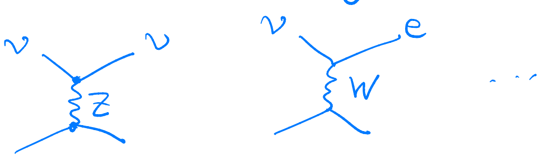
3 Neutrino decoupling & e^+e^- annihilation

So far we have assumed that neutrinos are in thermal equilibrium.

Neutrinos have only weak interactions



$m_Z = 90$ GeV , $m_W = 80$ GeV

For energies $\ll m_W$:

Scattering amplitude $\sim \dfrac{\alpha_W}{m_W^2}$ $\qquad \alpha_W \simeq \frac{1}{28}$

Scattering rate $\Gamma_W \sim \dfrac{\alpha_W^2}{m_W^4} T^5$

Hubble rate $H^2 = \dfrac{8\pi}{3 M_{pl}^2} \rho$, $M_{pl} = 1.2 \cdot 10^{19}$ GeV

$\rho = g_* \frac{\pi^2}{30} T^4$ $\qquad g_* = 10.8$

equilibrium requires $\dfrac{\Gamma_W}{H} \gg 1$

$$\frac{\Gamma_W}{H} = \left(\frac{8\pi^3}{90} g_*\right)^{-1/2} \left(\frac{1}{28}\right)^2 \frac{1.2 \times 10^{19}}{(80)^4} \left(\frac{T}{\text{GeV}}\right)^3$$

$$= 6.8 \cdot 10^{7} \cdot 10^{-9} \left(\frac{T}{\text{MeV}}\right)^3 = 6.8 \times 10^{-2} \left(\frac{T}{\text{MeV}}\right)^3$$

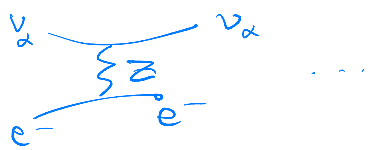
$\dfrac{\Gamma_W}{H} \sim 1$ for $T = T_{dec} \simeq 2.4$ MeV

A more detailed computation gives $T_{dec} \simeq 1.5$ MeV

T_{dec} : <u>decoupling temperature</u>

for $T \ll T_{dec}$ we have $\Gamma_W \ll H$, and neutrinos decouple from the $e^{\pm}\gamma$-plasma.

Since there are no $\mu^{\pm}$ or $\tau^{\pm}$ in the plasma, ν_μ and ν_τ only interact via Z-boson exchange



$$(\alpha = \mu, \tau)$$

$$m_Z = 90 \text{ GeV} > m_W$$

Therefore $\nu_\mu, \bar{\nu}_\mu, \nu_\tau, \bar{\nu}_\tau$ decouple a little earlier than $\nu_e, \bar{\nu}_e$

All neutrino species decouple <u>before</u> e^+ and e^- start to annihilate $(m_e = 511 \text{ MeV})$

After decoupling: free streaming, momenta are red-shifted : $p \to \frac{a_{dec}}{a} p$

phase-space density remains Fermi-Dirac

with $T = \frac{a_{dec}}{a} T_{dec}$

When e^+ & e^- annihilate they heat the photons. Thus the photon temperature T_γ becomes bigger than the neutrino temperature.

Quantitatively: s_ν and $s_e + s_\gamma$ are both conserved after ν - decoupling $\Rightarrow$

$$\frac{s_e + s_\gamma}{s_\nu} = \frac{s_\gamma}{s_\nu}\left(1 + \frac{s_e}{s_\gamma}\right) = \text{const}$$

$$s_\gamma \propto T_\gamma^3 \quad , \quad s_\nu \propto T_\nu^3 \quad \Rightarrow$$

$$\left(\frac{T_\gamma}{T_\nu}\right)^3 \left(1 + \frac{s_e}{s_\gamma}\right) = C \quad \text{another constant}$$

just before ν - decoupling:

$$T_\nu = T_\gamma \quad , \quad \frac{s_e}{s_\gamma} = \frac{\frac{7}{8} g_{*e}}{g_{*\gamma}} = \frac{7}{8}\,\frac{4}{2} = \frac{7}{4} \quad \Rightarrow$$

$$C = \frac{7}{4} + 1 = \frac{11}{4} \quad \Rightarrow$$

$$\frac{T_\gamma}{T_\nu} = \left(\frac{11}{4}\right)^{1/3} \left(1 + \frac{s_e}{s_\gamma}\right)^{-1/3}$$

After $e^+ e^-$ annihilation $s_e \ll s_\gamma$ and can be neglected.

Then

$$\frac{T_\gamma}{T_\nu} = \left(\frac{11}{4}\right)^{1/3} = 1.401$$

Today $\quad T_\nu = \dfrac{2.73\,K}{1.4} = 1.9\ K$

So far this cosmic neutrino background ($C\nu B$) has not been discovered directly. However, indirect evidence comes from its contribution to ρ and thus to H.