrsim m$ there are several possibilities.

When they are relativistic during structure formation they are called <u>hot DM</u>.

Examples: neutrinos, $T_d \sim 1 MeV$

When they become non-relativistic during structure formation They are called <u>warm DM</u>

<u>hot relics ψ</u>

after decoupling they propagate freely and $\frac{n_\psi}{s}$ is constant.

$$\Omega_\psi = m_\psi n_\psi / \rho_c = m_\psi s \frac{n_\psi}{s} \rho_c^{-1}$$

in equil: $n_\psi = \frac{3}{4} g_\psi \frac{\zeta(3)}{\pi^2} T^3$ for fermions

$$s = \frac{2\pi^2}{45} g_* T^3$$

here we must use the entropy that ends up in the photon entropy.

$$\frac{n_\psi}{s} = \left(\frac{n_\psi}{s}\right)_d = \frac{3}{4} g_\psi \frac{\zeta(3)}{\pi^2} \frac{45}{2\pi^2} g_{*d}^{-1} = \frac{135}{8\pi^4} \zeta(3) \frac{g_\psi}{g_{*d}}$$

at decoupling

astrophysical constants:

$n_{0\gamma} = 2891 \text{ cm}^{-3}$

$\rho_{c0} = 10^{-5} h^2 \text{ GeV cm}^{3} = 1 \cdot 10^{4} h^2 \text{ eV cm}^{-3}$ $\Rightarrow$

where $H_0 = h \cdot 100 \text{ km s}^{-1} \text{ Mpc}^{-1}$

$$\Omega_{\psi 0} = m_\psi \, 2891 \, \frac{135}{8\pi^4} \zeta(3) \frac{g_\psi}{g_*} 10^{-4} h^{-2} \text{eV}^{-1}$$

$$= \frac{g_\psi}{g_*} h^{-2} \frac{m_\psi}{17 \text{eV}}$$

in terms of $h_{75} = \frac{H_0 \text{ Mpc}}{75 \text{ km s}^{-1}}$: $h = \frac{75}{100} h_{75}$

$$\Omega_{\psi 0} = \frac{g_\psi}{g_*} h_{75}^{-2} \frac{m_\psi}{10 \text{eV}}$$

Example: 3 neutrino flavors with equal mass

$$g_\psi = \underset{\text{spin}}{2} \cdot \underset{\text{flavor}}{3} = 6$$

at ν decoupling: $g_* = 2 + \frac{7}{8} 4 = 5{,}5$

$$\Omega_{\psi 0} \approx h_{75}^{-2} \frac{m_\psi}{9 \text{eV}}$$

For $m_\nu \sim 9 \text{eV}$:

$\Omega_\nu \simeq 1$ "neutrinos would close the Universe"

$\Omega_m \lesssim 0{,}3 \Rightarrow m_\nu \lesssim 3 \text{eV}$

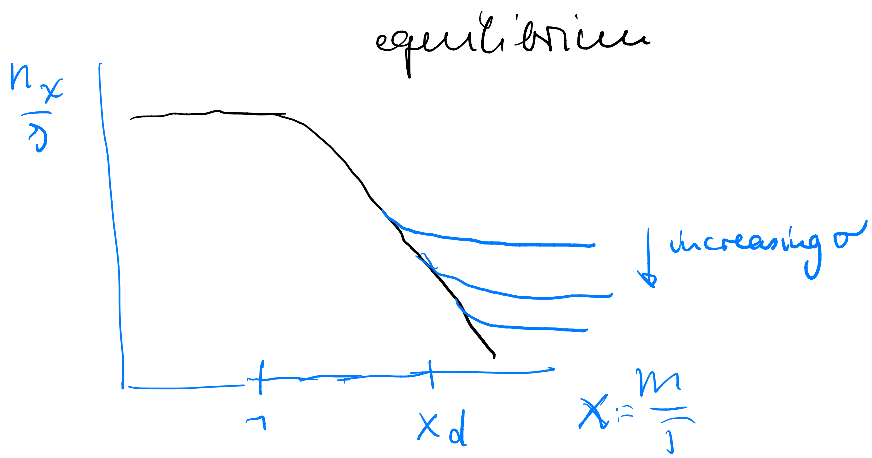
However, the bounds are even stronger, because hot DM cannot be all of the DM: If hot DM dominates, then inhomogeneities are washed out by free streaming, and the larger scale structure of the Universe cannot be explained.

Thermal cold relics

Assume particles χ that can be created or annihilated only in pairs like in

$$\chi\chi \leftrightarrow \text{SM particles}$$

Then χ will stable, and it will decouple when n_χ becomes very small. This will happen at some $T \ll m_\chi$.



number density at decoupling

$$n_{\chi d} \langle v\sigma \rangle_d = H_d = \left[\frac{8\pi}{3} G \rho_d\right]^{1/2} = \left[\frac{8\pi}{3} G \frac{\pi^2}{30} \tilde{g}_* T^4\right]^{1/2}$$

all relativistic degrees of freedom contributing to ρ

$$= \left(\frac{8\pi^3}{90}\right)^{1/2} \tilde{g}_*^{1/2} T_d^2 / m_{pe}$$

use $\langle v\sigma \rangle_d \sim \sigma_d \sqrt{T_d/m_\chi}$

$$(*) \quad n_{\chi d} = \left(\frac{8\pi^3}{90}\right)^{1/2} \tilde{g}_{*d}^{1/2} T_d^{3/2} m_\chi^{1/2} \sigma_d^{-1} m_{pe}^{-1}$$

equilibrium number density:

$$n_\chi = g_\chi \left(\frac{x}{2\pi}\right)^{3/2} e^{-x} T^3$$

$$\ln\left(n_{\chi d} T_d^{-3} g_\chi^{-1} x_d^{-3/2} (2\pi)^{3/2}\right) = -x_d$$

plug in $(*)$ $\Rightarrow$

$$x_d = \ln\left(0{,}038\, g_\chi \tilde{g}_{*d}^{-1/2} \sigma_d m_{pe} m_\chi x_d^{-1/2}\right)$$

$$\simeq \ln\left(0{,}038\, g_\chi \tilde{g}_{*d}^{-1/2} \sigma_d m_{pe} m_\chi\right)$$

This can be written as

$$x_d = 16{,}3 + \ln\left(g_\chi \tilde{g}_*^{-1/2} \frac{\sigma_d}{10^{-38}\,\mathrm{cm}^2} \frac{m_\chi}{\mathrm{GeV}}\right)$$

$10^{-38}\,\mathrm{cm}^3$ is a typical weak-interaction cross section

For the relics to be cold one needs $x_d \gg 1$,

Energy density today:

$$\rho_{\chi 0} = m_\chi n_{\chi 0} = m_\chi n_{\chi d} \frac{s_{0\gamma}}{s_d}$$

s_d : entropy density of the species that later transfer their entropy into photons, g_{*d} = their # of eff. degrees of freedom

$$\rho_{\chi 0} \simeq m_\chi n_{\chi d} \frac{2}{g_{*d}} \left(\frac{T_{0\gamma}}{T_d} \right)^3$$

$$\rho_{\chi 0} = m_\chi \left(\frac{8\pi^3}{90}\right)^{1/2} \tilde{g}_*^{1/2} T_d^{3/2} m_\chi^{1/2} \sigma_d^{-1} m_{pe}^{-1} \frac{2}{g_{*d}} \left(\frac{T_{0\gamma}}{T_d} \right)^3$$

$$= 3.32 \frac{\sqrt{\tilde{g}_*}}{g_{*d}} \underbrace{m_\chi^{3/2} T_d^{-3/2}}_{= x_d^{3/2}} \sigma_d^{-1} m_{pe}^{-1} T_{0\gamma}^3$$

This gives

$$\Omega_{\chi 0} = \frac{\rho_{\chi 0}}{\rho_{c0}} = h_{75}^{-2} \frac{\sqrt{\tilde{g}_*}}{g_{*d}} x_d^{3/2} \frac{3 \cdot 10^{-38} \mathrm{cm}^2}{\sigma_d}$$

It depends only logarithmically on m_χ through x_d.

Thus particles with $\ln \frac{m_\chi}{GeV} \sim 1$ and electroweak interaction strength naturally give $\Omega_{\chi 0} \sim 1$ and are ideal DM candidates. This is the so-called WIMP (weakly interacting massive particle) miracle.

Other popular candidates are non-thermal relics like axions or axion-like particles (ALPs). These are light pseudo-scalar particles that are produced with zero momentum so that they act similarly to CDM