5 Recombination [Mukhanov 3.6]

After BBN the Universe mainly contains γ, e, p and ^4He.

The abundance of other nuclei is very small can we neglect them.

When the temperature drops, the p and ^4He can capture electrons and eventually form neutral atoms.

(For historical reasons this process is called recombination, even though nuclei and electrons had not been combined before.)

Then there are no more free electric charges, and the Universe becomes transparent to photons, which then stream freely and are now observed as the CMB.

Since He has a much larger binding energy than hydrogen, He starts forming earlier.

Like BBN, recombination is a non-equilibrium process, which must be treated with Boltzmann equation.

However, one can get a first idea when it happens by assuming thermal equilibrium.

5.1 Equilibrium treatment

helium recombination

This happens in two steps. First the He nuclei capture one electron to form He^+ ions.

binding energy B for $H = -\frac{\nabla^2}{2m_e} - \frac{Z\alpha}{r}$:

$$B \sim \frac{1}{m_e}\frac{1}{r^2} \sim \frac{Z\alpha}{r} \quad \Rightarrow r^{-1} \sim Z\alpha m_e \,, \quad B \sim Z^2\alpha^2 m_e$$

Here we have $Z = 2$, so binding energy of He^+ is

$$B_+ = 4\, B_H = 4 \times 13.6\,eV = 54.4\,eV$$

Estimate the temperature at which most He-nuclei have been converted to He^+ ions.

Assume that the reaction

$$He^{++}\ e \leftrightarrow He^+\ \gamma$$

is fast enough to maintain chemical equilibrium.

Conserved are the numbers of He nuclei and of electrons, so we have chemical potentials μ_{He} and μ_e.

$$\mu_{++} = \mu_{He} \,, \quad \mu_+ = \mu_{He} + \mu_e$$

We can now use the same calculation that we did for $p\ n \leftrightarrow D\ \gamma$

$$n = g\left(\frac{mT}{2\pi}\right)^{3/2} \exp([\mu - m]/T)$$

$$n_+ = g_+\left(\frac{m_{He}T}{2\pi}\right)^{3/2} \exp([\mu_+ - m_{He}]/T)$$

(*) $$= \frac{g_+}{g_{++}g_e}\left(\frac{m_eT}{2\pi}\right)^{-3/2} n_{++}n_e \exp(B_+/T)$$

This type of eq. is called <u>Saha equation</u>

$$\frac{n_{++}}{n_+} = \frac{g_{++}g_e}{g_+}\left(\frac{m_eT}{2\pi}\right)^{3/2} n_e^{-1} \exp\left(-\frac{B_+}{T}\right)$$

$$\frac{g_+}{g_{++}g_e} = 1$$

$$\left(\frac{m_eT}{2\pi}\right)^{3/2} = \left(\frac{1}{2\pi}\frac{m_e}{B_+}\frac{B_+}{T}T^2\right)^{3/2} = 5.8\times 10^4 \left(\frac{B_+}{T}\right)^{3/2} T^3$$

n_e: density of free electrons

about 12% of electrons is captured $\Rightarrow$

$$n_e \simeq 0.88\, n_B = 0.88\, \eta_{10}\, 10^{-10} n_\gamma = 8.8\times 10^{-11}\eta_{10}\frac{2\zeta(3)}{\pi^2}T^3$$

$$\simeq 2\times 10^{-11}\eta_{10}\, T^3$$

$$\left(\frac{m_eT}{2\pi}\right)^{3/2} n_e^{-1} = 2.9\times 10^{15}\,\eta_{10}^{-1}\left(\frac{B_+}{T}\right)^{3/2}$$

Insert this into (*) $\Rightarrow$

$$\frac{n_{++}}{n_+} = \exp\left(35.6 + \frac{3}{2}\ln\frac{B_+}{T} - \frac{B_+}{T} - \ln\eta_{10}\right)$$

$n_{++} \sim n_+$ when the exponent vanishes

Logarithmic approximation: assume that the log is small compared to the rest of the exponent

$$\frac{B_+}{T_+} \simeq 35.6 - \ln \eta_{10} + \frac{3}{2} \ln (35.6 - \ln \eta_{10})$$

$$T_+ \simeq \frac{B_+}{41 - \ln \eta_{10}} \simeq 13.4 \left(1 + 2.4 \times 10^{-2} \ln \eta_{10}\right) eV$$

$1 eV = 1.16 \times 10^4 K$, $13.4 eV = 1.58 \times 10^5 eV$

At T_+ about half of the He nuclei has captured an electron while the rest is still completely ionized.

Expand around T_+ : $T = T_+ - \Delta T$

$$\frac{n_{++}}{n_+} \simeq \exp\left(-\frac{B_+}{T_+}\frac{\Delta T}{T_+}\right) \sim \exp\left(-41 \frac{\Delta T}{T_+}\right)$$

exponentially decreasing with increasing ΔT

At a lower temperature (see exercise) neutral helium forms

After that starts the

hydrogen recombination

Saha eq:

$$\frac{n_p n_e}{n_H} = \left(\frac{T m_e}{2\pi}\right)^{3/2} \exp\left(-\frac{B_H}{T}\right)$$

$B_H = 13.6 eV$, n_p : number density of free protons

charge neutrality : $n_e = n_p$

hydrogen ionization fraction

$$\frac{n_p}{n_p + n_H} =: X_p ,$$

$$n_p + n_H \simeq 0.75\, n_B = 0.75 \times 10^{-10} \eta_{10} \frac{2\zeta(3)}{\pi^2} T^3 = 1.8 \times 10^{-11} \eta_{10} T^3$$

$$n_p = X_p (n_p + n_H) , \quad n_H = (1 - X_p)(n_p + n_H)$$

$$\left(\frac{m_e T}{2\pi}\right)^{3/2} = \left(\frac{1}{2\pi} \frac{m_e}{B_H} \frac{B_H}{T} T^2\right)^{3/2} = 4.6 \times 10^5 \left(\frac{B_H}{T}\right)^{3/2} T^3$$

$$\frac{X_p^2}{1 - X_p} = \exp\left(37.8 + \frac{3}{2} \ln \frac{B_H}{T} - \frac{B_H}{T} - \ln \eta_{10}\right)$$

The exponent vanishes for $T = T_{rec}$ with

$$\frac{B_H}{T_{rec}} = 37.8 - \ln \eta_{10} + \frac{3}{2} \ln (37.8 - \ln \eta_{10}) \simeq 43.2 - \ln \eta_{10}$$

$$T_{rec} = 0.31 \times (1 + 2.3 \times 10^{-2} \ln \eta_{10})\ eV , \qquad 0.31\ eV = 3.6 \times 10^3 K$$

Then $X_p^2 \simeq 1 - X_p$, $X_p \simeq -\frac{1}{2} + \sqrt{\frac{1}{4} + 1} = 0.62$

again, X_p rapidly decreases below T_{rec}.

In reality, chemical equilibrium breaks down when X_p becomes too small.

the time of recombination

At T_{rec} we already have matter domination

$$\rho_m = \rho_{m0} (a_0/a)^3 = \Omega_m \rho_{0crit} (T/T_0)^3$$

$$H_0^2 = \frac{8\pi}{3} \frac{\rho_{0crit}}{m_{pl}^2} \qquad \rho_{0crit} = \frac{3}{8\pi} m_{pl}^2 H_0^2$$

$$H^2 = \Omega_m H_0^2 (T/T_0)^3$$

$$H^{-1} = H_0^{-1} \Omega_m^{-1/2} (T/T_0)^{-3/2}$$

$$= h^{-1} \, 9.8 \cdot 10^9 \pi 10^7 s \, (0.27)^{-1/2} \left(\frac{3.6 \times 10^3}{2.7}\right)^{-3/2}$$

$$= 1.7 \times 10^{13} s \sim \frac{1}{2} My$$

Boltzmann eq: $\partial_t f_\gamma = f_\gamma n_e \sigma_T \Rightarrow$

mean free time of a photon

$$\tau_\gamma = \frac{1}{n_e \sigma_T}$$

At T_{rec}: $n_e = 0.67 \times 1.8 \times 10^{-11} \eta_{10} \, (0.31\,eV)^3$

$$\simeq 2.1 \times 10^{-12} (eV)^3$$

$$\sigma_T = 1.7 \times 10^{-15} (eV)^{-2}$$

$$\tau_\gamma = 2.7 \; 10^{26} \, eV^{-1}$$

$$\hbar = 6.7 \times 10^{-22} \, MeV\,s = 6.7 \times 10^{-16} \, eV\,s$$

$$\tau_\gamma = 1.8 \times 10^{11} \, s \ll H_{rec}^{-1}$$

So then the photons are still in kin. equilibrium with electrons.

Therefore photons decouple later than recombination.

Since n_e is then no longer in equilibrium, one needs to solve Boltzmann equations to determine the decoupling time.

result: $T_{dec} \simeq 0.26\,eV$ [Gorbunov Rubakov 6.2]