6.2 Phase transitions

The fundamental building blocks of the SM are fields.
They determine energy density ρ, pressure P, ...

So far we only considered the contribution of particles.
There are also contributions from fields not associated with particles, e.g., from a constant Higgs field.

$$\rho = T^{00}, \qquad P = \frac{1}{3} T^{mn}$$

$T^{\mu\nu}$: energy-momentum tensor

Energy momentum tensor due to some scalar field χ
(without calculation, ignoring gravity):

$$T_{\mu\nu} = \partial_\mu \chi \, \partial_\nu \chi - \eta_{\mu\nu} \mathcal{L}$$

$$\partial_\mu \chi \equiv \frac{\partial \chi}{\partial x^\mu}$$

$$(\eta_{\mu\nu}) = (\eta^{\mu\nu}) = \begin{pmatrix} 1 & & & 0 \\ & -1 & & \\ & & -1 & \\ 0 & & & -1 \end{pmatrix}$$ Minkowski metric

$$\mathcal{L} = \frac{1}{2} \eta^{\mu\nu} \partial_\mu \chi \, \partial_\nu \chi - V(\chi) = \frac{1}{2}\left(\dot{\chi}^2 - (\nabla \chi)^2\right) - V(\chi)$$

$V(\chi)$: potential

This gives

$$\rho = T^{00} = \dot{\chi}^2 - \mathcal{L} = \tfrac{1}{2}\dot{\chi}^2 + \tfrac{1}{2}(\nabla\chi)^2 + V(\varphi)$$

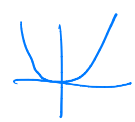
$$T^{mn} = \partial_m\chi\,\partial_n\chi + \delta_{mn}\left[\tfrac{1}{2}\dot{\chi}^2 - \tfrac{1}{2}(\nabla\chi)^2 - V\right]$$

$$P = \tfrac{1}{3}T^{mm} = \tfrac{1}{3}\left\{(\nabla\chi)^2 + 3\left[\tfrac{1}{2}\dot{\chi}^2 - \tfrac{1}{2}(\nabla\chi)^2 - V\right]\right\}$$

$$= \tfrac{1}{2}\dot{\chi}^2 - \tfrac{1}{6}(\nabla\chi)^2 - V$$

For constant χ : $\rho = V$, $P = -V$

example: $V(\chi) = \frac{m^2}{2}\chi^2$



for $m^2 > 0$ the minimum of the potential

= the maximum of the pressure

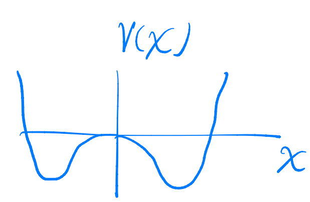
is at $\chi = 0$

This is the state of smallest ρ and largest P, the stable ground state of the system.

Now consider

$$(*) \qquad V(\chi) = -\frac{\mu^2}{2}\chi^2 + \frac{\lambda}{4}\chi^4$$

with $\mu^2, \lambda > 0$



The ground state is at $\chi = \pm v$ where

$$V'(v) = -\mu^2 v + \lambda v^3 = 0 \,, \qquad v = \frac{\mu}{\sqrt{\lambda}}$$

The potential for the Higgs field $\varphi = \begin{pmatrix} \varphi_1 \\ \varphi_2 \end{pmatrix}$
with complex φ_1, φ_2:

$$V(\varphi) = -\mu^2 \varphi^\dagger \varphi + \lambda (\varphi^\dagger \varphi)^2 \quad \text{with } \varphi^\dagger = (\varphi_1^*, \varphi_2^*)$$

define $\quad \chi := \sqrt{2 \varphi^\dagger \varphi}$

Then $V(\varphi)$ looks just like $(*)$.

At high temperature we must add minus the pressure of the particles to V, which defines the <u>effective potential</u>

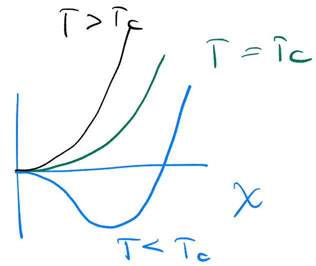
$$V_{eff}(\chi) = V(\chi) - P_{particles}(\chi)$$

We had $\quad P_{particles} = c_0 T^4 - \frac{c_2}{2} T^2 \chi^2 + \cdots$

The first term can be ignored when looking for the minimum of V_{eff}. This gives

$$V_{eff}(\chi) = \frac{1}{2}(c_2 T^2 - \mu^2)\chi^2 + \frac{\lambda}{4}\chi^4$$

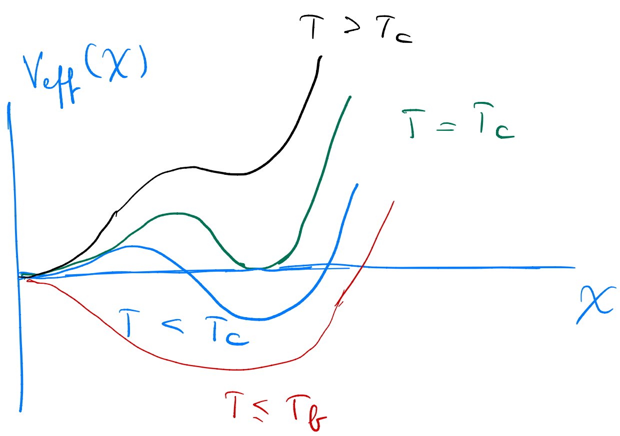
At $T = T_c := \mu/\sqrt{c_2}$ the quadratic term changes sign.



This looks like a second order phase transition.

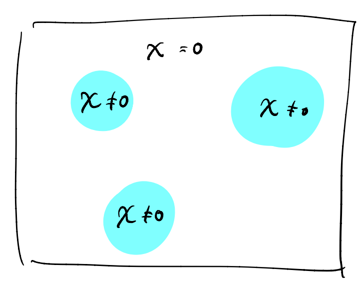
Including higher order in the expansion in m^2 one obtains a cubic term in V_{eff}:

$$V_{eff} = \frac{1}{2}(c_2 T^2 - \mu^2)x^2 - \frac{B}{3}x^3 + \frac{\lambda}{4}x^4$$



This suggests a first order phase transition, which would give a deviation from equilibrium:

- For $T_c \geq T > T_B$: supercooling
  nucleation of droplets with $x \neq 0$



Droplets expand and collide until the whole Universe is in the phase with $x \neq 0$.

However, it turns out that perturbation theory is not reliable and that there is no phase transition (PT) in the SM. Simple extensions of the SM have a first-order PT. This could have produced gravitational waves which might be detectable with LISA.

6.3 Baryon number violation

The Lagrangian of the SM is invariant under the $U(1)$ transformations

$$q_i \to e^{i\varepsilon_B} q_i \qquad \text{for quarks}$$

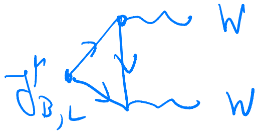
$$l_i \to e^{i\varepsilon_L} l_i \qquad \text{for leptons}$$

Naively this would imply baryon (B) and lepton number (L) conservation, described by the continuity equations

$$\partial_\mu J^\mu_B = \partial_\mu J^\mu_L = 0$$

[In components: $\partial_\mu J^\mu = \underbrace{\frac{\partial}{\partial t} J^0}_{\text{charge density}} + \nabla \cdot \underbrace{\vec{J}}_{\text{current}}$]

Quantum effects violate these symmetries



$\Rightarrow$

$$\partial_\mu J^\mu_B = \partial_\mu J^\mu_L = \frac{n_f}{16\pi^2} g^2 \vec{E}^a \cdot \vec{B}^a$$

$n_f = 3$ number of fermion families

g: weak $SU(2)$ gauge coupling

$\vec{E}^a, \vec{B}^a$: weak-$SU(2)$ electric and magnetic field

This violates B- and L conservation but still conserves $B-L$.

integrate:

$$\int_0^t dt' \int d^3x' \, \partial_\mu J^\mu_B = \int_0^t dt' \frac{\partial}{\partial t'} \underbrace{\int d^3x' \, J^0_B}_{= B(t')} \Rightarrow$$

$$B(t) - B(0) = n_f Q(t)$$

with

$$Q(t) = \int_0^t dt' \int d^3x' \frac{g^2}{16\pi^2} \vec{E}^a \cdot \vec{B}^a$$

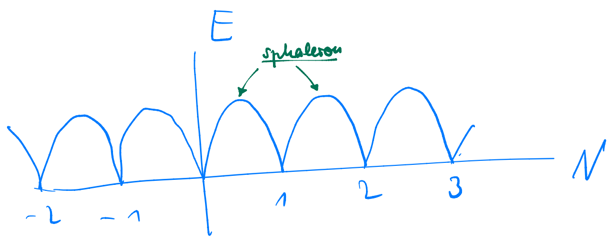
One can show that the integrand is also a total derivative $\partial_\mu K^\mu \Rightarrow$

$$Q(t) = N(t) - N(0)$$

with

$$N = \int d^3x \, K^0$$

In the vacuum N is an integer. The SM has "many vacua", separated by an energy barrier



A transition between two neighboring vacua changes B and L by n_f.

At $T=0$ the transition can only proceed through quantum tunneling with a tiny rate.
At high T one can hop over the barrier, the so called Sphaleron.

Sphaleron energy $E_{sph} \propto \frac{|\langle\varphi\rangle|}{g}$

Since $\varphi \to 0$ for $T > T_c$, the barrier vanishes above T_c and $B+L$ is rapidly violated.