4.4 Deuterium Bottleneck

How to make ^4He? Binding energy $B_{^4He} = 28.2\,MeV$

$p\,p\,n\,n \longrightarrow {}^4He$ not efficient, baryon concentration too low.

Instead: sequence of 2-body reactions

(*) $$p\,n \leftrightarrow D\,\gamma$$

$p\,D \leftrightarrow {}^3He\,\gamma$, $D\,D \leftrightarrow {}^3He\,n \leftrightarrow {}^3H\,p$

$^3H\,D \leftrightarrow {}^4He\,n$

first one needs D.

interaction rate of (*) $\gg H$ for $t < 10^3\,s$

determined by nuclear cross section (see below)

Compute equilibrium value of n_D

non-relativistic particles:

$$n = g\left(\frac{mT}{2\pi}\right)^{3/2} \exp\left(\frac{\mu - m}{T}\right)$$

D has spin 1 $\Rightarrow g_D = 3$

$\mu_D = \mu_p + \mu_n$

$m_D = m_p + m_n - B_D$, B: binding energy of nucleus.

$B_D = 2.23$ MeV

$$n_D \simeq 3\left(\frac{2m_pT}{2\pi}\right)^{3/2} \exp\left(\left[\mu_p + \mu_n - m_p - m_n + B_D\right]/T\right)$$

$$= 6 \times 2^{3/2}\left(\frac{2\pi}{m_pT}\right)^{3/2} \frac{1}{4}\, n_p n_n\, e^{B_D/T}$$

$n_p \sim n_n \sim 10^{-10}\, T$, so at neutron freeze-out ($T \simeq 0.8$ MeV)

this is still extremely small.

The reason why it is so small even for $T < B_D$

is the huge number of photons.

Even though when most photons have

$E \sim T \ll B_D$, there can still be enough

of them in the high-energy tail that have

$E > B_D$ and can break up the deuteron.

Define

$$X_i := \frac{A_i n_i}{n_B}$$

where A_i = atomic mass number of nucleus i

= # baryons in i

X_i: mass fraction of nucleus i:

= fraction of baryon density n_B carried by nucleus i

After e^+e^- annihilation

$\eta_{10} := 10^{10} \frac{n_B}{n_\gamma}$ is constant (measured: $\eta_{10} = 6.04$)

$$n_\gamma = \frac{2\zeta(3)}{\pi^2} T^3$$

$$X_D = 12\, \pi^{3/2} \left(\frac{T}{m_p}\right)^{3/2} \eta_{10}\, 10^{-10} \frac{2\zeta(3)}{\pi^2} X_p X_n e^{B_D/T}$$

$$= 24 \times 10^{-10} \frac{1}{\sqrt{\pi}} \zeta(3)\, \eta_{10} \left(\frac{T}{m_p}\right)^{3/2} X_p X_n e^{B_D/T}$$

$\zeta(3) = 1.202 \qquad m_p = 931\ \mathrm{MeV}$

$$(*) \quad \boxed{X_D = 5.7 \times 10^{-14}\, \eta_{10} \left(\frac{T}{\mathrm{MeV}}\right)^{3/2} X_p X_n e^{B_D/T}}$$

Fig: $10^{10}\,\mathrm{K} \approx 0.86\ \mathrm{MeV}$

When does it become of $\mathcal{O}(1)$?

Use $X_n \simeq X_n^* = 0.158$, $X_p \simeq 0.842$, $X_p X_n \simeq 0.133$

T/keV	X_D
90	7.1×10^{-5}
80	1.3×10^{-3}
[65	0.60]

X_D abruptly becomes of order unity in a small temperature interval.

When X_D becomes large enough, the reactions

$$D\,D \to {}^3He\; n \quad , \qquad D\,D \to T\,p$$

become important and nucleosynthesis begins

This happens when $T \simeq T_N = 80\ \mathrm{keV}$

$$\frac{t}{s} = 1.39 \left(\frac{\pi^2}{30} g_*\right)^{-1/2} (T/\mathrm{MeV})^{-2}$$

Here e^+e^- no longer contribute to g_*,

$g_* = 3.36$ (exercise) $\quad \frac{\pi^2}{30} g_* = 1.1 \quad \Rightarrow$

$t_N = 207\,s \gtrsim 3\ \mathrm{min}$

Due to the large binding energy, mostly ^4He is produced.

estimate ^4He abundance

most neutrons end up in ^4He

$$n_{^4He}(T_N) \simeq \frac{1}{2} n_n(T_N)$$

$$X_{^4He} = \frac{4\, n_{^4He}}{n_B} = \frac{2\, n_n}{n_B} = 2 X_n$$

$$X_n(T_N) = e^{-t/\tau_n} X_n^* = e^{-207/886} \times 0.158 \Rightarrow$$

$$\boxed{X_{^4He} \simeq 0.25}$$

N.B. Increasing η_{10} at fixed T increases X_D

$\rightsquigarrow$ BBN starts earlier $\rightsquigarrow$ less neutrons

have decayed $\rightsquigarrow$ more ^4He

Thus $X_{^4He}$ increases with increasing η_{10}.

Significant uncertainty due to uncertainty of τ_n