4 Nucleosynthesis [Mukhanov 3.5]

Most abundant nuclei in the Universe:

almost 75% hydrogen & ~ 25% ^4He

Estimate the amount of hydrogen that could have been produced in stars:

Binding energy of ^4He: 28.4 MeV

energy per nucleon from fusion to ^4He: 7.1 MeV

Assume $\frac{1}{4}$ of H was fused into ^4He in the past 10 Gy.

$$1\,y = \pi \cdot 10^7\,sec$$

$$\text{radiated energy} = 7\,MeV \cdot \frac{1}{4} \cdot \#\,\text{baryons} = \frac{7}{4}\,MeV\,\frac{M_b}{m_p}$$

M_b: total mass of baryonic matter

Luminosity / mass ratio

$$\frac{L}{M_b} = \frac{7\,MeV}{4\,m_p}\,\frac{1}{10\,Gy} = \frac{7 \cdot 10^{-3}\,GeV}{4 \cdot 1\,GeV/c^2}\,\frac{1}{3 \cdot 10^{17}\,s} \simeq \frac{7}{4 \cdot 3}\,10^{-20}\,s^{-1}\,c^2$$

$$c = 3 \cdot 10^8\,m/s = 3 \cdot 10^{10}\,cm/s$$

$$\frac{L}{M_b} = \frac{21}{4}\,\frac{g\,cm^2/s^2}{g\,s} \simeq 5\,\frac{erg}{g\,s} = 2.5\,\frac{L_\odot}{M_{b\odot}}$$

observations : $\frac{L}{M_b} < 0.05 \frac{L_\odot}{M_{\odot}}$

$\Rightarrow$ Stars could not produce 25% of ^4He. $\Rightarrow$

it must come from the early Universe.

Helium can form when $T \lesssim$ binding energy $\sim 28\,MeV$

Amount of ^4He depends on neutron abundance.

Below $T \sim 1\,MeV$ neutrons decouple because Γ_W drops below H. Then scatterings no longer change n_n/n_p. One says that neutrons freeze out.

4.1 Boltzmann equation (BE)

The BE is a quantitative description of non-equilibrium processes. It is differential/integral equation for the phase space density $f(t, \vec{x}, \vec{p})$.

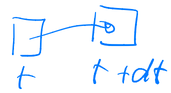
In general, there are two effects that change f:

(i) particle positions change with time

(ii) particle momenta change with time.

without scattering: particles follow classical trajectories in phase space

Liouville's theorem: f constant along such trajectories



$$f(t+dt, \vec{x} + \dot{\vec{x}}\,dt, \vec{p} + \dot{\vec{p}}\,dt) = f(t, \vec{x}, \vec{p}) \Rightarrow$$

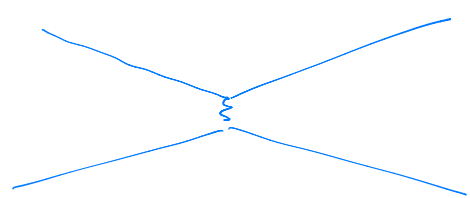
$$(*) \quad \partial_t f + \dot{\vec{x}} \cdot \frac{\partial f}{\partial \vec{x}} + \dot{\vec{p}} \cdot \frac{\partial f}{\partial \vec{p}} = 0$$

Now include scattering.

neglect $\dot{\vec{p}}$ for a moment.

$$\partial_t f + \vec{v} \cdot \frac{\partial f}{\partial \vec{x}} = C \qquad C: \text{ collision term}$$

We assume that interactions have short range, small compared over the typical length scale on which $f(t, \vec{x}, \vec{p})$ changes.



The interaction is approximated as taking place instantaneously at one point in space.

To compute C it is convenient to make use of Lorentz-covariance. f is Lorentz-invariant. One can formally show that, but we can simply check that in an expression that we used earlier:

energy-momentum tensor

$$T^{\mu\nu} = \int \frac{d^3p}{(2\pi)^3} \frac{p^\mu p^\nu}{E} f \qquad \text{with } E = \sqrt{\vec{p}^{\,2}+m^2}$$

$$\int \frac{d^3p}{2E} (\dots) = \underbrace{\int d^4p \, \delta(p^2-m^2)\, \Theta(p^0)\, (\dots)}_{\text{Lorentz-invariant}}$$

$p^\mu p^\nu$ = tensor $\Rightarrow$ f invariant <u>ok</u>

If we multiply (*) by $E = p^0$

$$p^\mu \partial_\mu f = p^0 \, C$$

then both sides are Lorentz-invariant. We may therefore evaluate it in the rest frame of p^μ where it reads

$$\partial_t f = C$$

To be specific, we evaluate C for 2 → 2 particle scattering

(*) $$1 + 2 \leftrightarrow 3 + 4$$

For simplicity assume that all particles are different.

For particle species 1 the process (*) gives rise to a <u>loss term</u> in C'. The inverse reaction gives rise to a <u>gain term</u>.

Consider the loss term. We need the <u>total cross section</u>

$$\sigma = \frac{\#\text{ scatterings off a single target 1 / time}}{\text{flux of particle species 2}}$$

flux = particle current density $\vec{j} = \int \frac{d^3p}{(2\pi)^3} \vec{v} f$, $\vec{v} = \frac{\vec{p}}{E}$

flux of 2-particles with momenta in the volume d^3p_2:

$$\vec{j}_2 = \frac{d^3p_2}{(2\pi)^3} \vec{v}_2 f_2$$

number of targets in the phase-space volume $d^3x_1 d^3p_1$ around $\vec{p}_1 = 0$ is

$$\#\text{ targets} = d^3x_1 \frac{d^3p_1}{(2\pi)^3} f_1$$

Then

$$\frac{\#\text{ scattering}}{\text{time}} = d^3x_1 \frac{d^3p_1}{(2\pi)^3} f_1 \frac{d^3p_2}{(2\pi)^3} |\vec{v}_2| f_2 \, \sigma$$

This should equal to the loss of the number of 1-particles inside $d^3x_1 d^3p_1$ per unit time

$$-\partial_t f_1 d^3x_1 \frac{d^3p_1}{(2\pi)^3}$$

due to scattering with 2-particles in d^3p_2 $\Rightarrow$

$$\partial_t f_1 = -f_1 \frac{d^3p_2}{(2\pi)^3} |\vec{v}_2| f_2 \sigma$$

Cross section [see particle/nuclear physics]

$$\sigma = \prod_{i=1}^{2}\left[\frac{1}{2p_i^0}\right]\frac{1}{v_{12}}\int\prod_{j=3}^{4}\left[\frac{d^3p_j}{(2\pi)^3 2p_j^0}\right]|\mathcal{M}|^2(2\pi)^4\delta^{(4)}(p_1+p_2-p_3-p_4)$$

v_{12} = relative velocity of particles 1 and 2.

In our frame $v_{12} = v_2$

Integrate over $\vec{p}_2$ $\Rightarrow$

$$C_{loss} = -\frac{1}{2p_1^0}\int\prod_{j=2}^{4}\left[\frac{d^3p_j}{(2\pi)^3 2p_j^0}\right] f_1 f_2 |\mathcal{M}|^2 (2\pi)^4 \delta(p_1+p_2-p_3-p_4)$$

The integral on the RHS is Lorentz invariant and can be evaluated in any frame.

This expression is valid when $f_3, f_4 \ll 1$, so that Pauli blocking or Bose enhancement can be neglected.

Otherwise they are additional factor $[1 \pm f_3][1 \pm f_4]$

upper sign: bosons

lower sign: fermions

Homogeneous & isotropic Universe: f is $\vec{x}$ - independent, only depends on t and $|\vec{p}|$.

With expansion $\vec{p}$ at $t \rightarrow \vec{p}\,'$ at t'

$$f(t, |\vec{p}|) = f(t', |\vec{p}\,'|)$$

$\vec{p}\, a(t) = \vec{p}\,' a(t')$ $\Rightarrow$ $f(t, |\vec{p}|) = f\left(t', \frac{a(t)}{a(t')} |\vec{p}|\right)$

$t' = t + dt$ $\qquad \frac{a(t)}{a(t')} = \frac{a(t)}{a(t) + \dot{a}(t) dt} = 1 - H\,dt$ $\Rightarrow$

$$\boxed{\partial_t f - H |\vec{p}| \frac{\partial f}{\partial |\vec{p}|} = C}$$